Universität Bonn
Physikalisches Institut
Theoretische Physik

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Example sheet 8
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Elementary Particle Physics II

Prof. Dr. H.-P. Nilles

1. **Orbifold symmetries**

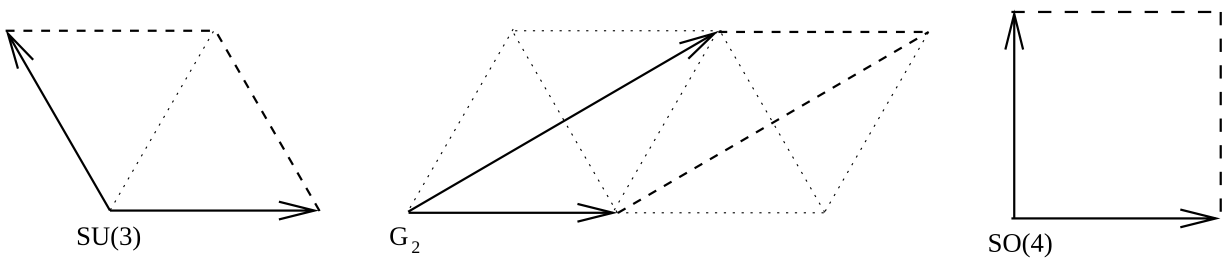

Figure 1: $SU(3)$, G_2 and $SO(4)$ torus lattices.

Determine the fixed points and the fundamental domains of the torus lattices in fig. 1:

(a) under a $\mathbb{Z}_2$ twist.

(b) under a $\mathbb{Z}_3$ twist.

2. **5d Orbifold gauge symmetry breaking**
We want to compactify a 5d, $N = 2$, $SU(3)$ gauge multiplet on $S^1/(\mathbb{Z}_2 \times \mathbb{Z}_2')$, where the S^1 radius is R. These spacetime symmetries identify points in the compact $x^4 \equiv y$ direction as follows:

$$S^1 : \quad y \equiv y + 2\pi R\,, \qquad \mathbb{Z}_2 : \quad y \to -y\,, \qquad \mathbb{Z}_2' : \quad y \to \pi R - y\,. \tag{1}$$

(a) Show that the fundamental domain is $y = [0, \frac{\pi R}{2}]$.

(b) Denoting the parities under $\mathbb{Z}_2$ and $\mathbb{Z}_2'$ by $(\pm, \pm)$, show that only a field with $(+, +)$-parity can have a massless mode in 4d.
(Take a field $\phi(x^\mu, y)$, expand it in Fourier components, check the parities, then check the equation of motion $\Box^{(5)}\phi = 0$.)

(c) The $\mathbb{Z}_2'$ parities of the gauge fields $A_M = A_M^a\, T^a$ (where T^a are the generators of $SU(3)$) can be expressed as

$$A_M(x,y) = \Lambda_M^N\, P\, A_N(x, \pi R - y)\, P^{-1} \tag{2}$$

where $\Lambda = \mathrm{diag}(1,1,1,1,-1)$ is required to reproduce the $\mathbb{Z}_2$ parity assignments for $P = 1$.

Check that the choice of $P = \mathrm{diag}(-1,-1,1)$ breaks the gauge symmetry from $SU(3)$ to $SU(2) \times U(1)$. *(Hint: Check $P\, T^a\, P^{-1}$.)*