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Exercise 6
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Exercises on 'Introduction to Strings and Branes'

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A9. Enhanced gauge symmetries in toroidal compactifications

Remember massformula and level-matching condition for a bosonic string with one dimension compactified on a circle (c.f. A7.):

$$m^2 = \frac{n^2}{R^2} + \frac{R^2}{\alpha'^2} w^2 + \frac{2}{\alpha'}(N + \tilde{N} - 2a) \ , \tag{1}$$

$$nw + N - \tilde{N} = 0 \ ,$$

where the zero-point energy is given by $a = 1$.

1) Write down the massless states for vanishing winding and KK momentum numbers (briefly!). What's the gauge symmetry?
2) Now choose the selfdual compactifaction radius, i.e. $R^2 = \alpha'$. Which additional massless states do you get for nonvanishing winding and/or KK momentum numbers (Hint: you should find eight more states)?
3) The states found in 1) and 2) combine into irreps of a nonabelian gauge group $SU(2) \times SU(2)$ (the gauge group is enhanced though the rank is not altered). Assign the states to irreps.

A10. The $S^1/\mathbb{Z}_2$ orbifold

As explained in the lectures orbifolds are obtained by modding out discrete symmetries of the compactified space manifold. We want to study an example where the compact space manifold is a cricle S^1 and the symmetry we would like to mod out a $\mathbb{Z}_2$.

1) Choose the fundamental domain for the 'flat' circle to be $-\pi R < x^{25} < \pi R$. Draw this flat circle. Apply the $\mathbb{Z}_2$ identification $x^{25} \equiv -x^{25}$. How does this look like in your sketch? Which points are invariant under the $\mathbb{Z}_2$? Transfer your results on the standard picture of a circle.

The spectrum of an orbifold seperates into different sectors, the untwisted sectors and the twisted sectors (for the $\mathbb{Z}_2$ we only get one twisted sector). We first concentrate on the untwisted sector, where you just have to project the toroidal compactified theory (A9.) on $\mathbb{Z}_2$ invariant states.

2) Since the construction of left and rightmoving states for the bosonic string is identical, just consider the leftmovers. Which states do you get for vanishing winding and momentum? What are their eigenvalues under a $\mathbb{Z}_2$ operation? If you now tensor left and rightmovers together, which states are $\mathbb{Z}_2$ invariant? Which states are projected out? What's about the tachyon?
3) Again choose the selfdual radius $R^2 = \alpha'$. What happens to winding and KK momentum numbers under a $\mathbb{Z}_2$. Which combinations of the massless states found in A9. 2) are $\mathbb{Z}_2$ invariant? How do they look like? Which gauge group do you get? Do you get additional tachyons? If yes, what's their mass?

The twisted sector is characterized by strings which are only periodic up to a nontrivial $\mathbb{Z}_2$ transformation. Therefore the center of mass position has to sit at a fixed point/plane of the orbifold. For the $\mathbb{Z}_2$ the situation is similar to the NS sector of type II superstring theory. The oscillator modes are halfintegered. Because of this the zeropoint energy of the twisted sector is different to the one of the untwisted sector, i.e. (using zeta function regularization) one finds for the $\mathbb{Z}_2$ twist: $a = \frac{15}{16}$. Therefore the massformula (1) changes. Here (and in most other examples) you can't get nonvanishing winding and KK momentum numbers in the twisted sector, so take $n = m = 0$.

4) What's the mass of the twisted groundstate? Can you reach massless states via $N \neq 0$ and/or $\tilde{N} \neq 0$? Do you get additional tachyons and if yes what are their masses? What's the influence of the fixed points/planes on the number of twisted states?

5) Repeat A9 for the radius $R = 2R_{sd}$ and compare the results with the ones found for the $S^1/\mathbb{Z}_2$ orbifold. What changes if you choose $R = \frac{1}{2}R_{sd}$ (Think or calculate!)?