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Exercise 7
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Exercises on 'Introduction to Strings and Branes'

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A11. Boundary conditions for open strings

Remember the gaugefixed worldsheet superstring action

$$S = \frac{1}{2\pi\alpha'} \int d\sigma^+ d\sigma^- \left(\partial_- X^\mu \partial_+ X_\mu + \frac{i}{2} \psi_+^\mu \partial_- \psi_{+\mu} + \frac{i}{2} \psi_-^\mu \partial_+ \psi_{-\mu} \right) . \tag{1}$$

Again $\sigma \in [0, \pi]$, but since we want to describe open strings $\sigma = 0$ and $\sigma = \pi$ are real boundaries.

1) Rewrite the bosonic part of (1) in τ and σ coordinates and vary the X^μ. You'll get the same eqm's as in the closed string case but additionally boundary conditions (bc's).
2) Are the bc's at $\sigma = 0$ and $\sigma = \pi$ correlated? What do you get from this?
3) Concentrate on the $\sigma = 0$ boundary first. You should find two possibilities to fullfill the bc's: *Neumann* (N) bc's $\partial_\sigma X^\mu = 0|_{\sigma=0}$ and *Dirichlet* (D) bc's $\partial_\tau X^\mu = 0|_{\sigma=0}$. Derive them.
4) Consider a situation where you got N bc's for $\mu = i \in \{0, \ldots, p\}$ and D bc's for $\mu = a \in \{p+1, \ldots, D\}$. Interpret the picture. What can you say about *Lorentz* invariance?
5) Use (1) to derive the bc's for the fermions (again the eqm's are the same as for the closed string). Are the bc's at $\sigma = 0$ and $\sigma = \pi$ correlated for the fermions? Concentrate on $\sigma = 0$ again. Which possibilities are there to fullfill the bc?

Remember the worldsheet SUSY transformations

$$\delta X^\mu = i\epsilon_+ \psi_-^\mu - i\epsilon_- \psi_+^\mu \ , \tag{2}$$

$$\delta \psi_-^\mu = -2\epsilon_+ \partial_- X^\mu \ , \tag{3}$$

$$\delta \psi_+^\mu = 2\epsilon_- \partial_+ X^\mu \ , \tag{4}$$

with ϵ being the SUSY parameter.

6) As we'll see later we have got the freedom to assign the fermionic bc $\psi_+^i = \psi_-^i$ at $\sigma = 0$ to the bosonic N directions. Use (3) and (4) to derive the fermionic bc's in D directions if we don't want the worldsheet SUSY to be broken completely. What happens to the worldsheet SUSY anyway?

Recall that for the closed string using the worldsheet SUSY transformations one finds that an antiperiodic ϵ_+ implies that the fermions are antiperiodic (so i.e. in the NS sector) iff the bosons are periodic. On the other hand a periodic ϵ_+ implies that the fermions are periodic (so i.e. in the R sector) iff the bosons are periodic.

Because of this it's very useful to introduce the doubling trick for the open strings. Define

$$\epsilon \equiv \begin{cases} \epsilon_+(\sigma) & , \quad 0 \leq \sigma < \pi \\ \pm\epsilon_-(2\pi - \sigma) & , \quad \pi \leq \sigma < 2\pi \end{cases} \tag{5}$$

7) Define ∂X in analogy to (5), where $\partial_\pm X$ takes the role of $\epsilon_\pm$. We introduce the following notation, a string is called DD if it fullfills D bc's at $\sigma = 0$ and D bc's at $\sigma = \pi$, DN if it fullfills D bc's at $\sigma = 0$ and N bc's at $\sigma = \pi$, ND and NN analogously. Which possibilities do you get for the bc's if ∂X is periodic (antiperiodic)?

8) Define Ψ in analogy to (5), where the role of $\epsilon_\pm$ is taken by the $\psi_\pm$. Consider the NS sector (i.e. ϵ antiperiodic for DD/NN strings. Which sign to you get in (5)? From this you can derive a condition between ϵ_+ and ϵ_- at $\sigma = \pi$. Using (3) and (4) you get the bc's at $\sigma = \pi$ for the $\psi_\pm$ in N and D directions. What are they? What does this mean for Ψ? Repeat the same for the R sector. If you like repeat the same for ND and DN strings in NS and/or R sector.