Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 3
May 16, 2001
SS 2001

Exercises on 'Introduction to Strings and Branes'

Dr. S. Förste

A6. Type II Superstrings

The lightcone gauge quantisation of the type II superstring works quiet analogous to the bosonic case (c.f. [A5.]). However there are some complications we want to concentrate on.

Remember the worldsheet action

$$S = \frac{1}{2\pi\alpha'} \int d\sigma^+ d\sigma^- \left(\partial_- X^\mu \partial_+ X_\mu + \frac{i}{2} \psi_+^\mu \partial_- \psi_{+\mu} + \frac{i}{2} \psi_-^\mu \partial_+ \psi_{-\mu} \right) . \tag{1}$$

The equations of motion for the bosons X^μ are identical to the bosonic string, so the whole discussion for them carries over to the type II superstring.

1) Vary the fermionic part of (1) to get the equations of motion for $\psi_\pm^\mu$. What surface terms do you get?

From the worldsheet SUSY transformations we know that $\delta\psi_+^\mu$ and $\delta\psi_-^\mu$ were independent of each other. Relating them via the surfaceterm would therefore break worldsheet SUSY at the boundary. We don't want this!

2) Use this to derive the periodicity conditions

$$\psi_+^\mu(\sigma = \pi) = \pm\psi_+^\mu(\sigma = 0) \quad \text{and} \quad \psi_-^\mu(\sigma = \pi) = \pm\psi_-^\mu(\sigma = 0) . \tag{2}$$

The plus and minus signs can be choosen independently. Periodic boundary conditions (+) correspond to the so called *Ramond* or R sector, antiperiodic boundary conditions (−) to the *Neveu-Schwartz* or NS sector. Depending on the sector the fermions get different *Fourier* expansions:

$$\text{R sector:} \qquad \psi_-^\mu = \sum_{n\in\mathbb{Z}} d_n^\mu e^{-2in\sigma^-} , \tag{3}$$

$$\psi_+^\mu = \sum_{n\in\mathbb{Z}} \tilde{d}_n^\mu e^{-2in\sigma^+} . \tag{4}$$

$$\text{NS sector:} \qquad \psi_-^\mu = \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^\mu e^{-2ir\sigma^-} , \tag{5}$$

$$\psi_+^\mu = \sum_{r\in\mathbb{Z}+\frac{1}{2}} \tilde{b}_r^\mu e^{-2ir\sigma^+} . \tag{6}$$

Again we use canonical quantization. The bosonic equal time commutators are unchanged (cf. [A5.]) and for the fermions from

$$\{\psi_+^\mu(\sigma), \psi_+^\nu(\sigma')\} = \{\psi_-^\mu(\sigma), \psi_-^\nu(\sigma')\} = \pi\eta^{\mu\nu}\delta(\sigma - \sigma') , \tag{7}$$

$$\{\psi_+^\mu(\sigma), \psi_-^\nu(\sigma')\} = 0 \tag{8}$$

one finds in the NS sector

$$\{b_r^\mu, b_s^\nu\} = \{\tilde{b}_r^\mu, \tilde{b}_s^\nu\} = \eta^{\mu\nu}\delta_{r,-s} \tag{9}$$

and in the R sector

$$\{d_m^\mu, d_n^\nu\} = \{\tilde{d}_m^\mu, \tilde{d}_n^\nu\} = \eta^{\mu\nu}\delta_{m,-n} . \tag{10}$$

As in the bosonic case one splits annihilators and creators via the corresponding index n or r being greater or less than zero.

3) What's about zero mode oscillators? Taking the usual vacuum, what can you say about the R sector states?

Pairing left and rightmovers there are four different sectors to be discussed: NSNS, NSR, RNS and RR. We want to examine the NSNS sector first. Analogous to [A5.] one finds for the mass spectrum

$$M^2_{\mathrm{NSNS}} = 8(N_{\mathrm{NS}} - a_{\mathrm{NS}}) = 8(\tilde{N}_{\mathrm{NS}} - a_{\mathrm{NS}}) \; , \tag{11}$$

where a_{NS} is a new normal ordering constant and

$$N_{\mathrm{NS}} = \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n + \sum_{r=\frac{1}{2}}^{\infty} r b^i_{-r} b^i_r \; , \tag{12}$$

and analogous for $\tilde{N}$.

4) What is the first excited state consistent with (11)? What can you say about M and a_{NS}?

Using zeta function regularization one finds the critical dimension to be 10.

If one continues to calculate the states coming from the NSR, RNS and RR sectors one finds that this stringtheory gets inconsistent. There are several reasons for this (e.g. scattering amplitudes) but we won't give a proof here. Instead one can truncate the spectrum by introducing a so called GSO (*Gliozzi-Scherk-Olive*) projection. The truncated theory is consistent and (even more striking) leads to target space SUSY.

To intruduce the GSO projection we define the fermion number operators F ($\tilde{F}$) counting the number of worldsheet fermionic NS right (left) handed creation operators acting on the vacuum. In addition we assign to the right (left) handed NS vacuum an F ($\tilde{F}$) eigenvalue of one. We can write

$$F = 1 + \sum_{r>0} b^i_{-r} b^i_r \tag{13}$$

and $\tilde{F}$ analogous. Now the GSO projection operators are given by

$$P_{\mathrm{GSO}} \equiv \frac{1+(-1)^F}{2}\frac{1+(-1)^{\tilde{F}}}{2} \; . \tag{14}$$

5) Which states of the NSNS sector are projected out, which are kept?

Now we turn to the R sector. From 3) we know that the d^μ_0's satisfy a *Clifford* algebra. Using lightcone gauge d^1_0 and d^2_0 are eliminated. We define

$$D_I \equiv \frac{1}{\sqrt{2}}(d^{2I}_0 + i d^{2I+1}_0) \; , \qquad\qquad I \in \{1, \ldots, 4\} \; . \tag{15}$$

6) Show that the only nonvanishing anticommutator is given by

$$\{D_I, D^\dagger_I\} = 1 \; . \tag{16}$$

What's about D^2_I and $D^{\dagger 2}_I$?

Construct the R vacuum by starting with a state which is annihilated by all the D_I

$$D_I|-,-,-,-\rangle = 0 \quad \forall I \; . \tag{17}$$

Acting with a $D^\dagger_I$ on the vacuum changes the Ith minus into a plus.

7) What is the degeneracy of the R vacuum?

In the R sector the GSO is defined by

$$(-1)^F = 2^4 d_0^2 d_0^3 d_0^4 d_0^5 d_0^6 d_0^7 d_0^8 d_0^9 (-1)^{\sum_{n>0} d_{-n}^i d_n^i} \tag{18}$$

8) What does (18) correspond to for the R groundstate?

Again we define the GSO projector by

$$P_{\mathrm{GSO}}^{\pm} = \frac{1 \pm (-1)^F}{2} \,. \tag{19}$$

9) Using a GSO projector, how many degrees of freedom are left?

Because of left and right movers, we've essentially got two different choices for the GSO projections. Equal signs on both sides or opposite ones. These different choices lead to different (consistent) superstring theories: type IIA (different signs) and type IIB (same signs).

The number operator in the R sector is straightforwardly given by

$$N_R = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i + \sum_{n=1}^{\infty} n d_{-n}^i d_n^i \tag{20}$$

and the normal ordering constant has to be $a_R = 0$.

10) Write down the mass shell and level matching conditions for all the different sectors: NSNS, NSR, RNS and RR.

Now you are completely equipped with all the knowledge to calculate the type IIA and IIB superstring spectra. Do so if you like... ;-)