An Introduction to

Superstring Theory

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Contents

1 Introduction

problems of **Bosonic String Theory**:

- tachyonic ground state
- no fermions in the spectrum

$\longrightarrow$ modify the theory

2 The Action

- action of Bosonic String Theory:

$$S = -\frac{1}{4\pi\alpha'}\int \mathrm{d}\tau\mathrm{d}\sigma\partial_\alpha X^\mu\partial^\alpha X_\mu \tag{1}$$

with X^μ bosonic fields

- now: include fermions and install **Supersymmetry**
$\Rightarrow \Psi_A$ two-component spinor, $\quad A \in \{-,+\}$

$$\Psi_A = \begin{pmatrix} \Psi_- \\ \Psi_+ \end{pmatrix}$$

- second index $\mu = 0, ...d-1: \quad \Psi_A^\mu$
$\Rightarrow$ vector index

- generalize the action:

$$S = -\frac{1}{4\pi\alpha'} \int \mathrm{d}\tau \mathrm{d}\sigma \left(\partial_\alpha X^\mu \partial^\alpha X_\mu + i \overline{\Psi}^\mu \varrho^\alpha \partial_\alpha \Psi_\mu \right) \quad (2)$$

with

$$\varrho^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \varrho^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$\{\varrho^\alpha, \varrho^\beta\} = -2\eta^{\alpha\beta} \quad \text{(Clifford Algebra)}$$

$\Rightarrow \Psi^\mu$ real $\qquad$ **Majorana Spinor**

- indizes: X^μ, Ψ^μ

vector index $\mu = 0, ..., d-1$

spinor index $A \in \{-,+\}$

3 Light-Cone Coordinates

- aim: physical degrees of freedom only

- reparametrization invariance:

$$\sigma^{\pm} = \tau \pm \sigma \tag{3}$$

$$\partial_{\pm} = \frac{1}{2}(\partial_\tau \pm \partial_\sigma) \quad \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}$$

$$X^{\pm} = \frac{1}{\sqrt{2}}(X^0 \pm X^1) \tag{4}$$

$$\Psi^{\pm} = \frac{1}{\sqrt{2}}(\Psi^0 \pm \Psi^1) \tag{5}$$

$$X^i, \Psi^i, \quad i = 2, ..., d-1$$

4 Symmetries

- use symmetries to reduce degrees of freedom

- **reparametrization invariance:**

$$(\tau, \sigma) \longrightarrow (\sigma^-, \sigma^+)$$

$$\frac{1}{2\pi\alpha'}\int \mathrm{d}\sigma^+\mathrm{d}\sigma^-(\partial_- X^\mu \partial_+ X_\mu + \frac{i}{2}(\Psi_+^\mu \partial_- \Psi_{+\mu} + \Psi_-^\mu \partial_+ \Psi_{-\mu})) \tag{6}$$

- **Poincaré-invariance:** in 2- and d-dimensions (index structure)

- **Supersymmetry:** invariance under

$$\begin{aligned} \delta X^\mu &= i(\epsilon_+ \Psi_-^\mu - \epsilon_- \Psi_+^\mu) &(7)\\ \delta \Psi_\mp^\mu &= \mp 2\epsilon_\pm \partial_\mp X^\mu &(8) \end{aligned}$$

  mixes X^μ, Ψ^μ

  checking:

  $S(X^\mu, \Psi^\mu) \longrightarrow S(X^\mu + \delta X^\mu, \Psi^\mu + \delta\Psi^\mu) = S(X^\mu, \Psi^\mu)$

  (henceforth: closed string only)

- **superconformal invariance:** $\sigma^\pm \longrightarrow \tilde{\sigma}^\pm(\sigma^\pm)$
  further: $\epsilon_- = \epsilon_-(\sigma^-), \epsilon_+ = \epsilon_+(\sigma^+)$
  (partially local symmetry)

5 Equations of motion

- variational principle

- bosonic: $X^\mu \to X^\mu + \delta X^\mu$

$$\partial_+\partial_- X^\mu = 0 \tag{9}$$

- fermionic: $\Psi^\mu \to \Psi^\mu + \delta\Psi^\mu$, $\delta\Psi^\mu(\tau \in [\tau_0, \tau_1]) = 0$

$$\delta S = \frac{i}{4\pi\alpha'} \int \mathrm{d}\tau\mathrm{d}\sigma(\partial_\alpha \overline{\Psi}^\mu \varrho^\alpha \delta(\Psi_\mu))$$

$$-\frac{i}{4\pi\alpha'} \int \mathrm{d}\tau(\begin{pmatrix} \overline{\Psi^\mu_+}, -\overline{\Psi^\mu_-} \end{pmatrix} \varrho^1 \begin{pmatrix} \delta\Psi^\mu_- \\ \delta\Psi^\mu_+ \end{pmatrix})|_{\sigma\in\partial S}$$

Thus:

$$\partial_+ \Psi^\mu_- = \partial_- \Psi^\mu_+ = 0 \tag{10}$$

$$(-\Psi_{+\mu}\delta\Psi^\mu_+ + \Psi_{-\mu}\delta\Psi^\mu_-)|^{\sigma=\pi}_{\sigma=0} = 0. \tag{11}$$

- solutions with negative norm $(X^{\pm}, \Psi^{\pm}) \Rightarrow$ negative norm states, called **ghosts** (after quantization)

6 Solutions and Boundary Conditions

- consider $\quad (-\Psi_{+\mu}\delta\Psi_{+}^{\mu} + \Psi_{-\mu}\delta\Psi_{-}^{\mu})|_{\sigma=0}^{\sigma=\pi} = 0$
  supersymmetry $\Rightarrow$ independent variation of $\Psi_{\pm}$
  $\Rightarrow$ two possible choices:

  Ramond-Sector: $\quad \Psi_{\pm}^{\mu}(\tau, \sigma + \pi) = \Psi_{\pm}^{\mu}(\tau, \sigma)$

  Neveu-Schwarz-Sector: $\quad \Psi_{\pm}^{\mu}(\tau, \sigma+\pi) = -\Psi_{\pm}^{\mu}(\tau, \sigma)$

  $\Rightarrow$ solutions can lie in two different sectors (independent choice for each component of Ψ_A

- mode expansion: (for R-Sector)

$$\Psi_{-}^{\mu} = \sum_{n\in\mathbb{Z}} d_n^{\mu} e^{-2in(\tau-\sigma)} \quad \text{R} \tag{12}$$

$$\Psi_{+}^{\mu} = \sum_{n\in\mathbb{Z}} \tilde{d}_n^{\mu} e^{-2in(\tau+\sigma)} \quad \text{R} \tag{13}$$

(for NS Sector)

$$\Psi^\mu_- = \sum_{n\in\mathbb{Z}+\frac{1}{2}} b^\mu_n e^{-2in(\tau-\sigma)} \quad \text{NS} \tag{14}$$

$$\Psi^\mu_+ = \sum_{n\in\mathbb{Z}+\frac{1}{2}} \tilde{b}^\mu_n e^{-2in(\tau+\sigma)} \quad \text{NS} \tag{15}$$

- for the bosonic coordinates:

$$X^\mu_R = \frac{1}{2}x^\mu + \frac{1}{2}p^\mu\sigma^- + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n e^{-2in\sigma^-} \tag{16}$$

$$X^\mu_L = \frac{1}{2}x^\mu + \frac{1}{2}p^\mu\sigma^+ + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}^\mu_n e^{-2in\sigma^+} \tag{17}$$

7 Constraint Equations

- ghosts $\Rightarrow$ problem for probabilistic interpretation of states remove them!
  $\Rightarrow$ constraints

- **Energy-momentum tensor:** $T_{\alpha\beta}$
  insert a vielbein (due to spinors) e^a_α and its superpartner χ_α

$$T_{\alpha\beta} = -\frac{2\pi}{e}\frac{\delta S}{\delta e^\beta_a}e_{\alpha a} \tag{18}$$

$$T_{\alpha\beta} = 0 \qquad \forall \alpha, \beta \tag{19}$$

- **Supercurrent:** J_α
  apply Noether's method to the supersymmetry transf.

$$[J_\pm(\sigma), J_\pm(\sigma')]_{P.B.} = \pi\delta(\sigma - \sigma')T_{\pm\pm}(\sigma)$$
$$[J_+(\sigma), J_-(\sigma')]_{P.B.} = 0$$

  thus demand:

$$J_\alpha = 0$$

- constraints: $T_{++} = T_{--} = J_+ = J_- = 0$

- **superconformal invariance:** $\sigma^\pm \longrightarrow \tilde{\sigma}^\pm(\sigma^\pm)$, $\epsilon_- = \epsilon_-(\sigma^-), \epsilon_+ = \epsilon_+(\sigma^+))$ $\longrightarrow$ fix X^+, Ψ^+
  as in the bosonic case:

$$\Rightarrow \tau \longrightarrow \frac{1}{2}(\tilde{\sigma}^+(\sigma^+) + \tilde{\sigma}^-(\sigma^-)$$
$$\Rightarrow \partial_+\partial_-\tau = 0$$
$$X^+ = x^+ + p^+\tau \tag{20}$$

  choose:

$$\begin{pmatrix} \Psi_- \\ \Psi_+ \end{pmatrix}^{\mu=+} = 0 \tag{21}$$

- constraints explicitly:

$$
\begin{aligned}
-\frac{p^+}{2}\Psi^-_+ + \Psi^i_+\partial_+ X^i &= 0 \\
-\frac{p^+}{2}\Psi^-_- + \Psi^i_-\partial_- X^i &= 0 \\
-p^+\partial_+ X^- + (\partial_+ X)^i - \frac{i}{2}\Psi^i_+\partial_+\Psi^i_+ &= 0 \\
-p^+\partial_- X^- + (\partial_- X)^i - \frac{i}{2}\Psi^i_-\partial_-\Psi^i_- &= 0
\end{aligned}
$$

they can be solved:

$$
(\Psi_\pm)^- = \frac{2}{p^+}\Psi^i_\pm\partial_\pm X^i \tag{22}
$$

$$
\partial_\pm X^- = \frac{1}{p^+}((\partial_+ X^i)^2 + \frac{i}{2}\Psi^i_\pm\partial_\pm\Psi^i_\pm). \tag{23}
$$

- imposing of constraints $\Rightarrow$ reduces the number of degrees of freedom
Henceforth:

$$
\begin{aligned}
&X^i \qquad i = 2, ..., d-1 \\
&\Psi^i \qquad i = 2, ..., d-1
\end{aligned}
$$

8 Quantization

- **quantization:** regard X^μ, Ψ^μ as operators and perform the replacement:

$$[\quad,\quad]_{P.B.} \longrightarrow \frac{1}{i}[\quad,\quad] \quad \text{(bosonic)} \tag{24}$$

$$[\quad,\quad]_{P.B.} \longrightarrow \frac{1}{i}\{\quad,\quad\} \quad \text{(fermionic)} \tag{25}$$

- for classical solutions (due to def. of Poisson Brackets):

$$[\Psi_+^\mu(\sigma), \Psi_+^\nu(\sigma')]_{P.B.} = [\Psi_-^\mu(\sigma), \Psi_-^\nu(\sigma')]_{P.B.} = i\pi\eta^{\mu\nu}\delta(\sigma-\sigma')$$

$$[\Psi_+^\mu(\sigma), \Psi_-^\nu(\sigma')]_{P.B.} = 0.$$

- quantized version

$$\{\Psi_+^\mu(\sigma), \Psi_+^\nu(\sigma')\} = \{\Psi_-^\mu(\sigma), \Psi_-^\nu(\sigma')\} = \pi\eta^{\mu\nu}\delta(\sigma - \sigma') \tag{26}$$

$$\{\Psi_+^\mu(\sigma), \Psi_-^\nu(\sigma')\} = 0. \tag{27}$$

for bosons the same results as in the Bosonic String Theory

- for the Fourier-modes: insert the formula for X^μ, Ψ^μ into the brackets

$$\{b^\mu_r, b^\nu_s\} = \{\tilde{b}^\mu_r, \tilde{b}^\nu_s\} = \eta^{\mu\nu}\delta_{r+s,0} \qquad \text{NS} \tag{28}$$

$$\{d^\mu_r, d^\nu_s\} = \{\tilde{d}^\mu_r, \tilde{d}^\nu_s\} = \eta^{\mu\nu}\delta_{r+s,0} \qquad \text{R} \tag{29}$$

- reality of Majorana spinors: $\quad (b^\mu_r)^\dagger = b^\mu_{-r}$ for $r > 0$

  harmonic oscillator algebra $\quad \{b^{\mu\,\dagger}_r, b^\nu_s\} = \eta^{\mu\nu}\delta_{r,s}$

- **second quantization:**

  b^μ_r lowering operators for $r > 0$

  b^μ_r raising operators for $r < 0$

- construct states by acting with $b^\mu_r,\ r < 0$ on a vacuum state $|k\rangle$ (*second quantization*)

- number-operator:

$$N = N^{(a)} + N^{(b)} = \sum_{m=1}^{\infty} \alpha_{-m} \cdot \alpha_m + \sum_{r=\frac{1}{2}}^{\infty} r b_{-r} \cdot b_r \tag{30}$$

- in terms of oscillators (for the NS-sector for example):

$$\alpha_n^- = \frac{1}{p^+}\left(\sum_{m=-\infty}^{\infty} :\alpha_{n-m}^i\alpha_m^i: + \sum_{r\in\mathbb{Z}+\frac{1}{2}} r : b_{m-r}^i b_r^i : -2\, a_{NS}\, \delta_n\right)$$

$$b_r^- = \frac{1}{p^+}\sum_{s=-\infty}^{\infty} \alpha_{r-s}^i b_s^i$$

(with normal-ordering constant a_{NS})

- formula for the mass-operator:

$$\alpha_n^- = \frac{1}{p^+}\left(\sum_{m=-\infty}^{\infty} :\alpha_{n-m}^i\alpha_m^i: + \sum_{r\in\mathbb{Z}+\frac{1}{2}} r : b_{m-r}^i b_r^i : -2\, a_{NS}\, \delta_n\right)$$

use $p^\mu = 2\alpha_0^\mu$

$$\Rightarrow\ m^2 = 8(N_{NS} - a_{NS})$$

- zero-modes in the R-Sector:

$$\{d_0^\mu, d_0^\nu\} = \eta^{\mu\nu} \quad \text{Clifford Algebra} \tag{31}$$

- general state: pairing left- and right movers (taken each from R- or NS-sector)

9 Review

- action:

$$S = -\frac{1}{4\pi\alpha'}\int \mathrm{d}\tau\mathrm{d}\sigma\,\left(\partial_\alpha X^\mu \partial^\alpha X_\mu + i\overline{\Psi}^\mu \varrho^\alpha \partial_\alpha \Psi_\mu\right) \tag{32}$$

indizes: two fields: X^μ, Ψ^μ

vector index $\mu = 0, ..., d-1$
spinor index $A \in \{-,+\}$

- equations of motion:

$$\partial_+\partial_- X^\mu = 0 \tag{33}$$

$$\partial_+\Psi^\mu_- = \partial_-\Psi^\mu_+ = 0 \tag{34}$$

- two sectors:

Ramond-Sector: $\Psi^\mu_\pm(\tau, \sigma+\pi) = \Psi^\mu_\pm(\tau, \sigma)$
Neveu-Schwarz-Sector: $\Psi^\mu_\pm(\tau, \sigma+\pi) = -\Psi^\mu_\pm(\tau, \sigma)$

$\Rightarrow$ solutions can lie in two different sectors (independent choice for each component of Ψ_A)

- imposing of constraints $\Rightarrow$ reduces the number of degrees of freedom
  Henceforth:

$$X^i \qquad i = 2, ..., d-1$$
$$\Psi^i \qquad i = 2, ..., d-1$$

- **quantization:** regard X^μ, Ψ^μ as operators and perform the replacement:

$$[\quad , \quad]_{P.B.} \longrightarrow \frac{1}{i}[\quad , \quad] \quad \text{(bosonic)} \tag{35}$$

$$[\quad , \quad]_{P.B.} \longrightarrow \frac{1}{i}\{\quad , \quad\} \quad \text{(fermionic)} \tag{36}$$

- for the Fourier-modes:

$$\{b^\mu_r, b^\nu_s\} = \{\tilde{b}^\mu_r, \tilde{b}^\nu_s\} = \eta^{\mu\nu}\delta_{r+s,0} \qquad \text{NS} \tag{37}$$

$$\{d^\mu_r, d^\nu_s\} = \{\tilde{d}^\mu_r, \tilde{d}^\nu_s\} = \eta^{\mu\nu}\delta_{r+s,0} \qquad \text{R} \tag{38}$$

$\Rightarrow$ can be seen as creation and annihilation operators:

b^μ_r lowering operators for $r > 0$

$$b_r^\mu \quad \text{raising operators for } r < 0$$

(for the NS right-moving part)

- construct states by acting with $b_r^\mu,\ r < 0$ on a vacuum state $|k\rangle$ (*second quantization*)

  general state: pairing left- and right movers (taken each from R- or NS-sector)

- groundstates of each sector will determine states to be vectors or spinors:
  $\Rightarrow$ oscillators are space-time bosons; won't change the vector/spinor features of the groundstate

- zero-modes:

  NS bosonic α_0^μ

  R bosonic (α_0^μ) and fermionic (d_0^μ)

- mass-operator:

$$m^2 = 8(N_{NS} - a_{NS})$$

10 The Critical Dimension

- interpretation as particles $\Leftrightarrow$ states must be irreducible representations of little group

- consider NS-Sector (only for right movers):
  no fermionic zero-modes $\Rightarrow$ vacuum $|k\rangle$ is eigenstate of bosonic zero-modes
  $\Rightarrow$ vacuum is a d-dim. vector

- first excited state: $b^i_{-\frac{1}{2}}|k\rangle$
  vector under representations of $SO(d-2)$
  $\Rightarrow$ massless

$$0 = m^2 = 8(\frac{1}{2} - a_{NS})$$

$$a_{NS} = \frac{1}{2} \tag{39}$$

- demand: ordering in quantum expressions must be symmetric

$$N_{NS} - a_{NS} = \frac{1}{2}\left(\sum_{n=-\infty, n\neq 0}^{\infty} \alpha^i_{-n}\alpha^i_n + \sum_{r\in\mathbb{Z}+\frac{1}{2}} r b^i_{-r} b^i_r\right)$$

combine this with the old result:

$$N_{NS} - a_{NS} = \frac{1}{2}\left(\sum_{n=1}^{\infty}\alpha_{-n}^{i}\alpha_{n}^{i} + \sum_{r=\frac{1}{2}}^{\infty} r b_{-r}^{i} b_{r}^{i}\right)$$

$$+\frac{1}{2}\left(\sum_{n=1}^{\infty}\alpha_{n}^{i}\alpha_{-n}^{i} - \sum_{r=\frac{1}{2}}^{\infty} r b_{r}^{i} b_{-r}^{i}\right)$$

$$= N_{NS} + \frac{1}{2}\sum_{i=2}^{d-1}\left(\sum_{n=1}^{\infty} n - \sum_{r=\frac{1}{2}}^{\infty} r\right)\eta^{ii} = N_{NS}$$

$$+\frac{1}{2}(d-2)\left(\sum_{n=1}^{\infty} n - \sum_{r=\frac{1}{2}}^{\infty} r\right)$$

trick: zeta-function regularization

$$\sum_{n=0}^{\infty} n = -\frac{1}{12}$$

$$\sum_{n=0}^{\infty}(n+c) = \zeta(-1,c) = -\frac{1}{2}(6c^2 - 6c + 1)$$

then we find ($c = \frac{1}{2}$)

$$\alpha_{NS} = \frac{d-2}{16} \tag{40}$$

- critical dimension:

$$d = 10$$

- more rigid calculations (Lorentz Algebra) $\Rightarrow$ gives the same result

Noether-currents for Poincaré-transf.: $x'^{\mu} = a^{\mu}_{\nu} x^{\nu} + b^{\mu}$

b^{μ}**:**

$$P^{\mu}_{\alpha} = \frac{1}{\pi} \partial_{\alpha} X^{\mu}$$

$$P^{\mu} := \int_0^{\pi} \mathrm{d}\sigma P^{\mu}_{\tau}$$

a^{μ}_{ν}**:**

$$J^{\mu\nu}_{\alpha} = \frac{1}{\pi} (X^{\mu} \partial_{\alpha} X^{\nu} - X^{\nu} \partial_{\alpha} X^{\mu} + i \overline{\Psi}^{\mu} \varrho_{\alpha} \Psi^{\nu})$$

$$J^{\mu\nu} := \int_0^{\pi} \mathrm{d}\sigma J^{\mu\nu}_{\tau}$$

$\Rightarrow$ they satisfy the usual commutation relations for the Lorentz-Algebra, except $[J^{i-}, J^{j-}]$: anomaly-term
$\Rightarrow$ vanishes only (for NS) if $a_{NS} = \frac{1}{2}$, $d = 10$

- string propagates in 10 dimensions

11 The Spectrum

separate discussion of NS/R-sector
$\Rightarrow$ GSO-projection to get rid of unwanted states

NS

- (only for the right movers)
  vacuum $|k\rangle$ is eigenstate of bosonic zero-modes
  $\Rightarrow$ states describe bosons

- for vacuum $|k\rangle$: $\quad m^2 = -4$
  for first excited state $b^i_{-\frac{1}{2}}|k\rangle$: $\quad m^2 = 0$
  $\Rightarrow$ vector representation of $SO(8)$
  for second excited state $b^i_{-\frac{1}{2}}b^j_{-\frac{1}{2}}|k\rangle$: $m^2 = 4$

  $\Rightarrow$ these and all following combine to irred. represent. of $SO(9)$

- project tachyon out:
  $\Rightarrow$ def. fermionic number operator F ($F = 1$ for vacuum)

$$P_{GSO} = \frac{1 + (-1)^F}{2} \cdot \frac{1 + (-1)^{\tilde{F}}}{2} \qquad (41)$$

multiply each state with P_{GSO}

$\Rightarrow$ half of the states are removed, no more tachyon!!

(same procedure for left-movers)

R • (only right movers) eight fermionic zero-modes d_0^i

$$\{d_0^\mu, d_0^\nu\} = \eta^{\mu\nu}$$

$\Rightarrow \quad d_0^\mu = \frac{1}{\sqrt{2}}\Gamma^\mu$
show: they form a spinor representation

• Therefore redefine

$$\begin{aligned} D_1 &= d_0^2 + id_0^3 && (42)\\ D_2 &= d_0^4 + id_0^5 && (43)\\ D_3 &= d_0^6 + id_0^7 && (44)\\ D_4 &= d_0^8 + id_0^9. && (45) \end{aligned}$$

then

$$\{D_I, D_I^\dagger\} = 2 \tag{46}$$

all other anti-commutators vanish

• system of 4 creation and annihilation operators

take a state, which is annihilated by all D_I:

$$D_I|----\rangle = 0 \quad \forall I$$

and

$$D_3^\dagger|----\rangle = |--+-\rangle$$

$$D_3^\dagger|--+-\rangle = D_3^\dagger D_3^\dagger|----\rangle = 0$$

- system of the $D_I, D_I^\dagger$ can be represented in a Hilbert space of dimension $2^4 = 16$

- due to $D_1 = (d_0^2 + i d_0^3) = \frac{1}{\sqrt{2}}(\Gamma^2 + i\Gamma^3), \ldots$
  the representation of D_I is given by the representation of Γ^μ
  generator of the representation: $\Sigma^{\mu\nu} = \frac{i}{4}[\Gamma^\mu, \Gamma^\nu]$

  $\Rightarrow$ spinor representation of $SO(8)$ (16-component spinor)

- this method to construct spinor representations can be generalized to $SO(2n)$

- vacuum is a Majorana-spinor of $SO(10)$
  $(\frac{1}{2} \cdot 2^{\frac{10}{2}} = 16$ components)

- GSO-projection:
  $$(-1)^F := 2^4 d_0^2 d_0^3 d_0^4 d_0^5 d_0^6 d_0^7 d_0^8 d_0^9 (-1)^{\tilde{N}} \tag{47}$$
  with $\tilde{N} = \sum_{n>0} d_{-n}^i d_n^i$
  then
  $$P_{GSO}^\pm := \frac{1 \pm (-1)^F}{2} \tag{48}$$

only even or odd chirality states survive the projection
$\Rightarrow$ Majorana spinor becomes a Majorana-Weyl spinor

- same for left-movers

- combine right- and left-movers $\Rightarrow$ two possibilities:
  - different sign: type IIA strings
  - same sign: type IIB strings

- tensor right and left movers together $\Rightarrow$ four possibilities:

NSNS, NSR, RNS, RR

- NSNS: bosons

- NSR: lowest state $b^i_{-\frac{1}{2}}|k\rangle u_\alpha$
  $\Rightarrow$ decomposes to 8-dim. representation and 56-dim. representation
  (dilatino, gravitino)

12 Summary

- generalized the action
- found supersymmetry
- reduced number of free fields (constraints)
- solutions of the equations of motion
- quantization
- critical dimension $d = 10$
- projected tachyon out of the spectrum
- fermions in the spectrum
  (spinor-states) !!