Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 8
11.12.02
WS 02/03

Exercises on 'Elementary Particle Physics'

Prof. H. Dreiner

1. *Spontaneous symmetry breakdown*

   This is supposed to be a basic introductory example to show what happens in a theory where we have a classical ground state which exhibits less symmetry than the Lagrangian.

   (a) Let $\phi = (\phi_1, \dots \phi_n)$ be an n component scalar field with Lagrangian density

   $$\mathscr{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - V(\phi) \quad , \quad V(\phi) = \frac{1}{8} g(\phi^2 - v^2)^2 \quad , g > 0 \, .$$

   Why is the symmetry group O(n)? Expand the potential and identify the standard ingredients to a Lagrangian density (kinetic term, mass term, interactions..) – what is different? Draw the potential in a suitable projection!

   (b) Calculate the equation which determines the classical ground state(s) (states with lowest energy, hence potential). This equation defines an $n-1$ dimensional sphere (S^{n-1}), which breaks the higher symmetry of the Lagrangian.

   (c) Pick one ground state $\phi_0 = (0, \dots, 0, v)$ and calculate the effective Lagrangian with low excitations around that ground state by expanding around it

   $$\phi = (\tilde{\phi}, v + f) \quad , \quad \tilde{\phi} = (\phi_1, \dots, \phi_{n-1}) \, .$$

   Give an interpretation for the different modes you find in the effective Lagrangian. What is the link to the Goldstone theorem.

   (d) What happens if one has the opposite sign in $V(\phi)$, i.e. $V' = \frac{1}{8} g(\phi^2 + v^2)^2$?

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1. Gauge invariance of non-abelian SU(2) gauge theory

   For general non-abelian SU(N) gauge theories we define the covariant derivative

   $$\mathcal{D}_\mu = \partial_\mu + ig\frac{T^a}{2}W^a_\mu \, ,$$

   where T^a are the generators of the gauge group (a runs from 1 to the number of generators), g is the gauge coupling and W^a_μ are the gauge fields (gauge connection). Under a local gauge transformation with gauge parameter $\alpha(x)$ the gauge fields transform as

   $$W^a_\mu \to W^a_\mu - \tfrac{1}{g}\partial_\mu\alpha(x)^a - f^a_{\ bc}\alpha(x)^b W^c_\mu \, ,$$

   with $f^a_{\ bc}$ the structure constants, and the fields as

   $$\phi \to e^{i\alpha(x)^a T^a/2}\phi \approx (1 + i\alpha^a\frac{T^a}{2})\phi \, ,$$

   with the last equation valid for infinitesimal gauge transformations only.

   The field strengths of non-abelian gauge fields are defined by

   $$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^a_{\ bc} W^b_\mu W^c_\nu \, .$$

   i. How many generators are there for SU(N) and i.e. for SU(2)? Write down a set of generators for SU(2). What are the structure constants of SU(2)? (No derivations needed...)

   ii. Again for the SU(2) case, show that under a gauge transformation $h : \phi \to h\phi$ it follows that $\mathcal{D}_\mu\phi \to h(\mathcal{D}_\mu\phi)$. You can restrict to infinitesimal gauge transformations if you like.

   iii. Use the result of ii. to show that the kinetic term $(\mathcal{D}^\mu\phi)^\dagger(\mathcal{D}_\mu\phi)$ is gauge invariant.

   iv. How does the SU(2) field strenghts transform under infinitesimal gauge transformations?

   v. Use the result of iv. to show that the kinetic term $W^a_{\mu\nu}W^{a\,\mu\nu}$ is gauge invariant.