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Exercise 4
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SS 06

Exercises on Elementary Particle Physics II

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1. *The Superpotential*

   The highest component of a superfield transforms under a SUSY transformation into a total derivative. A space-time integral $\int \mathrm{d}^4x$ of such a quantity is thus invariant under supersymmetry transformations. This is the basic observation for the construction of a SUSY invariant action S. By definition, the part of the Lagrange density containing only one type of chiral superfields (e.g. left-chiral) is called the superpotential $\mathcal{L}_F$. Then, integration over $\int \mathrm{d}^2\theta$ gives the highest component (called the F-term) and +h.c. makes the term real:

$$S \supset \int \mathrm{d}^4x \left[\int \mathrm{d}^2\theta \; \mathcal{L}_F + \mathrm{h.c.} \right]$$

   Evaluate the following superpotential of a left-chiral superfield ($\bar{D}\phi = 0$):

$$\mathcal{L}_F = m\phi^2 + \lambda\phi^3$$

   Hint: use the left-chiral representation:

$$\phi_L(x,\theta) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x)$$

2. *Kinetic terms of chiral superfields*

   In general, a vector superfield V is defined through $V^\dagger = V$. By definition, the part of the Lagrange density containing only vector superfields is called $\mathcal{L}_\mathcal{D}$. The highest component (called the D-term) is obtained by a $\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}$ integration:

$$S \supset \int \mathrm{d}^4x \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \; \mathcal{L}_D$$

   In this part we will write down a SUSY invariant term for $\mathcal{L}_\mathcal{D}$ which will give rise to kinetic terms for both, a left-chiral fermion $\psi(x)$ and its bosonic partner $\varphi(x)$. Therefore, we start with a left-chiral superfield $\phi_L(x,\theta)$.

   (a) Show that $[\phi_L(x,\theta)]^\dagger$ transforms in the right-chiral representation of the SUSY algebra.

(b) Using the relations between S, S_L and S_R derived in Ex.3.1(f), what are the relations between ϕ, ϕ_L and ϕ_R?

(c) Evaluate $\phi\phi^\dagger$. Why does this term belong to $\mathcal{L}_\mathcal{D}$? What is the $\theta\theta\bar{\theta}\bar{\theta}$ coefficient?

3. *The Wess-Zumino Model*

Next, we combine the results of the two previous parts and get the model of Wess + Zumino.

(a) Assume the Lagrange density

$$\mathcal{L} = \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\ \phi\phi^\dagger + \left[\int \mathrm{d}^2\theta\ m\phi^2 + \lambda\phi^3 + \text{h.c.}\right]$$

Solve the equation of motion for the $F(x)$ field. Insert the result into the Lagrangian $\mathcal{L}$.

(b) Show that the scalar potential $V(\varphi)$ can be obtained from the superpotential

$$V(\varphi) = \left|\frac{\partial W(\varphi)}{\partial\varphi}\right|^2$$

where $W(\varphi)$ denotes the superpotential $\mathcal{L}_F(\phi)$ with the superfield ϕ replaced by its scalar component φ.

(c) Compare the masses of the fermion and the boson. (Take care that the kinetic energies of the fields are normalized appropriately)