Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 13
29.01.03
WS 02/03

Exercises on 'Elementary Particle Physics'

Prof. H. Dreiner

1. *The See-Saw Mechanism (finally!) and CP-violating Phases*

   (a) Restrict to one generation. Remember the *Yukawa*-couplings. How would such a term look like for right-handed neutrinos? What kind of massterm do you get from this if the Higgs gets a VEV?

   (b) Assume that there is a $SO(10)$ GUT-theory at a scale of about $M_{\mathrm{GUT}} \approx 10^{16} GeV$. To fill an $SO(10)$ irrep **16** then a right-handed neutrino comes naturally into play (why?). What can you say about the quantum numbers of the right-handed neutrino, i.e. the hypercharge? Which other massterm can you write down?

   (c) Assume that the masses are real (not complex) and diagonalize the mass-matrix. Which eigenvalues do you get? Interpret the result.

   (d) Now consider three generations and allow the masses to be complex. What can you say about the scales appearing in the matrices you use to diagonalize the mass-matrix? One can derive the so called *Maki-Nakagawa-Sakata*-matrix, which is the leptonic analogon to the CKM-matrix of the quarks. What changes now due to the mixing of the neutrinos? What can you say about the CP-violating phases?

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2. Nuclear β-Decay and Massive Neutrinos

   (a) Consider the process ${}^{14}O \to {}^{14}N^* + e^+ + \nu_e$. By simplifying assumptions like small momentum transfer, conserved vector current hypothesis, neglecting the nuclear wavefunction, using non-relativistic spinors for the nucleons, etc (for the interested student c.f. e.g. Halzen-Martin 12.3), one can show that the corresponding amplitude can be approximated by

   $$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\bar{u}(p_\nu)\gamma^0(1-\gamma_5)v(p_e)\right)(2m_N)(\sqrt{2}) \,, \qquad (1)$$

   with m_N the nucleon mass.

   Average the spins and derive the decay rate $d\Gamma$ from this, but **do not(!!!)** negelect electron and neutrino masses.

   (b) In a so called *Kurie*-plot one plots $p_e^{-1}(d\Gamma/dp_e)^{1/2}$ against the energy E_e. Sketch the Kurie-plots for vanishing and non-vanishing neutrino masses.