Universität Bonn
Physikalisches Institut
Theoretische Physik

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Example sheet 1
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# General Relativity

Prof. Dr. H.-P. Nilles

1. ***Lorentz transformations***
   Let us write a linear transformation of the space-time coordinates as a matrix $\Lambda$ which acts on the coordinate 4-vectors as $x'^\mu = \Lambda^\mu{}_\nu \, x^\nu$. The metric $\eta$ is defined as $-\eta_{\mu\nu} \equiv \mathrm{diag}(1, -1, -1, -1)$.

   (a) Requiring the length element $ds^2 = -\eta_{\mu\nu} dx^\mu dx^\nu$ to be constant leads to a constraint equation for $\Lambda$, which defines $\Lambda$ as a Lorentz transformation. What does this constraint look like?

   (b) What form does $\Lambda$ take for purely spatial rotations? Check that this $\Lambda$ satisfies the condition in (a).

   (c) Now consider a *boost* along the z-axis (*i.e.* the origin of the $x'^\mu$-system moves along the z-axis with constant speed $v$). Use condition (a) to derive $\Lambda$ for this situation.
   *(Hint: the only interesting components are* $\Lambda^0{}_0$*,* $\Lambda^0{}_3$*,* $\Lambda^3{}_0$ *and* $\Lambda^3{}_3$*.*
   *Also,* $\cosh x = \frac{1}{\sqrt{1-\tanh^2 x}}$, $\sinh x = \frac{\tanh x}{\sqrt{1-\tanh^2 x}}$ *and* $\cosh^2 x - \sinh^2 = 1$*)*

2. ***Time and space dilations***
   Have a look at figure 1: it shows the following situation from the viewpoint of an observer $R$ at rest: At event $A$, two spaceships $S$ and $T$ are launched with a velocity such that $\gamma = 100$. At event $B$, the ships encounter a planet, where one ship ($T$) remains, while the other ship ($S$) continues on its old course. At event $C$, ship $T$ begins its return journey to Earth, with the same velocity as before. Its arrival back on Earth is event $D$.

   (a) Calculate the velocity of the ships from $\gamma = 100$ as a percentage of $c$.

   (b) Calculate the distance $d$ from Earth to the planet, in the frame of $R$. How far is it from the spaceships' point of view?

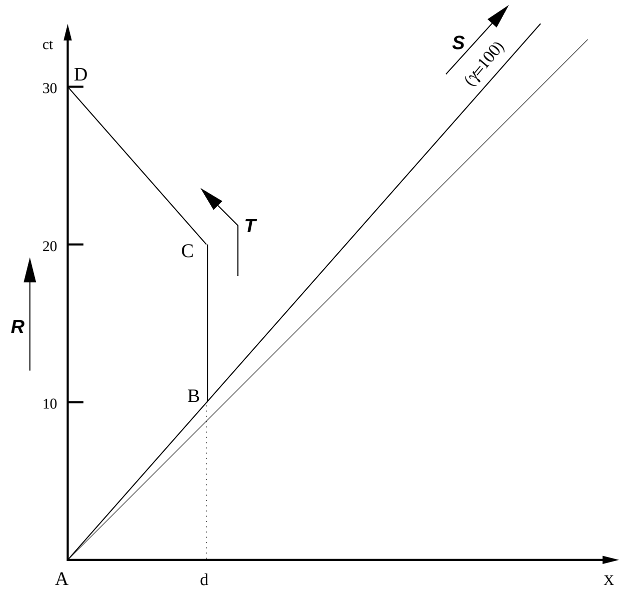


Figure 1: Space-time diagram for Question 2.

(c) Complete the following table:

| | A | B | C | D |
|---|---|---|---|---|
| $(t,x)_R$ | (0,0) | (10, ) | (20, ) | (30,0) |
| $(t,x)_S$ | (0,0) | ( ,0) | ( , ) | ( , ) |
| $(t,x)_T$ | (0,0) | ( ,0) | ( ,0) | ( ,0) |

(d) In the frame of $R$, calculate $\tau_R(A \to D)$, $\tau_T(A \to D)$ and $\tau_S(t_R = 30)$.

(e) Sketch a space-time diagram in which $S$ is at rest. Use the table to mark the events $A$–$D$.

(f) In the frame of $S$, calculate $\tau_R(A \to D)$, $\tau_T(A \to D)$ and $t_S(D)$.

## *3. Accelerated objects*

(a) Sketch the rest-frame space-time diagram of an object $O$ moving along the path

$$x(t) = \sin t \tag{1}$$

from $t = 0$ until $O$ returns to the starting point.
Let's call the event at the origin $A$, the one at the return point $B$.

(b) How much time elapses between $A$ and $B$ for an observer $R$ at rest?

(c) Integrate over $d\tau$ to calculate the time between $A$ and $B$ for the co-moving observer $O$. ($\beta \equiv v/c$)