Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 7
November 27^{th}, 2002
WS 2002/2003

Exercises on *Theoretical Elementary Particle Physics*

Prof. H. Dreiner

In the lecture it was shown that when calculating $\langle|\mathcal{M}|^2\rangle$ it is useful to know[1]

$$\sum_{all\ \ spins}\Big(\bar{u}(a)\Gamma_1 u(b)\Big)\Big(\bar{u}(a)\Gamma_2 u(b)\Big)^* = \mathrm{Tr}\Big[\Gamma_1(\not{p}_b + m_b)\gamma_0{\Gamma_2}^\dagger\gamma_0(\not{p}_a + m_a)\Big].$$

There are no spinors on the RHS, so all one is left with is mere matrix multiplications and taking a trace (that's great!). Thus it is useful to know various **trace- and contraction theorems**. Thus show that

1. Tr$[AB]$=Tr$[BA]$
2. Tr$[M^{-1}AM]$ =Tr$[A]$
3. $-\gamma_\mu = \gamma_5\gamma_\mu(\gamma_5)^{-1}$, with $\gamma_5 = \frac{-i}{4!}\varepsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta$
4. $\mathrm{Tr}[\gamma^{\mu_1}...\gamma^{\mu_{2n+1}}] = 0$
5. $\mathrm{Tr}[\gamma^\mu\gamma^\nu] = 4\eta^{\mu\nu}$
6. $\not{a}\ \not{b} + \not{b}\ \not{a} = 2a\cdot b\ \ \mathbb{1}$
7. $\gamma^\mu\gamma_\mu = 4\ \ \mathbb{1}$
8. $\gamma^\mu\gamma^\lambda\gamma_\mu = -2\gamma^\lambda$

[1] Warning: In *Griffiths* this formula is called "Casimir's trick", but nobody else uses this name.

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1. $\mathrm{Tr}[\gamma^5\gamma^{\mu_1}...\gamma^{\mu_{2n+1}}] = 0$
2. $\mathrm{Tr}[\gamma^5] = 0$ (without going to a particular representation)
3. $\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu] = 0$ (without going to a particular representation)
4. (optional, for freaks) $\mathrm{Tr}[\gamma^5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta] = 4i\epsilon^{\mu\nu\alpha\beta}$
5. (optional, for freaks) $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma] = 4(\eta^{\mu\nu}\eta^{\lambda\sigma} - \eta^{\mu\lambda}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\lambda})$
6. $\gamma_\mu\gamma^\nu\gamma^\lambda\gamma^\mu = 4\eta^{\nu\lambda}$
7. $\gamma_\mu\gamma^\nu\gamma^\lambda\gamma^\sigma\gamma^\mu = -2\gamma^\sigma\gamma^\nu\gamma^\lambda$
8. $\gamma_\mu \not a \not b \not c \gamma^\mu = -2 \not c \not b \not a$
9. For $e^+e^- \longrightarrow e^+e^-$ calculate (using Feynman rules for spinors) at tree-level $\mathcal{M}$, $\langle|\mathcal{M}|^2\rangle$, $\left.\frac{d\sigma}{d\Omega}\right|_{CM}$, σ (for the last two you may consider only the ultrarelativistic case). Note that $e^+e^- \longrightarrow e^+e^-$ contains more diagrams than $e^+e^- \longrightarrow \mu^+\mu^-$