Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 11
January 15^{th}, 2003
WS 2002/2003

Exercises on *Theoretical Elementary Particle Physics*

Prof. H. Dreiner

1. **Introduction to the homework: Meson decay – How to deal with the fact that the quarks are bound together?** Draw the Feynman diagram for "quark-antiquark scattering" giving a lepton ℓ and a massless antineutrino $\overline{\nu_\ell}$. What is the corresponding $\mathcal{M}$? Draw a second diagram, wrapping our ignorance about QCD, and make yourself clear that in this case one has

$$\mathcal{M} = \frac{{g_W}^2}{8\ {m_W}^2} \left[\overline{u_\ell}\ \gamma_\mu\ (1-\gamma^5)\ v_{\nu_\ell}\ \right]\ f_{meson}\ {p_{meson}}^\mu . \tag{1}$$

2. **The GIM Mechanism** Using the technique of Feynman diagrams, explain why the decay of K^0 in to $\mu^+\mu^-$ is extremely slow.

3. **The KM-Matrix with N Generations** How does the KM matrix arise in the Lagrangian? How many phases can be absorbed by field-refinitions? How many generations do we need to have at least one phase?

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1. **Meson Decay** Name all six mesons made of $\overline{u^j}$ and d^i, i, j being generational indices (e.g. $\overline{u^1}d^1$ gives a π^-, $\overline{u^1}d^2 \equiv \overline{u^1}s$ gives a K^-); why are there only six and not nine mesons? Look up the masses of the six mesons and thus verify that all but K^- and π^- can decay into a τ.

Consider $\pi^-{}_{1i} \equiv \pi_i$ $(i = 1,2)$ decaying into ℓ_l and $\overline{\nu_{\ell_l}}$ $(l = 1,2$, also a generational index) and calculate the decay rate $\Gamma_{\pi_i \to \ell_l \overline{\nu_l}} = \frac{|\vec{p_{\overline{\nu_{\ell_l}}}}|}{8\pi\ m_{\pi_i}{}^2} \langle |\mathcal{M}|^2 \rangle$. To do so, use the $\mathcal{M}$ of the previous page and show that

$$\langle |\mathcal{M}|^2 \rangle = \left[\frac{f_{\pi_i}}{2\sqrt{2}} \left(\frac{g_W}{m_W} \right)^2 \right]^2 \ \left[2(p_{\pi_i} p_{\overline{\nu_{\ell_l}}})(p_{\pi_i} p_{\ell_l}) - m_{\pi_i}{}^2 (p_{\ell_l} p_{\overline{\nu_{\ell_l}}}) \right]. \qquad (2)$$

summing over the outgoing spins and using thrace theorems. Rewrite the expression above: to do so, show that from $p_{\pi_i} = p_{\ell_l} + p_{\overline{\nu_{\ell_l}}}$ one has $p_{\pi_i} p_{\overline{\nu_{\ell_l}}} = p_{\ell_l} p_{\overline{\nu_{\ell_l}}}$, $p_{\pi_i} p_{\ell_l} = m_{\ell_l}{}^2 + p_{\ell_l} p_{\overline{\nu_{\ell_l}}}$, $p_{\ell_l} p_{\overline{\nu_{\ell_l}}} = \frac{1}{2}(m_{\pi_i}{}^2 - m_{\ell_l}{}^2)$; also employ the expression for the Fermi constant G_F. Now you can calculate $\Gamma_{\pi_i \to \ell_l \overline{\nu_l}}$. For this you first have to prove that $|\vec{p_{\overline{\nu_{\ell_l}}}}| = \frac{m_{\pi_i}{}^2 - m_{\ell_l}{}^2}{2\ m_{\pi_i}}$. In order to cancel the troublesome f_{π_i} one considers $\frac{\Gamma_{\pi_i \to e \overline{\nu_e}}}{\Gamma_{\pi_i \to \mu \overline{\nu_\mu}}}$. Calculate the numerical value of this expression. Why is it a small number? Intuitively one would guess it's a large number!!

2. **No leptonic KM-Matrix** Describe in your own words why (for massless neutrinos) there is no leptonis analogon of the KM-matrix.