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Exercise 8
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Exercises on 'Introduction to Strings and Branes'

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A12. Open strings in non-trivial backgrounds

For our calculations we will perform a *Wick* rotation such that the worldsheet is of *euclidean* signature. Choose the coordinates

$$z = z^1 + iz^2 \equiv e^{\tau + i\sigma} \ . \tag{1}$$

1) The parameter space is $\tau \in [-\infty, \infty]$ and $\sigma \in [0, \pi]$. What is the corresponding parameter space for the z's? Where are the boundaries of the open string mapped to?

We can restrict our discussion to the bosonic string, since all results carry over to the supersymmetric case. We consider a situation where the metric $G_{\mu\nu}$ and the antisymmetric tensor $B_{\mu\nu}$ are constant. For the open string we choose *Neumann* boundary conditions, i.e.

$$\partial_\sigma X^\mu|_{\sigma=0} = \partial_\sigma X^\mu|_{\sigma=\pi} = 0$$

at first. Now consider a single brane setup. As shown in the lecture the massless open string excitations then correspond to a $U(1)$ gauge field A_μ. Consider the fieldstrength $F_{\mu\nu}$ of A to be slowly varying, i.e. neglect second or higher order derivatives of F. The gauge fixed worldsheet action in z coordinates reads

$$S = \frac{1}{2\pi\alpha'}\left[\int d^2z \frac{1}{2}\left(\partial_\alpha X_\mu \partial^\alpha X^\mu + i\epsilon^{\alpha\beta} B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu\right) + i\int_{z^2=0} dz^1 A_\mu \partial_1 X^\mu\right] \ . \tag{2}$$

In order to proceed we specify a classical configuration $\bar{X}^\mu$ around which we are going to expand.

2) What are the eqm's and the boundary conditions for the $\bar{X}^\mu$? Interpret the boundary conditions.

The term with the B field can be absorbed into the fieldstrength of A. Therefore we can just set $B = 0$ and recover it via $F \to F + B$.

3) Since the metric is constant we are allowed to *Taylor*-expand the action (2) around the classical configuration. Write down $S[\bar{X} + \zeta]$. Split the result into a free and an interacting piece. The free piece determines the propagator whereas the interacting piece determines the vertices.

4) Vary S_{free} with respect to ζ^μ to find the defining equations for the propagator.

A solution of the boundary value problem is given by

$$\Delta_{\mu\nu}(z,z') = \\ -\alpha' \left[G^{\mu\nu} \log \frac{|z-z'|}{|z-\bar{z}'|} + (\hat{G}^{-1})^{\mu\nu} \log |z-\bar{z}'|^2 + \theta^{\mu\nu} \log \frac{z-\bar{z}'}{\bar{z}-z'} + D^{\mu\nu} \right] , \tag{3}$$

with

$$\begin{aligned} (\hat{G}^{-1})^{\mu\nu} &= \left(\frac{1}{G+F}\right)_S^{\mu\nu} = \left(\frac{1}{G+F} G \frac{1}{G-F}\right)^{\mu\nu} , \\ \hat{G}_{\mu\nu} &= G_{\mu\nu} - (FG^{-1}F)_{\mu\nu} , \\ \theta^{\mu\nu} &= \left(\frac{1}{G+F}\right)_A^{\mu\nu} = \left(\frac{1}{G+F} F \frac{1}{G-F}\right)^{\mu\nu} , \end{aligned}$$

A/S stands for (anti-)symmetrization, $D^{\mu\nu}$ is an additive constant.

5) Draw the vertices coming from the interacting piece of the action.

In the lectures you saw, that continuing with regularization, renormalization and computing the β-function for A one finds the *Dirac-Born-Infeld* or D-Brane action. Here we want to concentrate on another effect. Because the vertex operators always have to be inserted at the boundary, we restrict the propagator to the real axis. Set $z = \bar{z} = \tau$ and $z' = \bar{z}' = \tau'$.

6) Introduce
$$D^{\mu\nu} = -\frac{1}{\alpha'}\frac{i}{2}\theta^{\mu\nu}\epsilon(\tau-\tau') ,$$
where ϵ is the step function, i.e. $+1$ (-1) for positive (negative) arguments. The introduction of D here is very ad hoc, but can be motivated by methods of conformal field theories. However, what do you get for the propagator on the boundary?

7) Again from conformal field theories we borrow the definition of the equal 'time' commutator
$$[X^\mu(\tau), X^\nu(\tau)] = T\left\{\Delta^{\mu\nu}(\tau,\tau^-) - \Delta^{\mu\nu}(\tau,\tau^+)\right\} ,$$
where T denotes time ordering, and $\tau^\pm = \tau \pm \delta\tau$ with $\delta\tau$ positive and running towards 0. What do you get for the commutator? Interpret the result.