Physikalisches Institut
Universität Bonn
Theoretische Physik

Exercise 3
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Exercises on 'Introduction to Strings and Branes'

Dr. S. Förste

A7. Toroidal compactification

We want to extend the results for the one-dimensional circle compactification found in the lecture to more-dimensional toroidal compactifications. It's enough to consider the bosonic string (we don't bother about the tachyon). The critical dimension for the bosonic string is $D = 26$. The number of uncompactified dimensions will be denoted by d.

Uppercase latin letters run from 0 to 25 (or from 2 to 25 if we choose lightcone gauge), greek indices from 0 to $d-1$ and lowercase latin indices from d to $D-1=25$.

We compactifiy all dimensions on the same radius R, i.e. we identify

$$X^m \cong X^m + 2\pi R \; . \tag{1}$$

The geomtery of the compactified dimensions is a k-torus (T^k) and we will denote its metric by G_{mn}. We restrict on the case that G_{mn} is constant.

1) From the one-dimensional case we know that for vanishing *Kaluza-Klein* (momentum) and winding numbers the massless states are those with $N = \tilde{N} = 1$. For arbitrary R (i.e. not a kind of selfdual radius) there are no other massless states. In lightcone gauge the states are given by
$$\alpha^M_{-1}\tilde{\alpha}^N_{-1}|0;k\rangle \; .$$
How many states are this? Group this states seperately in spacetime and internal indices and assign the following particle interpretations: graviton, dilaton, antisymmetric tensor, KK gauge bosons, antisymmetric tensor gauge bosons and scalars. The scalars can be split again. Into what?

Now we concentrate on the changes in the string spectrum. Therefore we just consider the compactified part of the gaugefixed worlsheet action (G and B constant). It's given by

$$S^{\text{comp}} = \frac{1}{4\pi\alpha'}\int d^2\sigma \left(\eta^{\alpha\beta}G_{mn} + \epsilon^{\alpha\beta}B_{mn}\right)\partial_\alpha X^m \partial_\beta X^n \; . \tag{2}$$

2) Why is the B-term in (2) a topological term?

Because of that there is no local contribution to the oscillators. However the zero mode spectrum changes!

3) The zero modes of the internal dimensions are given by
$$X^m(\tau,\sigma) = x^m(\tau) + w^m R\sigma$$
with w^m being the winding numbers. Plug this into the action (2) and extract the *Lagrange* density. Derive the canonical momenta P_n. Like in the one-dimensional case the momenta are quantised, i.e. $p_m = n_m/R$ (why?). Use this to derive
$$\dot{x}^n = \frac{\alpha'}{R}G^{nm}n_m - B^n_m R w^m \; .$$

What do you get for $X^n(\tau,\sigma)$? Split this into dependencies of $\sigma^\pm$ and use $X^n(\tau,\sigma) = X_L(\sigma^+) + X_R(\sigma^-)$ to show that the left (right) moving momenta are given by

$$p_{Lm} = \frac{\alpha'}{R} n_m + \frac{w^n R}{\alpha'}(G_{mn} + B_{mn}) \, , \tag{3}$$

$$p_{Rm} = \frac{\alpha'}{R} n_m + \frac{w^n R}{\alpha'}(-G_{mn} + B_{mn}) \, . \tag{4}$$

4) Use

$$m^2 = \frac{1}{2}(p_{Ln}p_L^n + p_{Rn}p_R^n) + \frac{2}{\alpha'}(N + \tilde{N} - 2)$$

to derive the spectrum of the toroidal compactified theory and

$$\alpha'(p_{Ln}p_L^n - p_{Rn}p_R^n) + 4(N - \tilde{N}) = 0$$

to derive the level-matching condition.