UNIVERSITÄT BONN
Physikalisches Institut

Neutrino Masses in R-Parity Violating Supersymmetry

by

Christoph Luhn

The possibility of generating neutrino masses is examined in this thesis. Considering a supersymmetric theory, Majorana type neutrino masses can be obtained most naturally by allowing for R-parity violating couplings. The brief discussion of the tree level neutrino mass is followed by a thorough analysis of the one loop contributions to the neutrino mass matrix. The resulting mass spectra are shown for different phenomenological scenarios.

Post address:
Nussallee 12
53115 Bonn
Germany

BONN-IB-2002-10
Bonn University
July 2002

UNIVERSITÄT BONN
Physikalisches Institut

Neutrino Masses in R-Parity Violating Supersymmetry

by

Christoph Luhn

Dieser Forschungsbericht wurde als Diplomarbeit von der Mathematisch - Naturwissenschaftlichen Fakultät der Universität Bonn angenommen.

Angenommen am: 10. Mai 2002
Referent: Prof. Dr. Herbert K. Dreiner
Korreferent: Prof. Dr. Hans-Peter Nilles

Zu aller erst möchte ich mich bei Prof. Herbert Dreiner bedanken. Unter seiner Betreuung hatte ich die Möglichkeit, mich mit einem interessanten Gebiet der theoretischen Physik zu beschäftigen. Seine Tipps haben mein Verständnis vergrößert und mein Vorankommen unterstützt. Desweiteren möchte ich mich bei meinem Koreferenten Prof. Hans-Peter Nilles bedanken, dessen Vorlesung in mir das Interesse für Teilchenphysik weckte. Die durch ihn zur Verfügung gestellten finanziellen Mittel ermöglichten mir im Verlaufe des vergangenen Jahres die Teilnahme an einer Tagung und einer Sommerschule.

Allen Institutsangehörigen, die mich während meiner Diplomarbeit unterstützt haben, gilt mein herzlicher Dank; insbesondere Pascal Vaudrevange und Marc Thormeier, die meine Fragen stets geduldig ertragen haben. Zudem war Pascal immer zur Stelle, wenn ich im Kampf mit dem Computer keinen Rat mehr wusste.

Durch die konstruktive Kritik und die nützlichen Anmerkungen der Korrekturleser Pascal Vaudrevange, Marc Thormeier und Rolf Kreyer, konnten viele inhaltliche sowie sprachliche Unklarheiten und Fehler aus dieser Arbeit beseitigt werden. Vielen Dank!

Schließlich danke ich von Herzen meinen Eltern, meinen Geschwistern und allen meinen Freunden. Ihre Liebe und Zuneigung waren mir gerade im letzten Jahr unheimlich wichtig.

Contents

Overview

Neutrino masses can be generated within the framework of the R-parity violating MSSM. After presenting the experimental situation, a short introduction to the theoretical background will be given. The brief discussion of the tree level neutrino mass will be followed by a thorough analysis of the one loop contributions to the neutrino mass matrix. The resulting neutrino mass spectra will be shown for different phenomenological scenarios.

Chapter 1

Introduction

Up to the present day the neutrino and its physics have given rise to many questions and speculations [1, 2, 3]. Although much effort has been made in this field and some brilliant ideas and concepts have been developed, the nature of neutrinos still remains a puzzle and a challenge for today's physicists.

The neutrino appeared on the stage of history in 1930 when Pauli suggested the introduction of a neutral spin one-half particle [4] in order to explain the continuous energy spectrum of beta decay [5]. Assuming the neutrino's existence, Fermi formulated a theory of beta decay [6] long before the neutrino was actually discovered in the experiments by Cowan and Reines in 1956 [7, 8]. A decade later Glashow, Weinberg and Salam [9, 10, 11] implemented the neutrino and other particles into a $SU(2)_L \times U(1)_Y$ gauge field theory which became the basis of the standard model of particle physics. Within this theory the neutrino holds a special status since it is the only fermion without a right chiral spinor component. This fact implies that the neutrino cannot have a so-called Dirac mass. In order to understand this point one should have a look at the generic Lagrangian density for a free Dirac particle[1]

$$\mathcal{L}_{\mathrm{D}} = \overline{\psi_{\mathrm{D}}}(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi_{\mathrm{D}}. \tag{1.1}$$

Working in the chiral basis[2] the Dirac spinor is given by $\psi_{\mathrm{D}} = \begin{pmatrix} \psi_{\mathrm{L}} \\ \psi_{\mathrm{R}} \end{pmatrix}$ and the mass term can be written as

$$-\mathcal{L}_{\mathrm{mass}} = m(\bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}} + \bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}}). \tag{1.2}$$

Hence, it is evident that without any right chiral spinor component there is no way of giving neutrinos a Dirac mass.

It is possible to think of an alternative type of mass term, namely the Majorana mass. Here the right chiral part of the spinor is not independent of the left chiral one. Actually the two

[1] For notations and other conventions concerning Weyl and Dirac spinors see Appendix A.

[2] Throughout this text, four component spinors are written in the chiral basis.

are related through charge conjugation, denoted by an upper index c. Thus the Majorana mass term for a left chiral spinor[3] $\psi_{\mathrm{D}} = \begin{pmatrix} \psi \\ 0 \end{pmatrix}$ takes the form[4]

$$-\mathcal{L}_{\mathrm{mass}} = \frac{1}{2} m \overline{\psi_{\mathrm{D}}^{c}} \psi_{\mathrm{D}} + \frac{1}{2} m^{*} \overline{\psi_{\mathrm{D}}} \psi_{\mathrm{D}}^{c}. \tag{1.3}$$

Since the standard model does not contain right chiral neutrinos it does not have Dirac masses for neutrinos. Also Majorana masses are ruled out in this theory due to gauge invariance and renormalizability: A $SU(2)_L \times U(1)_Y$ gauge invariant term generating a Majorana mass for the neutrino is automatically non-renormalizable since two lepton doublets have to be contracted with two Higgs doublets leading to a dimension five operator in the Lagrangian density. Thus, a Majorana mass for the neutrino can only appear in an extension of the standard model. However, lepton number, which is conserved in the standard model, has to be violated by two units as can be seen in the diagram of Fig.1.1.

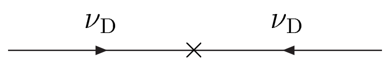


Figure 1.1: Diagram of a Majorana mass term

In order to complete the subject of neutrinos in the standard model it should be mentioned that there are three generations of neutrinos: the electron, the muon and the tau neutrino. For each of them the above considerations hold. So our theoretical understanding of particle physics based on the standard model predicts massless neutrinos. They have neither Dirac nor Majorana mass.

Up to now, direct mass measurements like the Mainz Neutrino Mass Experiment [12] only set upper limits on the neutrino masses[5]. Experiments of this kind, in which beta decay spectra are examined carefully near their maximum energy, are difficult to carry out. Their success depends crucially on the accuracy of the measurement as the spectra of the massive neutrino scenario and those resulting from the standard model, i.e. massless neutrinos, differ only very slightly. So far these experiments do not provide any evidence for non-zero neutrino masses.

However, oscillation experiments strongly suggest the existence of neutrino masses. The basic idea is quite simple: Take a well known neutrino source, place a detector at some distance of it, measure the neutrino flux and compare the data with the expected values. If, for example, a deficit of neutrinos is observed, some of them must have "oscillated away".

[3]The index D indicates that the spinor has four components and satisfies the Dirac equation. Imposing additional constraints on such a spinor reduces its degrees of freedom. This however, will *not* be expressed notationally in this text. So a four component left chiral spinor can be denoted by ψ_{D}. Also Majorana spinors are written with and index D!

[4]In general, the Majorana mass m will be complex. However its phase can be absorbed by a redefinition of the fields.

[5]The numerical values are given in Chapter 11. Better results are expected from KATRIN [13].

In the early 1970's Ray Davis was the first to study the flux of electron neutrinos produced in the sun's nuclear fusion reactions. With a chlorine detector, located 4,850 feet below the surface in the Homestake Mine, Davis and his collaborators measured less neutrinos than the standard solar model would predict [14, 15]. Many years later, other experiments followed: Kamiokande [16], SAGE [17], GALLEX [18], SNO [19] - all giving the same result: On their way from the sun's core to the earth a fairly high fraction of neutrinos seems to disappear. Moreover, the suppression ratios are different for all experiments due to the fact that the detectors are sensitive to different parts of the neutrino energy spectrum. These data have been compared with variations of the standard solar model but there has not been found a way to explain all results simultaneously [2]. By now, it is therefore a widespread opinion that the solar neutrino puzzle must find its solution in the nature of neutrinos themselves.

Our atmosphere provides another source for neutrinos. The protons of the cosmic rays interact with the upper atmosphere's nuclei producing pions which in turn decay into muons and electrons, thereby emitting two muon and one electron type neutrino per pion. Experimentalists measured this ratio and since the early 1990's it has been known that there is a distinct deficit in the ratio $\frac{\nu_\mu}{\nu_e}$ which cannot be explained within the standard model. But it was not until 1998 that the discussion of the atmospheric neutrino puzzle was revived. In the summer of that year the SuperKamiokande collaboration reported an asymmetry in the neutrino flux depending on the direction of incidence [20]. They measured more events for muon neutrinos traveling downwards than in the opposite direction. This result, fortifying the problem of atmospheric neutrinos, again shows that the neutrino is theoretically not well understood.

In order to reveal the nature of the neutrino, other experiments, including also neutrino beams from accelerators, have been performed and are still going on to collect more data. So far, only the LSND experiment [21] provides hints for non-zero neutrino mass. But as other experiments have not confirmed or excluded these results, it is difficult to make strong statements at this stage.

The prevailing explanation for the experimental situation illustrated briefly above is the concept of neutrino oscillations. For this, the first thing to accept is the possibility of neutrinos having a mass. Then one has to assume that the interaction eigenstates do not correspond to the mass eigenstates. In the quark sector this fact is represented by the CKM matrix. Since neutrinos are produced in weak processes, they come into existence as weak eigenstate $|\nu_l\rangle$, where l is a generation index. This can be written as a superposition of mass eigenstates $|\nu'_j\rangle$ with masses m_j

$$|\nu_l\rangle = \sum_j U_{lj} |\nu'_j\rangle, \tag{1.4}$$

where U is the unitary mixing matrix[6]. Considering the case with fixed 3-momentum[7] $\mathbf{p}$ the time evolution of the initial state is given by

$$|\nu_l(t)\rangle = \sum_j e^{-\mathrm{i}\sqrt{\mathbf{p}^2+m_j^2}\,t} U_{lj} |\nu'_j\rangle. \tag{1.5}$$

[6] The unitarity of U assures that the transformation diagonalizes the symmetric neutrino mass matrix without changing the kinetic terms.

[7] A discussion of the more general case, i.e. the wave packet treatment, can be found in [2, 22]

It should be noted that, in general, the three mass eigenstates of the emitted neutrino propagate differently. The process of measuring the particle is governed by the weak eigenstates again. So starting with an electron neutrino, a detector at some distance, which corresponds to a certain time elapsed, would count less electron neutrinos. The reason for this is simply that the particle's composition out of mass eigenstates has changed. This change can be expressed as a conversion of electron into muon or tau neutrinos. The amplitude for finding $|\nu_{l'}\rangle$ in the original $|\nu_l\rangle$ beam is given by

$$\begin{aligned} \langle \nu_{l'} | \nu_l(t) \rangle &= \sum_{j'j} \langle \nu'_{j'} | U^\dagger_{j'l'} e^{-\mathrm{i}\sqrt{\mathbf{p^2}+m_j^2}\,t} U_{lj} | \nu'_j \rangle \\ &= \sum_j e^{-\mathrm{i}\sqrt{\mathbf{p^2}+m_j^2}\,t} U_{lj} U^*_{l'j}. \end{aligned} \tag{1.6}$$

Considering neutrinos traveling through matter, there is a subtle but important additional effect, which was first discussed by Mikheev, Smirnov and Wolfenstein (MSW) [23]. They realized that one has to treat muon and tau type neutrinos differently from the electron type as matter is only constituted out of particles of the first generation, i.e. electrons and up/down quarks. Thus electron neutrinos can interact in matter also through the charged current interaction with the electrons, whereas muon and tau neutrinos cannot. This gives a different index of refraction for the electron neutrino which alters the pure vacuum oscillations of Eq.(1.6).

As the evidence for neutrino oscillations grows and the concept becomes more settled, the question of the neutrino's mass must be answered also theoretically. It has been mentioned before that there are two types of fermionic mass terms. This fact suggests two different ways for neutrinos to obtain masses.

The first idea is to introduce a new particle, the right chiral neutrino $N_\mathrm{D} = \begin{pmatrix} 0 \\ N \end{pmatrix}$. This concept, in which a Dirac mass term would be possible, is employed by the seesaw mechanism [24]. In addition to the Dirac mass m also a Majorana type mass M for the right chiral neutrino is allowed, whereas the left chiral one cannot acquire a mass due to gauge invariance and renormalizability. Thus the neutrino mass term of the Lagrangian density takes the form[8]

$$-\mathcal{L}_\mathrm{mass} = \frac{1}{2} \begin{pmatrix} \overline{\nu_\mathrm{D}} & \overline{N^c_\mathrm{D}} \end{pmatrix} \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu^c_\mathrm{D} \\ N_\mathrm{D} \end{pmatrix} + \mathrm{h.c.}, \tag{1.7}$$

where $\nu_\mathrm{D} = \begin{pmatrix} \nu \\ 0 \end{pmatrix}$. Assuming $m \ll M$, a biunitary transformation[9] will generate two Majorana particles with masses

$$m_1 \approx \frac{m^2}{M} \quad , \quad m_2 \approx M. \tag{1.8}$$

The light one would be interpreted as the standard model neutrino which detectors measure. The heavy neutrino has not been observed so far because, in this theory, its mass is far beyond the energy range of existing accelerators.

[8] A short discussion of Majorana and Dirac masses is given in Chapter 2.

[9] See Appendix C. Since the mass matrix of Eq.(1.7) is symmetric, the two unitary matrices of the biunitary transformation are identical.

The other possibility for neutrinos gaining a mass is achieved without an enlargement of the particle content but allowing for lepton number violation through a Majorana mass term for the left chiral neutrino. In the framework of the minimal supersymmetric extension of the standard model (MSSM) this can be readily done by introducing R-parity violating terms [25]. I will pursue this concept in the following.

However, first a closer look at the difference between Dirac and Majorana masses in the context of neutrino oscillations needs to be taken in the next chapter.

Chapter 2

Dirac and Majorana Masses

First I will pedagogically introduce the notion of Majorana mass as a generalization of Dirac mass. Then an amusing fact, the connection between one Dirac and two Majorana particles, will be presented. This will be useful for understanding how Majorana particles oscillate, compared to the more familiar Dirac particles.

2.1 Majorana Masses

For a Dirac spinor $\psi_{\rm D} = \begin{pmatrix} \psi \\ \chi \end{pmatrix}$, where ψ is a left chiral Weyl spinor and χ a right chiral one, the mass term in the Lagrangian density is given by

$$-\mathcal{L}_{\rm mass} = m\overline{\psi_{\rm D}}\psi_{\rm D}. \tag{2.1}$$

Defining the left and right chiral Dirac spinors $L_{\rm D} \equiv P_{\rm L}\psi_{\rm D}$ and $R_{\rm D} \equiv P_{\rm R}\psi_{\rm D}$, this then takes the form

$$-\mathcal{L}_{\rm mass} = m\overline{L_{\rm D}}R_{\rm D} + \text{h.c.}, \tag{2.2}$$

which, by the use of the simple relation $\overline{L_{\rm D}}R_{\rm D} = \overline{R^c_{\rm D}}L^c_{\rm D}$, can be written as a matrix equation[1]

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}\begin{pmatrix} \overline{L_{\rm D}} & \overline{R^c_{\rm D}} \end{pmatrix}\begin{pmatrix} & m \\ m & \end{pmatrix}\begin{pmatrix} L^c_{\rm D} \\ R_{\rm D} \end{pmatrix} + \text{h.c.}. \tag{2.3}$$

Note that on the right-hand side of the mass matrix there are only right chiral spinors, since $L^c_{\rm D} = \begin{pmatrix} 0 \\ \bar{\psi} \end{pmatrix}$, whereas on the left solely left chiral spinors appear. In the above form the difference between the two types of masses can be seen explicitely: Dirac masses connect left and right chiral fields which are independent of one another. Majorana masses on the other hand result from non-zero entries on the diagonal of the mass matrix, thus coupling a chiral field to its charge conjugate[2].

[1]Zero-entries of a matrix are omitted here and in the following.
[2]Compare with Eq.(1.7).

2.2 From Dirac to Majorana Fields

One might ask the question: Is it possible to rewrite the Lagrangian density of a Dirac particle such that one obtains Majorana mass terms? The answer is yes. First it is necessary to apply a biunitary transformation in order to diagonalize the mass matrix of Eq.(2.3). This will mix the right chiral fields independently from the left chiral ones on the other side of the matrix. Thus, four new fields are defined

$$\begin{pmatrix} \phi_{\mathrm{D}}^{r1} \\ \phi_{\mathrm{D}}^{r2} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} L_{\mathrm{D}}^{c} \\ R_{\mathrm{D}} \end{pmatrix}, \tag{2.4}$$

$$\begin{pmatrix} \phi_{\mathrm{D}}^{l1} \\ \phi_{\mathrm{D}}^{l2} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} L_{\mathrm{D}} \\ R_{\mathrm{D}}^{c} \end{pmatrix}. \tag{2.5}$$

In the new basis the mass term of the Lagrangian density is given by

$$-\mathcal{L}_{\mathrm{mass}} = \frac{1}{2} \begin{pmatrix} \overline{\phi_{\mathrm{D}}^{l1}} & \overline{\phi_{\mathrm{D}}^{l2}} \end{pmatrix} \begin{pmatrix} m & \\ & m \end{pmatrix} \begin{pmatrix} \phi_{\mathrm{D}}^{r1} \\ \phi_{\mathrm{D}}^{r2} \end{pmatrix} + \text{h.c.}. \tag{2.6}$$

Introducing

$$\phi_{\mathrm{D}}^{k} \equiv \phi_{\mathrm{D}}^{rk} + \phi_{\mathrm{D}}^{lk}, \tag{2.7}$$

with $k = 1, 2$, as the combination of right and left chiral fields one can rewrite Eq.(2.6)

$$-\mathcal{L}_{\mathrm{mass}} = \frac{1}{2} m \overline{\phi_{\mathrm{D}}^{k}} P_{\mathrm{R}} \phi_{\mathrm{D}}^{k} + \text{h.c.} = \frac{1}{2} m \overline{\phi_{\mathrm{D}}^{k}} \phi_{\mathrm{D}}^{k}. \tag{2.8}$$

Similarly, the kinetic term must be modified. The resulting Lagrangian density then reads

$$\mathcal{L}_{\mathrm{D}} = \frac{1}{2} \overline{\phi_{\mathrm{D}}^{k}} (\mathrm{i}\gamma^{\mu}\partial_{\mu} - m) \phi_{\mathrm{D}}^{k}, \tag{2.9}$$

which can interpreted as the expression for two fields obeying[3]

$$(\phi_{\mathrm{D}}^{k})^{c} = \mp \phi_{\mathrm{D}}^{k}, \tag{2.10}$$

where the sign depends on k. These fields, for which $\phi_{\mathrm{D}}^{k} = \begin{pmatrix} \varphi^{k} \\ \mp\bar{\varphi}^{k} \end{pmatrix}$, are called Majorana particles because they carry Majorana type masses.
It follows that a Dirac particle can be thought of as two Majorana particles with the same mass.

[3]This is a direct consequence of definitions (2.4), (2.5) and (2.7).

2.3 Oscillations

In the last section it was shown that one Dirac particle can be regarded as a composition out of two Majorana fields with the same mass. Oscillations of a Dirac particle will therefore correspond to two Majorana particles oscillating. The question arises: How does a Majorana particle oscillate? Actually, the brief derivation given in the introduction still holds for Majorana particles since the time evolution of any one-particle quantum state Ψ_p, being an eigenvector of the four-momentum p, is given by the Hamiltonian [26] which corresponds to the particle's energy $E_{\mathbf{p}}$. Thus Ψ_p evolves in time as

$$\Psi_p \quad \longrightarrow \quad e^{-\mathrm{i}E_{\mathbf{p}}t}\Psi_p, \tag{2.11}$$

with the relativistic dispersion relation

$$E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}. \tag{2.12}$$

In general, the mass m of Majorana particles is complex valued. But its phase must not change physics because, as mentioned before, it can be hidden by a redefinition of fields. So the mass m in Eq.(2.12) has to be replaced by $|m|$. Alternatively and more formally one can solve the equation of motion for a Majorana field leading to the same result.

All in all, Majorana particles oscillate just like other particles do. The general case for oscillations of spin one-half particles is Majorana type. Two Majorana fields with equal mass have the same oscillation frequency. Hence, it is possible to combine both fields into one Dirac particle.

Chapter 3

Supersymmetry

Returning to the suggested framework of introducing Majorana masses, I will motivate the consideration of low energy supersymmetry and supersymmetry in general. Then the main aspects of the mathematical treatment of supersymmetry in standard gauge field theories will be presented in the superfield formalism.

3.1 Why Supersymmetry?

The standard model successfully describes all phenomena of particle physics accessible to existing accelerators. Yet, the standard model is usually understood as a low-energy approximation to a more fundamental theory in which gravitation is unified with the strong and the electroweak interactions at a high energy scale of about 10^{16} to 10^{18} GeV [27]. This scale, where new physics enters, raises the hierarchy problem [28, 29] which is two-fold: (1) What accounts for the enormous ratio of the fundamental high energy scale at around $10^{16} - 10^{18}$ GeV and the weak scale at around 100 GeV? (2) Why is this hierarchy stable?

The second question is sometimes called the technical hierarchy problem, which shall now be illustrated in more detail. In the standard model the weak scale is set by the quadratic coupling m_H^2 of the Higgs field which acquires a vacuum expectation value, provided m_H^2 is negative. If the standard model is valid up to any very high cutoff Λ where new physics occurs, the radiative corrections Δm_H^2 to the bare m_H^2 are of $\mathcal{O}(\Lambda^2)$, due to the quadratic divergence in the scalar self energy. A miraculously precise cancellation between m_H^2 and Δm_H^2 is required in order to obtain a vacuum expectation value of $\mathcal{O}(100\,\mathrm{GeV})$. In other words, the fundamental parameters of the microscopic world would have to be fine tuned [30, 28, 29, 31]. This scenario seems unnatural as small perturbations destroy the hierarchy, i.e. the hierarchy is unstable.

Problem (2) concerning the *stability* of the hierarchy can be solved by imposing a new symmetry[1]. This "super-symmetry" implies that each particle possesses a supersymmetric partner with equal mass, but opposite statistics. For example, the left chiral electron will have a spin zero partner particle, called the left chiral selectron[2]. The doubling of the particle spectrum

[1] An interesting comparison with another hierarchy problem can be found in [32]. There the charge conjugation symmetry solves the question of the electron being a light particle.

[2] "Chiral" in the context of scalars is merely a label and does not refer to chirality.

effects the calculation of the Higgs self energy in a way that the quadratically divergent part of a diagram with an intermediate fermion is canceled by the corresponding diagrams where the supersymmetric partners are running in the loop. Thus, the radiative corrections to m_H^2 are at most logarithmically sensitive to the high energy scale Λ and the hierarchy is stabilized. This better high energy behavior is generic to supersymmetric theories. However, if supersymmetry is realized in nature, it has to be broken, as e.g. selectrons with a mass of 0.511 MeV are not observed. Assuming that also broken supersymmetry solves the hierarchy problem, one obtains an upper bound for the scale of supersymmetry breaking, phenomenologically defined by the masses of the new particles. This can be understood by noting that the self energy correction of the Higgs(es) is proportional to the squared mass splitting between particles and their superpartners which run in the loop[3] [32, 27]. If this mass splitting was large, the hierarchy would be unstable again. Therefore, the supersymmetry breaking scale is tied to the weak scale and the supersymmetric partners of the standard model particles are to be found in the 1 TeV region [31]. This is an exciting prospect because new accelerators are being built which will be able to probe the critical energy scale. It should, however, be emphasized that supersymmetry does not answer the question about the *origin* of the hierarchy[4].

Another reason for considering supersymmetry is given by the hope of finding a theory that unifies strong and electroweak interactions. In this context, one expects the three coupling constants of $SU(3)_C \times SU(2)_L \times U(1)_Y$ to be related by $g_3^2 = g^2 = \frac{5}{3}g'^2$ at and above the unification scale [27]. To examine this hypothesis it is necessary to apply the renormalization group equations which govern the running of the gauge coupling constants, expressed by so-called beta functions, from their measured values at low energies to higher energies. The calculation within the standard model shows no unification. But taking supersymmetry into account, the supersymmetric particles have to be included in the calculations of the beta functions, leading to one single gauge coupling constant at around 10^{16}GeV [36, 37, 27]. As these calculations assume a supersymmetry breaking scale of order 1 TeV, this grand unification might be regarded an independent hint for low energy supersymmetry besides the hierarchy problem.

To a large extent the persistent belief in supersymmetry is based on the fact that the form of supersymmetry algebra is almost uniquely fixed. Haag, Lopuszanski and Sohnius proved that of all graded Lie algebras only the supersymmetry algebra generates symmetries of the S-matrix consistent with a non-trivial relativistic quantum field theory [38]. This result is an extension of the Coleman-Mandula theorem [39].

In addition to the motivations for supersymmetry given above, it might well be that the astrophysical question of cold dark matter will find its solution in supersymmetry as well. The main idea is that there exists an electrically neutral and colorless stable supersymmetric particle which is responsible for missing mass in the universe. As for the electron, which is the lightest lepton and therefore does not decay if lepton number is conserved, one could assign a conserved quantum number to all superpartners (e.g. R-parity). The lightest supersymmetric particle would then automatically be stable.

---

[3] In this context, it is interesting to note that supersymmetry breaking can induce electroweak symmetry breaking by rendering $m_H^2 + \Delta m_H^2$ negative [33].

[4] Of course, there are other solutions to the hierarchy problem like Technicolor [34] and the idea of large extra spatial dimensions [35]

In conclusion, despite missing direct evidence for its realization in nature, supersymmetry is a promising candidate for physics beyond the standard model.

3.2 Ingredients of a Supersymmetric Theory

It has already been stated that supersymmetry connects particles of different spin. In other words, the generators of supersymmetry convert fermions into bosons and vice versa. Thus the needed operators must have half-integer spin and will be denoted by Q^i_A, with $A = 1, 2$ indicating the components of the left chiral spinors Q^i, enumerated by $i = 1, ..., N$. The supersymmetry algebra is given by the Poincaré algebra plus the following anticommutation and commutation relations

$$\begin{aligned} \{Q^i_A, \bar{Q}^j_{\dot{B}}\} &= 2\delta^{ij}\sigma^{\mu}_{A\dot{B}}P_{\mu} \quad , \quad \{Q^i_A, Q^j_B\} = \epsilon_{AB}Z^{ij} \quad , \\ \left[P^{\mu}, Q^i_A\right] &= 0 \qquad\qquad\quad , \quad \left[M^{\mu\nu}, Q^i_A\right] = -\mathrm{i}(\sigma^{\mu\nu})_A{}^B Q^i_B \quad , \end{aligned} \tag{3.1}$$

where the notations of Appendix A for Weyl spinors are applied. P_μ represents the 4-momentum operator, $M^{\mu\nu}$ is the generator of a Lorentz transformation and Z^{ij} are so-called central charges. Furthermore, $\sigma^{\mu\nu} \equiv \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$. As $N \geq 2$ supersymmetries are automatically “non-chiral” [40] as opposed to the experimental fact that the weak interaction treats left and right chiral fields differently, $N = 1$ supersymmetry is the only one that is thought to be relevant for low energy physics[5]. Therefore only $N = 1$ supersymmetry will be considered in the following.

Maybe the most natural setting for studying the action of supersymmetry generators on physical fields is given in the superfield formalism introduced by Salam and Strathdee [41]. Superfields, in this context, are functions of Minkowski space x^μ and four anticommuting coordinates θ_A, $\bar{\theta}_{\dot{A}}$, with $A, \dot{A} = 1, 2$. Note that the new parameters of this superspace are Weyl spinors, i.e. they behave spinor-like under Poincaré transformation. Considering a supersymmetry transformation on these superfields the generators can be represented as differential operators of the form[6]

$$Q_A = \mathrm{i}\frac{\partial}{\partial\theta^A} - (\sigma^\mu\bar{\theta})_A\frac{\partial}{\partial x^\mu} \quad , \quad \bar{Q}_{\dot{A}} = -\mathrm{i}\frac{\partial}{\partial\bar{\theta}^{\dot{A}}} + (\theta\sigma^\mu)_{\dot{A}}\frac{\partial}{\partial x^\mu}. \tag{3.2}$$

Now one can construct the covariant derivatives

$$D_A = \frac{\partial}{\partial\theta^A} - \mathrm{i}(\sigma^\mu\bar{\theta})_A\frac{\partial}{\partial x^\mu} \quad , \quad \bar{D}_{\dot{A}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{A}}} + \mathrm{i}(\theta\sigma^\mu)_{\dot{A}}\frac{\partial}{\partial x^\mu}, \tag{3.3}$$

which commute with supersymmetry transformations $e^{\mathrm{i}(\xi Q + \bar{\xi}\bar{Q})}$, where ξ is a shift in the new coordinates of superspace.

---

[5] Still $N \geq 2$ supersymmetries could be realized in nature because supersymmetry must be broken. Thus, the low energy world could appear chiral even in these cases. However, such a scenario seems rather unattractive.

[6] There has been some confusion due to different sign conventions and inconsistency in the literature. I will adopt H. Haber's generators of supersymmetry given in [42].

The covariant derivatives are useful for finding irreducible representations of the supersymmetric extension of the Poincaré group. This can be understood in analogy to the following well known statement: Elementary particles are irreducible representations of the Poincaré group. In order to obtain these, one has to consider a covariant operator, e.g. $(\partial_\mu \partial^\mu + m^2)$ in the Klein-Gordon case, and find the solutions to it. These then form the irreducible representation of the Poincaré group with spin zero and mass m. In supersymmetry, the covariant constraints are imposed on superfields. Thus three different sorts of superfields result: left chiral fields $\phi_{\rm L}$, right chiral fields $\phi_{\rm R}$ and vector superfields V which obey

$$\bar{D}_{\dot{A}}\phi_{\rm L} = 0, \quad D_A \phi_{\rm R} = 0, \quad V = V^\dagger. \tag{3.4}$$

In terms of component fields the solutions are given as

$$\phi_{\rm L}(y,\theta) = \varphi_{\rm L}(y) + \sqrt{2}\theta\psi_{\rm L}(y) + \theta\theta F_{\rm L}(y), \tag{3.5}$$

$$\phi_{\rm R}(\bar{y},\bar{\theta}) = \varphi_{\rm R}(\bar{y}) + \sqrt{2}\bar{\theta}\psi_{\rm R}(\bar{y}) + \bar{\theta}\bar{\theta} F_{\rm R}(\bar{y}), \tag{3.6}$$

$$V(x,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}A_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \tag{3.7}$$

where $\varphi_{\rm L}, \varphi_{\rm R}, F_{\rm L}, F_{\rm R}$ are complex scalar fields; $\psi_{\rm L}, \lambda$ left chiral and $\psi_{\rm R}$ right chiral complex Weyl spinors; A_μ a real vector field and D a real scalar field. The arguments y and $\bar{y}$ of the chiral superfields are a generalization of Minkowski space and defined by $y^\mu \equiv x^\mu - {\rm i}\theta\sigma^\mu\bar{\theta}$ and $\bar{y}^\mu \equiv x^\mu + {\rm i}\theta\sigma^\mu\bar{\theta}$. It can be shown that the complex conjugate of a right chiral superfield satisfies the covariant constraint for a left chiral superfield, thus complex conjugation changes the chirality of a chiral superfield. In addition, it should be mentioned that the vector superfield V of Eq.(3.7) is given in the Wess-Zumino gauge, but written in a slightly different form: Here the usual factor of $-i$ in front of the Weyl spinor λ is defined into the field[7].

The next step towards a physical theory is to formulate a supersymmetric gauge field theory [45, 46]. The matter sector of such a theory is given by chiral superfields whereas the vector superfields incorporate the gauge fields. In order to obtain a theory invariant under gauge and supersymmetry transformations[8], it is necessary to construct appropriate expressions for the Lagrangian density. The resulting terms are

$$\mathcal{L}_{\rm mat} \equiv \phi^\dagger e^{2gV^aT^a}\phi \,\Big|_{D-{\rm term}}, \tag{3.8}$$

$$\mathcal{L}_{\rm gau} \equiv \frac{1}{64}{\rm tr}(W^A W_A)\,\Big|_{F-{\rm term}} + {\rm h.c.}, \tag{3.9}$$

$$\mathcal{L}_{\rm int} \equiv W\,\Big|_{F-{\rm term}} + {\rm h.c.}. \tag{3.10}$$

[7]By this convention the gaugino will be defined directly as the Weyl spinor of the vector superfield, without a factor of $-i$ [43, 44]

[8]Poincaré invariance is included in supersymmetry.

To each generator T^a of the chosen gauge group there is one vector superfield V^a; g is the gauge coupling constant. W_A denotes the spinor type field strength superfield given by $W_A \equiv \frac{1}{2g}\bar{D}_{\dot{A}}\bar{D}^{\dot{A}}(e^{-2gV^aT^a}D_A e^{2gV^bT^b})$. Note that W_A contains the generators of the gauge group, to which the trace refers. Finally the superpotential W has been introduced which defines the non-gauge interactions of the theory and has to be a gauge invariant product of left chiral superfields. The restriction to F- and D-terms is related to Eqs.(3.5) and (3.7), filtering out the F and D component of the corresponding superfields. In that way supersymmetry invariance of the theory is ensured.

Before focusing on the explicit form of the model considered in this thesis, namely the R-parity violating MSSM, I should remark that the fields F in Eq.(3.5) and D in Eq.(3.7) can be removed from the theory as their equations of motion do not include derivatives. Thus they have no dynamics and are merely auxiliary type. Because of this fact I will rewrite the Lagrangian density in a different form. $\mathcal{L}_{\mathrm{int}}$ will be divided into a scalar and a Yukawa part; the auxiliary fields appearing in $\mathcal{L}_{\mathrm{mat}}$ and $\mathcal{L}_{\mathrm{gau}}$ are purely scalar and therefore collected together with the scalar interactions coming from the superpotential. Thus the Lagrangian density changes its form

$$\mathcal{L}_{\mathrm{mat}} + \mathcal{L}_{\mathrm{gau}} + \mathcal{L}_{\mathrm{int}} \quad \longrightarrow \quad \mathcal{L}_{\mathrm{matter}} + \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{scalar}} + \mathcal{L}_{\mathrm{Yukawa}}. \tag{3.11}$$

Further and more detailed discussions of the topic can be found abundantly in the literature, e.g. [40, 47, 48, 49].

Chapter 4

The R-Parity Violating MSSM

In this chapter, the minimal supersymmetric extension of the standard model will be presented. Starting with the particle content, the idea of R-parity will be discussed. Finally, the complete Lagrangian density will be given, which serves as basis for the rest of this diploma thesis.

4.1 Particle Content

In the last chapter I have argued that it is reasonable to consider a supersymmetric extension of the standard model [28]. Thereby one is forced to introduce a supersymmetric partner for each standard model particle. Additionally, a second Higgs doublet is required in order to obtain up and down type quark masses simultaneously[1] [43]. The minimal particle content in terms of superfields is thus given by:

Left Chiral Matter Fields				
Field	$Q_k^{r\rho} = \begin{pmatrix} u_k^\rho \\ d_k^\rho \end{pmatrix}$	$L_k^r = \begin{pmatrix} \nu_k \\ e_k \end{pmatrix}$	$H_1^r = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}$	$H_2^r = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}$
Hypercharge Y	$\frac{1}{3}$	-1	-1	1
Right Chiral Matter Fields				
Field	E_k	U_k^ρ	D_k^ρ	
Hypercharge Y	-2	$\frac{4}{3}$	$-\frac{2}{3}$	
Gauge Fields				
Gauge Group	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$	
Field	B	W^a	G^a	

Here $k = 1, 2, 3$ denotes the generations; $r = 1, 2$ labels the components of an $SU(2)_L$ doublet; $\rho = 1, 2, 3$ is the color index and a enumerates the generators of the gauge groups.

[1] Another reason for introducing a second Higgs doublet is the need of a theory without anomalies [50].

Note that L_k and H_1 have the same gauge quantum numbers and thus can be combined to $L_\alpha \equiv (H_1, L_k)$, with $\alpha = 0,..,3$. Of course, in writing L_α into the superpotential, lepton number will, in general, no longer be a conserved quantity since the Higgs doublet and the leptonic doublets will be treated identically in the theory unless additional symmetries are imposed.

The decomposition of the superfields into their component fields is defined in Eqs.(3.5), (3.6) and (3.7). For the sake of clearness in notation I will denote the standard model particles by the same letter as the superfields whereas their superpartners (sparticles) carry a tilde[2]. The auxiliary fields F and D do not correspond to physical particles as they can be eliminated by their equations of motion. Thus for the electron we obtain

$$e = \tilde{e} + \sqrt{2}\theta e + \theta\theta F_e, \tag{4.1}$$

$$E = \tilde{E} + \sqrt{2}\bar{\theta} E + \bar{\theta}\bar{\theta} F_E. \tag{4.2}$$

In contrast to Eqs.(3.5) and (3.6) the indices L, R are omitted. In order to distinguish between left and right chiral superfields the following convention is introduced. Small letters (e) denote left chiral superfields, whereas capital letters (E) symbolize right chiral ones.

The decomposition of the gauge superfields, e.g. B, will be

$$B = \theta\sigma^\mu\bar{\theta} B_\mu + \theta\theta\bar{\theta}\bar{\tilde{B}} + \bar{\theta}\bar{\theta}\theta\tilde{B} + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta} D_B, \tag{4.3}$$

where B_μ is the standard model gauge boson and $\tilde{B}$ the gaugino[3].

The gauge group of the MSSM is the same as in the standard model, i.e. $SU(3)_C \times SU(2)_L \times U(1)_Y$. The following table shows the notation.

Gauge Group	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$
Coupling Constant	g'	g	g_3
Generator	$\frac{Y}{2}$	$\frac{\sigma^a}{2}$	$\frac{\lambda^a}{2}$
Structure Constant	0	ϵ^{abc}	f^{abc}

Here Y is the hypercharge; σ^a, $a = 1, 2, 3$, are the Pauli matrices; λ^a, $a = 1,..8$, the Gell-Mann matrices. The structure constants define the Lie algebra of each gauge group; ϵ^{abc} is the totally antisymmetric tensor with $\epsilon^{123} = +1$. See [51] for the definitions of the $SU(3)_C$ generators and the corresponding structure constants.

[2]Special care has to be taken for the case $H_1 \equiv L_0$. The Higgs particle (Higgs boson) is thought of as a boson and does not carry a tilde. However written in the L_0-form the scalar field is denoted by a tilde. Thus $h_1^0 = \tilde{\nu}_0$ and $h_1^- = \tilde{e}_0$. Analogously one has for the spinor fields (higgsino) $\tilde{h}_1^0 = \nu_0$ and $\tilde{h}_1^- = e_0$.

[3]"Gaugino" is used for both two and four component spinors. $\tilde{B}$ is a left chiral Weyl spinor.

4.2 R-Parity

Apart from the particle content and the gauge symmetry it is the superpotential which fixes a global supersymmetric theory. The most general renormalizable gauge invariant superpotential with the assumed particle content is given by [52, 25, 53]

$$\begin{aligned} W &= \epsilon_{rs}\Big(- \mu_\alpha L^r_\alpha H^s_2 + \frac{1}{2}\lambda_{\alpha\beta k} L^r_\alpha L^s_\beta E^*_k + \lambda'_{\alpha ij} L^r_\alpha Q^{\rho s}_i D^{\rho *}_j - h_{ij} H^r_2 Q^{\rho s}_i U^{\rho *}_j \Big) \\ &\quad + \frac{1}{2}\lambda''_{ijk}\epsilon^{\rho\sigma\tau} U^{\rho *}_i D^{\sigma *}_j D^{\tau *}_k. \end{aligned} \tag{4.4}$$

h_{ij}, $\lambda_{\alpha\beta k}$, $\lambda'_{\alpha ij}$ and λ''_{ijk} are Yukawa couplings; μ_α are mass parameters. The coefficients $\lambda_{\alpha\beta k}$ and λ''_{ijk} are antisymmetric under the interchange of the indices $\alpha \leftrightarrow \beta$ and $j \leftrightarrow k$, respectively. As required, only the complex conjugates of the right chiral superfields (E, U, D) appear in the superpotential. In terms of component fields the superfield E of Eq.(4.2) enters in the following form

$$E^* = \tilde{E}^* + \sqrt{2}\theta\bar{E} + \theta\theta F^*_E. \tag{4.5}$$

If all operators in Eq.(4.4) were present and unsuppressed, the proton would decay rapidly [25, 54]. This, however, has not been observed [55]. The bound is so strict that the only natural way to ensure the stability of the proton lies in imposing a new symmetry which renders some couplings to zero. Comparing this situation with the standard model one would guess that baryon number B and lepton number L should be conserved quantities. Thus $\lambda_{ijk} = \lambda'_{ijk} = \lambda''_{ijk} = \mu_k = 0$, with i, j, k running from 1 to 3. The assumption of B and L conservation can be summarized in the conservation of R-parity, a discrete multiplicative symmetry defined as

$$R_P \equiv (-1)^{3B+L+2S}, \tag{4.6}$$

where S denotes the spin of the particle. By this convention, each standard model particle carries $R_P = +1$ and its superpartner has $R_P = -1$. Thus, at colliders superpartners can only be produced in pairs[4] if nature conserves R-parity. It should be emphasized that R-parity is imposed by hand to protect the proton from decaying. There are also other discrete symmetries which ensure the stability of the proton [25].

The decision between models with conserved R_P and those with R-parity violation ($\not{R}_P$) shall be made by a more fundamental theory at higher energies. Until this question is answered, one should take into consideration the idea of $\not{R}_P$, at least as a phenomenological approach. A good reason in favor of R-parity violation is the natural way in which it introduces neutrino masses. One does not have to enlarge the particle spectrum but simply allows for L-violation through $\not{R}_P$. However, as experiments do not observe L-violating interactions, the corresponding coupling constants must be small [56, 57].

[4] This also implies that the lightest supersymmetric particle (LSP) has to be stable.

4.3 The Lagrangian Density

If nature obeys supersymmetry at a fundamental level, it is obvious that supersymmetry has to be broken. Otherwise accelerators would, in addition to the standard model particles, also produce their superpartners at the same energy. In the context of the hierarchy problem, I mentioned that the supersymmetry breaking scale is expected in the TeV range. Thus the scalar partners of quarks and leptons would have masses at about this scale[5]. To obtain an effective theory with these properties one adds the so-called soft supersymmetry breaking terms to the Lagrangian density. The most general set of terms which break supersymmetry explicitely without generating unwanted quadratic divergences (soft breaking) is given by [58, 53]

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} &= (M^2_{\tilde{Q}})_{ij}\tilde{Q}^\dagger_i\tilde{Q}_j + (M^2_{\tilde{U}})_{ij}\tilde{U}^T_i\tilde{U}^*_j + (M^2_{\tilde{D}})_{ij}\tilde{D}^T_i\tilde{D}^*_j \\
&\quad + (M^2_{\tilde{L}})_{\alpha\beta}\tilde{L}^\dagger_\alpha\tilde{L}_\beta + (M^2_{\tilde{E}})_{ij}\tilde{E}_i\tilde{E}^*_j + (M^2_{H_2})H^\dagger_2 H_2 \\
&\quad - \epsilon_{rs}\Big(\mu_\alpha(B)_\alpha\tilde{L}^r_\alpha H^s_2\Big) + \text{h.c.} \\
&\quad + \epsilon_{rs}\Big(\frac{1}{2}\lambda_{\alpha\beta k}(A_E)_{\alpha\beta k}\tilde{L}^r_\alpha\tilde{L}^s_\beta\tilde{E}^*_k + \lambda'_{\alpha ij}(A_D)_{\alpha ij}\tilde{L}^r_\alpha\tilde{Q}^{\rho s}_i\tilde{D}^{\rho *}_j - h_{ij}(A_U)_{ij}H^r_2\tilde{Q}^{\rho s}_i\tilde{U}^{\rho *}_j\Big) + \text{h.c.} \\
&\quad + \epsilon^{\rho\sigma\tau}\Big(\frac{1}{2}\lambda''_{ijk}(A_{DU})_{ijk}\tilde{U}^{\rho *}_i\tilde{D}^{\sigma *}_j\tilde{D}^{\tau *}_k\Big) + \text{h.c.} \\
&\quad + \frac{1}{2}\Big(M_1\tilde{B}\tilde{B} + M_2\tilde{W}^a\tilde{W}^a + M_3\tilde{G}^a\tilde{G}^a\Big) + \text{h.c.},
\end{aligned} \tag{4.7}$$

where $(M^2_{\tilde{\dots}})$ are mass matrices of the sparticles and M_k the gaugino masses parameters. All other terms correspond to the superpotential: The A-parameters are connected to the trilinear, the B-parameters to the bilinear terms. Sometimes it is convenient to define

$$\begin{aligned}
a_{\alpha\beta k} &\equiv \lambda_{\alpha\beta k}(A_E)_{\alpha\beta k}, \quad a'_{\alpha ij} \equiv \lambda'_{\alpha ij}(A_D)_{\alpha ij}, \quad a''_{ijk} \equiv \lambda''_{ijk}(A_{DU})_{ijk}, \\
(a_U)_{ij} &\equiv h_{ij}(A_U)_{ij}, \qquad\qquad b_\alpha \equiv \mu_\alpha(B)_\alpha,
\end{aligned} \tag{4.8}$$

where repeated indices are *not* summed over.

After having explained the conceptual procedure of deriving the supersymmetric Lagrangian density and adding the soft breaking terms, we are ready to write down the effective theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{soft}}. \tag{4.9}$$

The explicit form of each part of the Lagrangian density is given in Appendix B.

[5]Concerning the gauginos it is more difficult to compare their mass to that of the gauge bosons since the $W^\pm$ and the Z bosons have masses of about 100 GeV which is comparable to the assumed supersymmetry breaking scale. In addition, it is not even possible to talk about the mass of colorless gauginos as these can mix with Higgsinos.

Chapter 5

Choice of Basis

Now that the theory is given, I will focus on the freedom of rotating superfields. After some general remarks, the choice made in this thesis is presented.

5.1 General Considerations

In the last chapter it has already been stated that the leptonic superfields L_k and the down type Higgs H_1 have identical gauge quantum numbers. Both are $SU(3)_C$ singlets and $SU(2)_L$ doublets with $U(1)_Y$ hypercharge -1. Although the Higgs does not carry lepton number it is convenient to denote all four superfields by one symbol L_α, with $\alpha = 0, 1, 2, 3$. Moreover one can define four new superfields emerging from a unitary transformation U

$$L_\alpha \quad \longrightarrow \quad U_{\alpha\alpha'} L_{\alpha'} . \tag{5.1}$$

Thus, lepton number is manifestly violated. By the redefinition of fields, the coefficients in the superpotential (4.4) have to be changed as well[1]. Thus, e.g.

$$\mu_\alpha \quad \longrightarrow \quad U^*_{\alpha\alpha'} \mu_{\alpha'} . \tag{5.2}$$

Obviously, it is possible to choose the coefficients in a form which is most suitable for the considered calculation. But once the choice is made one has to work consistently in that basis. For μ_α one often finds the convention of fixing $\mu_k = 0$, $k = 1, 2, 3$. The choice made in this thesis will be given in the next section.

Of course the rotation of superfields is not limited to L_α. Similar redefinitions can be made for Q_k, E_k, U_k and D_k, where k labels the generations.

5.2 Special Choice

In order to discuss which choice is most convenient for calculating neutrino masses, it is necessary to have a look at the spontaneous breaking of the $SU(2)_L \times U(1)_Y$ gauge symmetry. In

[1] The kinetic terms on the other hand do not change, which is the reason why one in the first place considers only a unitary transformation.

the standard model this is achieved by giving the neutral field of the Higgs doublet a vacuum expectation value (VEV). This is the way fermion masses are generated. In the supersymmetric extension there are two neutral scalar Higgses, h_1^0 and h_2^0. In addition, the superpartners of the three neutrinos, $\tilde{\nu}_k$, are also neutral and scalar. Thus, five fields can acquire vacuum expectation values altogether. These are defined as follows [53]

$$\langle h_2^0 \rangle \equiv \frac{1}{\sqrt{2}} v_u \quad , \quad \langle \tilde{\nu}_\alpha \rangle \equiv \frac{1}{\sqrt{2}} v_\alpha, \tag{5.3}$$

with $\alpha = 0, .., 3$. Agreement with the standard model requires the normalization

$$v \equiv \sqrt{|v_u|^2 + |v_\alpha|^2} = 246 \text{ GeV}. \tag{5.4}$$

In this diploma thesis the sneutrino VEVs will be rotated away by a unitary transformation, i.e. $v_k = 0$, with $k = 1, 2, 3$ [53]. This choice is compatible with the idea that lepton number should not be violated too strongly since a VEV for a sneutrino would still only be a number which therefore cannot contribute to lepton number[2]. In general, the remaining vacuum expectation values are complex, but they can be rendered real by a phase shift for both Higgs superfields. Thus, it is possible to define the angle β

$$\tan\beta \equiv \frac{v_u}{v_d}, \tag{5.5}$$

where $v_d \equiv v_0$.

Having solved the question concerning the VEVs, further choices for the other fields are made. Concentrating on the λ and λ' term of the superpotential (4.4) one notices that these are responsible for giving masses to the charged leptons and down type quarks. The relevant expressions are

$$\epsilon_{rs} \lambda_{0ij} H_1^r L_i^s E_j^* \quad , \quad \epsilon_{rs} \lambda'_{0ij} H_1^r Q_i^{\rho s} D_j^{\rho *}. \tag{5.6}$$

Regarding the coefficients as 3×3 matrices we conclude, that these fermion mass matrices can be diagonalized by biunitary transformations acting on L_i, E_j^* and Q_i, D_j^*, respectively. One convenient result of biunitary transformations is that the eigenvalues become real and non-negative[3]. One might argue that a unitary rotation has already been applied to L_α. Despite this, a second one affecting only L_k, with $k = 1, 2, 3$, is allowed as it does not change the above result of vanishing sneutrino VEVs.

The same procedure cannot be used to diagonalize the up quark mass matrix anymore since a second rotation of Q_k would be necessary, which however would effect the down quark sector. These considerations follow similar patterns as in the standard model, where the CKM matrix has to be introduced.

In conclusion, I will work in a basis with only two real vacuum expectation values, $\frac{v_u}{\sqrt{2}}$ and $\frac{v_d}{\sqrt{2}}$. Furthermore, the coefficients λ_{0ij} and λ'_{0ij}, which are connected to the masses of the charged leptons and the down type quarks, are diagonal, real and non-negative.

[2] If a sneutrino $\tilde{\nu}_k$ has a VEV $\frac{1}{\sqrt{2}} v_k$, it can be decomposed as follows: $\tilde{\nu}_k = \frac{1}{\sqrt{2}} v_k + \tilde{\nu}_k^{phys}$. $\tilde{\nu}_k$ and $\tilde{\nu}_k^{phys}$ have lepton number 1, whereas v_k has no lepton number.

[3] Biunitary transformations are discussed in more detail in Appendix C.

Chapter 6

Tree Level Neutrino Mass

The possibility of tree level neutrino mass will be described in this chapter. After giving the relevant terms, a discussion of the neutral fermion mass matrix will show that one neutrino can become massive in the theory considered.

6.1 Relevant Mass Terms

Spontaneous breaking of $SU(2)_L \times U(1)_Y \longrightarrow U(1)_{EM}$ gauge symmetry introduces the charge Q as a remaining conserved quantity, defined by

$$Q \equiv T_3 + \frac{Y}{2}, \tag{6.1}$$

where Y is the particle's hypercharge and T_3 the eigenvalue of the $SU(2)_L$ generator $\frac{\sigma^3}{2}$ acting on it. Thus, only color and electric charge are good quantum numbers and all colorless neutral spin one-half fermions can mix. In Weyl spinor notation these are

$$\psi^0_K \equiv \left(\tilde{B}\ , \quad \tilde{W}^3\ , \quad \tilde{h}^0_2\ , \quad \tilde{h}^0_1\ , \quad \nu_k\right)^T, \tag{6.2}$$

with $k = 1, 2, 3$ and $K = B, W, h_2, h_1, k$. So ψ^0 is defined as a vector of seven neutral Weyl spinors. Note that the index K (capital letter) includes the neutral gauginos and higgsinos, together called neutralinos, whereas k (small letter) symbolizes the neutrinos only[1].

The mass terms can be read off the Lagrangian density[2] after gauge symmetry breaking. All relevant terms are listed in the following

$$\begin{aligned}
&\frac{g'}{\sqrt{2}}(\tilde{L}^\dagger_\alpha \tilde{B} L_\alpha - H^\dagger_2 \tilde{B} \tilde{H}_2) + \text{h.c.} \\
&-\frac{g}{\sqrt{2}}(\tilde{L}^\dagger_\alpha \sigma^3 \tilde{W}^3 L_\alpha + H^\dagger_2 \sigma^3 \tilde{W}^3 \tilde{H}_2) + \text{h.c.} \\
&+\Big(\epsilon_{rs}\mu_\alpha (L^r_\alpha \tilde{H}^s_2) - \frac{1}{2}(M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}^3 \tilde{W}^3)\Big) + \text{h.c.}.
\end{aligned} \tag{6.3}$$

[1]Remember this convention for the upcoming chapters: The extension of a generation index is expressed by a capital letter.

[2]See Appendix B.

Introducing the VEVs[3] of Def.(5.3) leads to

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= \frac{g'}{2}(v_\alpha^* \tilde{B}\nu_\alpha - v_u^* \tilde{B}\tilde{h}_2^0) + \text{h.c.} \\ &-\frac{g}{2}(v_\alpha^* \tilde{W}^3\nu_\alpha - v_u^* \tilde{W}^3\tilde{h}_2^0) + \text{h.c.} \\ &+\Big(\mu_\alpha\nu_\alpha\tilde{h}_2^0 - \frac{1}{2}(M_1\tilde{B}\tilde{B} + M_2\tilde{W}^3\tilde{W}^3)\Big) + \text{h.c..} \qquad (6.4)\end{aligned}$$

6.2 Neutral Fermion Mass Matrix

Before proceeding with the explicit form of the mixing matrix, I should briefly comment on the subject of representing mass terms for 2-component Weyl and 4-component Majorana spinors. In Chapter 2 the Majorana mass term for a left chiral field $L_{\rm D} = \begin{pmatrix}\psi \\ 0\end{pmatrix}$ has been defined by

$$-\mathcal{L}_{\text{Majorana mass}} = \frac{1}{2}m\overline{L^c_{\rm D}}L_{\rm D} + \text{h.c..} \qquad (6.5)$$

This can be rewritten in terms of Weyl spinors

$$-\mathcal{L}_{\text{Majorana mass}} = \frac{1}{2}m\psi^A\psi_A + \text{h.c.,} \qquad (6.6)$$

where A is the spinor index. With the definition of a Majorana[4] spinor $S_{\rm D} \equiv \begin{pmatrix}\psi_A \\ \bar{\psi}^{\dot{A}}\end{pmatrix}$ this changes to

$$-\mathcal{L}_{\text{Majorana mass}} = \frac{1}{2}m\overline{S_{\rm D}}S_{\rm D}. \qquad (6.7)$$

The following question arises: What is the difference between chiral ($L_{\rm D}$) and Majorana ($S_{\rm D}$) type particles? The answer is quite simple but nonetheless noteworthy: It does not matter which philosophy one follows as long as one stays consistent. In the context of masses the difference between Eqs.(6.5) and (6.7) is that the hermitian conjugate is directly incorporated in the term $\overline{S_{\rm D}}S_{\rm D}$. To avoid such problems I will work in the 2-component Weyl spinor formalism, where both viewpoints meet. Thus, Eq.(6.4) can be expressed utilizing a 7×7 matrix

$$-\mathcal{L}_{\text{mass}} = \frac{1}{2}(\psi^0)^T \begin{pmatrix} M_1 & & \frac{g'}{2}v_u^* & -\frac{g'}{2}v_0^* & -\frac{g'}{2}v_l^* \\ & M_2 & -\frac{g}{2}v_u^* & \frac{g}{2}v_0^* & \frac{g}{2}v_l^* \\ \frac{g'}{2}v_u^* & -\frac{g}{2}v_u^* & & -\mu_0 & -\mu_l \\ -\frac{g'}{2}v_0^* & \frac{g}{2}v_0^* & -\mu_0 & & \\ -\frac{g'}{2}v_k^* & \frac{g}{2}v_k^* & -\mu_k & & \end{pmatrix} \psi^0 + \text{h.c.,} \qquad (6.8)$$

[3] According to the choice of basis made in the previous chapter, $v_k = 0$ for $k = 1,2,3$. However, these values will not be inserted until Eq.(6.10)

[4] Note that I do *not* follow the common notation for Majorana spinors. The index D is used to distinguish any four component spinor from a two component Weyl spinor.

where $k,l=1,2,3$. Applying the choice of basis made in Chapter 5, i.e. $v_k=0$ and v_u,$v_d\equiv v_0$ real, together with the standard model definitions

$$\tan\theta_W \equiv \frac{g'}{g} \quad , \quad m_Z \equiv \frac{gv}{2\cos\theta_W}, \tag{6.9}$$

one ends up with the neutral fermion mass matrix m_ν^{tree}

$$m_\nu^{\text{tree}} = \begin{pmatrix} M_1 & & m_Z\sin\theta_W\frac{v_u}{v} & -m_Z\sin\theta_W\frac{v_d}{v} & \\ & M_2 & -m_Z\cos\theta_W\frac{v_u}{v} & m_Z\cos\theta_W\frac{v_d}{v} & \\ m_Z\sin\theta_W\frac{v_u}{v} & -m_Z\cos\theta_W\frac{v_u}{v} & & -\mu_0 & -\mu_l \\ -m_Z\sin\theta_W\frac{v_d}{v} & m_Z\cos\theta_W\frac{v_d}{v} & -\mu_0 & & \\ & & -\mu_k & & \end{pmatrix}. \tag{6.10}$$

The mixing between neutralinos and neutrinos is controlled by the R-parity violating parameters μ_k. Since these are small[5] compared to the entries in the 4×4 neutralino block, continuity of basis transformations ensures that this mixing is very weak. Thus, the mass eigenvalues are almost the same as in the R-parity conserving model, where the neutrinos are massless. In order to extract the neutrino masses from the seven eigenvalues of the matrix (6.10) it is necessary to find three small masses. This can be achieved by calculating the characteristic polynomial of the matrix m_ν^{tree}, which has no linear and no constant contribution. Therefore two masses have to be zero. The third small mass can be computed in good approximation by dividing the coefficient of the quadratic term by the determinant of the 4×4 neutralino mass matrix. The first is the product of the five non-zero eigenvalues of m_ν^{tree}, the latter yields the product of the four neutralino masses in the R-parity conserving theory. Thus, we can conclude, that two neutrinos are massless whereas one acquires a mass of [59]

$$\frac{m_Z^2 M_{\tilde\gamma}\cos^2\beta}{m_Z^2\mu_0 M_{\tilde\gamma}\sin 2\beta - M_1 M_2\mu_0^2}\sum_k \mu_k^2, \tag{6.11}$$

with $M_{\tilde\gamma}\equiv M_1\cos^2\theta_W + M_2\sin^2\theta_W$. Note that this value can be complex. In conclusion, due to mixing with the neutralinos one neutrino becomes massive at tree level if $\sum_k \mu_k^2 \neq 0$.

[5] The upper bounds on the bilinear R-parity violating parameters μ_k are of order MeV [57], whereas the lower bounds on the masses of the neutralinos are at the GeV scale [55].

Chapter 7

One Loop Neutrino Mass

Radiative corrections contribute to the neutrino mass as well. In this context, the impossibility of having a Dirac mass for neutrinos will be considered more carefully, showing that the former result still holds. Then I will focus on loop diagrams generating Majorana masses. To make sure that no contribution is omitted, all necessary vertices will be derived, followed by a discussion of possible combinations to obtain loop diagrams. Finally, the connection between the matrix element of such a graph and a Majorana mass term will be established.

7.1 Majorana Masses for Neutrinos

It has been argued in Chapter 1 that one cannot have a Dirac mass for a left chiral particle like the neutrino. This is due to the definition of a mass term in the Lagrangian density $m\overline{\psi_{\mathrm{D}}}\psi_{\mathrm{D}}$, which is represented diagrammatically in Fig.7.1.

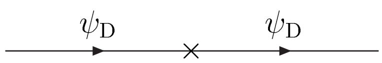


Figure 7.1: Diagram of a Dirac mass term

However, one can ask the question whether the introduction of a loop can generate Dirac mass. Fig.7.2 shows this process. The following considerations do not depend on the sort of particles in the loop. For example, it could be a neutrino and a photon, or a down quark and its superpartner. Note that the boson in the loop is symbolized by a dashed line in Fig.7.2 although it could as well be a gauge boson.

Now, there is no reason to believe that these diagrams should disappear. Actually, they give corrections to the propagator of the neutrino. Since the mass is defined as the pole of the propagator, a shift in this pole corresponds to the generation of a Dirac mass for the neutrino. However, it will be shown in the following that the propagator is corrected only multiplicatively. Therefore, the Dirac mass of the neutrino stays zero.

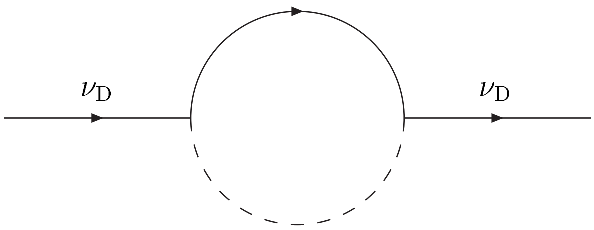


Figure 7.2: Correction to the propagator of a neutrino

In order to keep the discussion general, I regard the loop as a bilinear mapping in 4-component Dirac spinor space. This can be expressed in terms of the complete set of 4×4 matrices

$$\mathbf{1} \quad , \quad \gamma_5 \quad , \quad \gamma^\mu \quad , \quad \gamma^\mu\gamma_5 \quad , \quad \Sigma^{\mu\nu} \equiv \frac{\rm i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu), \tag{7.1}$$

where the Lorentz indices have to be contracted with the 4-momentum p^μ of the neutrino which is the only Lorentz vector occurring in the process. Thus, Lorentz invariance as a basic feature of every relativistic theory is ensured. Remembering the fact that the neutrino is a left chiral particle, the projection operator $P_{\rm L}$ has to be inserted for the incoming and $P_{\rm R}$ for the outgoing neutrino. The most general loop correction to the propagator of the neutrino then takes the form

$$\frac{\not p}{p^2} P_{\rm R}(c_1\mathbf{1} + c_2\gamma_5 + c_3\not p + c_4\not p\gamma_5 + c_5[\not p, \not p]) P_{\rm L}\frac{\not p}{p^2}. \tag{7.2}$$

Here c_i, $i = 1, .., 5$, are functions of p^2; $\not p \equiv p_\mu\gamma^\mu$ and $[\ldots, \ldots]$ is the commutator. Due to the projection operators only the terms c_3 and c_4 survive. Furthermore, as $\gamma_5 = P_{\rm R} - P_{\rm L}$, these two can be combined to one term $c\not p$, where $c \equiv c_3 - c_4$. Thus, one ends up with

$$\frac{\not p}{p^2} P_{\rm R} c \not p P_{\rm L} \frac{\not p}{p^2} \quad = \quad c\, P_{\rm L}\frac{\not p}{p^2} P_{\rm R}, \tag{7.3}$$

which is just the propagator of the neutrino multiplied by a factor of $c(p^2)$. As the pole of the propagator is not shifted by a loop correction, no Dirac mass is generated for the left chiral neutrinos.

7.2 All Vertices with Incoming Neutrinos

The previous section showed that left chiral neutrinos can only have a Majorana mass. The corresponding Feynman diagram is given in Fig.7.3. Note that the arrows on both neutrino lines point towards the loop. For convenience, I will call particles with arrows pointing into the vertex *incoming* particles. Equivalently, particles are called *outgoing* if the arrows point

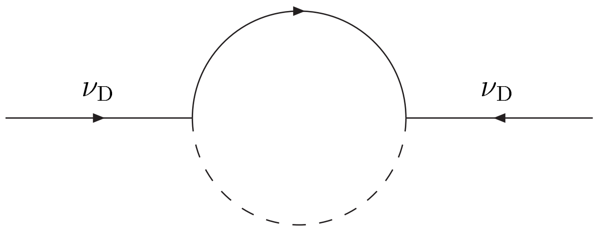


Figure 7.3: One loop contribution to Majorana mass

away from the vertex. For reasons of systematicity, I first extract all tree level interactions that include neutrinos. The relevant terms of the Lagrangian density are

$$\begin{aligned}
&\frac{g'}{2}\bar{\nu}_k\bar{\sigma}^\mu B_\mu\nu_k - \frac{g}{2}\bar{\nu}_k\bar{\sigma}^\mu W^3_\mu\nu_k - \frac{g}{\sqrt{2}}\bar{e}_k\bar{\sigma}^\mu W^-_\mu\nu_k - \frac{g}{\sqrt{2}}\bar{\nu}_k\bar{\sigma}^\mu W^+_\mu e_k \\
&+\frac{g'}{\sqrt{2}}\tilde{\nu}^*_k\tilde{B}\nu_k - \frac{g}{\sqrt{2}}\tilde{\nu}^*_k\tilde{W}^3\nu_k - g\tilde{e}^*_k\tilde{W}^-\nu_k + \mathrm{h.c.} \\
&-\lambda_{ijk}\nu_i e_j\tilde{E}^*_k + \lambda_{ijk}\tilde{e}_i\nu_j\bar{E}_k + \mathrm{h.c.} \\
&-\lambda'_{kij}\nu_k d^\rho_i\tilde{D}^{\rho*}_j - \lambda'_{kij}\nu_k\bar{D}^\rho_j\tilde{d}^\rho_i + \mathrm{h.c.},
\end{aligned} \tag{7.4}$$

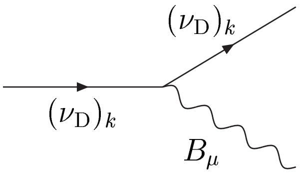
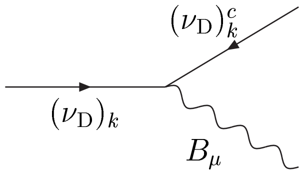
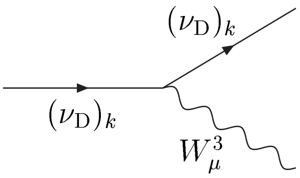
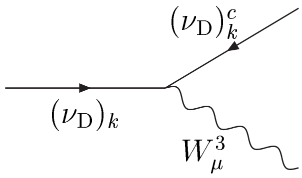
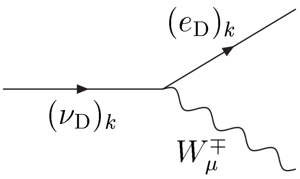
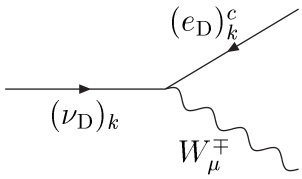
where $W^\pm \equiv \frac{1}{\sqrt{2}}(W^1 \mp \mathrm{i}W^2)$ and $\tilde{W}^\pm \equiv \frac{1}{\sqrt{2}}(\tilde{W}^1 \mp \mathrm{i}\tilde{W}^2)$. With the definition of Dirac spinors

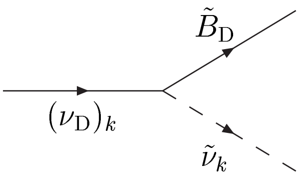
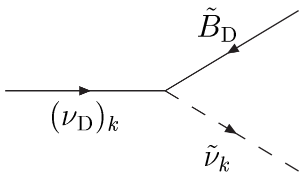
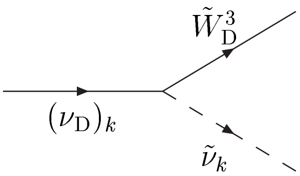
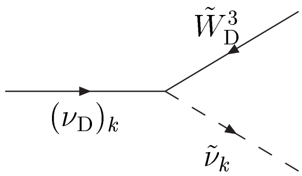
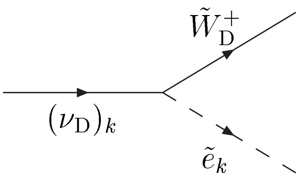
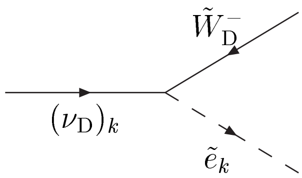
$$\begin{aligned}
&\tilde{B}_{\mathrm{D}} \equiv \begin{pmatrix}\tilde{B}\\ \bar{\tilde{B}}\end{pmatrix}, \qquad \tilde{W}^a_{\mathrm{D}} \equiv \begin{pmatrix}\tilde{W}^a\\ \bar{\tilde{W}}^a\end{pmatrix}, \qquad \tilde{W}^\pm_{\mathrm{D}} \equiv \tilde{W}^1_{\mathrm{D}} \mp \mathrm{i}\tilde{W}^2_{\mathrm{D}}, \\
&(\nu_{\mathrm{D}})_k \equiv \begin{pmatrix}\nu_k\\ 0\end{pmatrix}, \quad (e_{\mathrm{D}})_k \equiv \begin{pmatrix}e_k\\ E_k\end{pmatrix}, \quad (d_{\mathrm{D}})^\rho_k \equiv \begin{pmatrix}d^\rho_k\\ D^\rho_k\end{pmatrix},
\end{aligned} \tag{7.5}$$

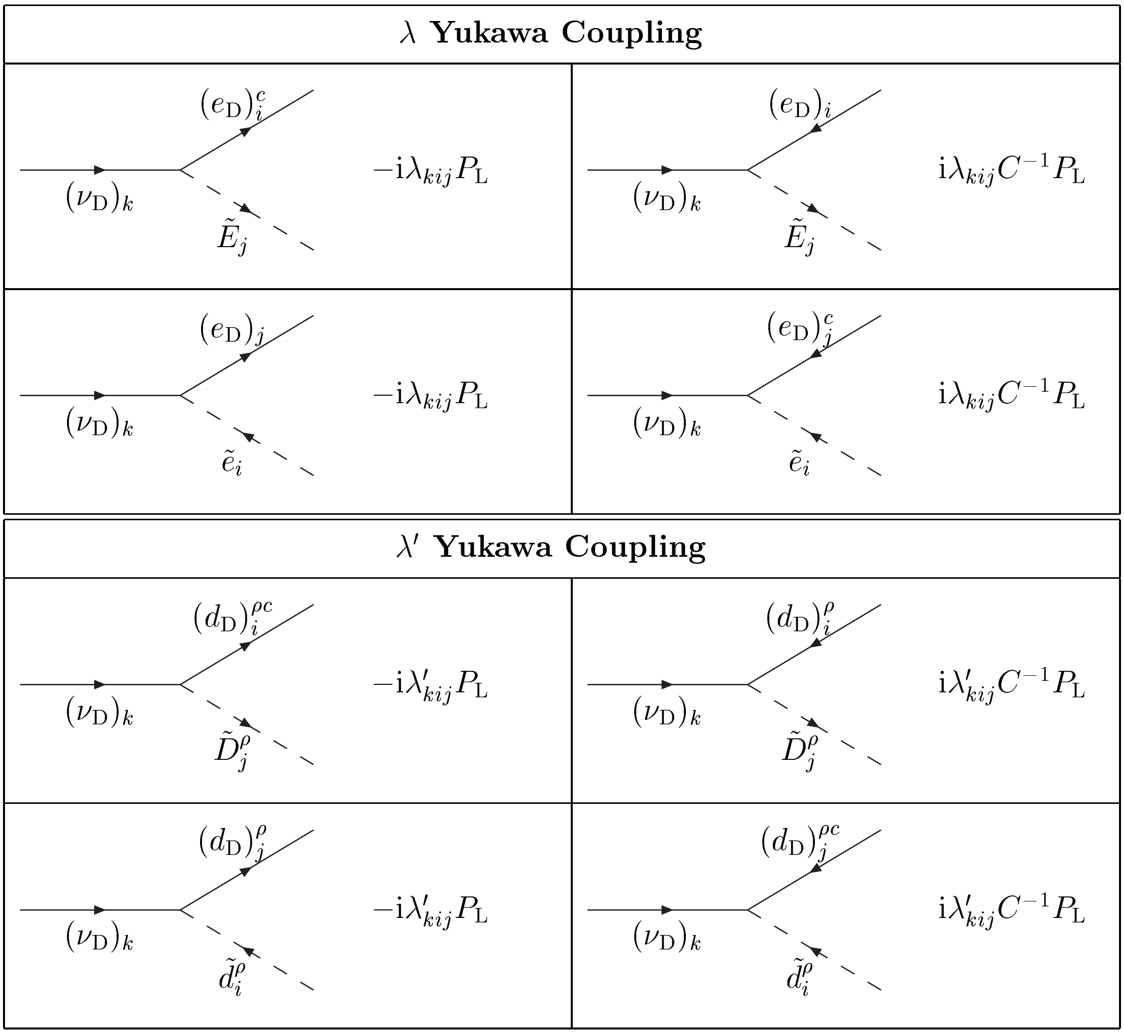
it is possible to rewrite the terms in (7.4) as

$$\begin{aligned}
&\frac{g'}{2}B_\mu\overline{(\nu_{\mathrm{D}})_k}\gamma^\mu(\nu_{\mathrm{D}})_k - \frac{g}{2}W^3_\mu\overline{(\nu_{\mathrm{D}})_k}\gamma^\mu(\nu_{\mathrm{D}})_k - \frac{g}{\sqrt{2}}W^-_\mu\overline{(e_{\mathrm{D}})_k}\gamma^\mu(\nu_{\mathrm{D}})_k - \frac{g}{\sqrt{2}}W^+_\mu\overline{(\nu_{\mathrm{D}})_k}\gamma^\mu(e_{\mathrm{D}})_k \\
&+\frac{g'}{\sqrt{2}}\tilde{\nu}^*_k\overline{\tilde{B}_{\mathrm{D}}}(\nu_{\mathrm{D}})_k - \frac{g}{\sqrt{2}}\tilde{\nu}^*_k\overline{\tilde{W}^3_{\mathrm{D}}}(\nu_{\mathrm{D}})_k - g\tilde{e}^*_k\overline{\tilde{W}^+_{\mathrm{D}}}(\nu_{\mathrm{D}})_k + \mathrm{h.c.} \\
&-\lambda_{ijk}\tilde{E}^*_k\overline{(e_{\mathrm{D}})^c_j}(\nu_{\mathrm{D}})_i - \lambda_{jik}\tilde{e}_i\overline{(e_{\mathrm{D}})_k}(\nu_{\mathrm{D}})_j + \mathrm{h.c.} \\
&-\lambda'_{kij}\tilde{D}^{\rho*}_j\overline{(d_{\mathrm{D}})^{\rho c}_i}(\nu_{\mathrm{D}})_k - \lambda'_{kij}\tilde{d}^\rho_i\overline{(d_{\mathrm{D}})^\rho_j}(\nu_{\mathrm{D}})_k + \mathrm{h.c.}.
\end{aligned} \tag{7.6}$$

In this form the vertex factors can be read off directly. Only the vertices with incoming neutrinos, i.e. ν_{D} as opposed to $\overline{\nu_{\mathrm{D}}}$, are necessary for calculating the contribution of Fig.7.3 to the Majorana mass. A list of all these diagrams is given in the following table.

Gauge Boson at the Vertex			
	$\mathrm{i}\frac{g'}{2}\gamma^\mu P_{\mathrm{L}}$		$-\mathrm{i}\frac{g'}{2}C^{-1}\gamma^\mu P_{\mathrm{L}}$
	$-\mathrm{i}\frac{g}{2}\gamma^\mu P_{\mathrm{L}}$		$\mathrm{i}\frac{g}{2}C^{-1}\gamma^\mu P_{\mathrm{L}}$
	$-\mathrm{i}\frac{g}{\sqrt{2}}\gamma^\mu P_{\mathrm{L}}$		$\mathrm{i}\frac{g}{\sqrt{2}}C^{-1}\gamma^\mu P_{\mathrm{L}}$

Gaugino at the Vertex			
	$\mathrm{i}\frac{g'}{\sqrt{2}}P_{\mathrm{L}}$		$-\mathrm{i}\frac{g'}{\sqrt{2}}C^{-1}P_{\mathrm{L}}$
	$-\mathrm{i}\frac{g}{\sqrt{2}}P_{\mathrm{L}}$		$\mathrm{i}\frac{g}{\sqrt{2}}C^{-1}P_{\mathrm{L}}$
	$-\mathrm{i}gP_{\mathrm{L}}$		$\mathrm{i}gC^{-1}P_{\mathrm{L}}$



The vertices in the right column are not independent. They are the same as those of the left side. The only difference lies in the interpretation of an outgoing fermion as its incoming antiparticle. I have listed both versions in order not to restrict myself to one viewpoint. This change, however, modifies the vertex factor by the matrix of charge conjugation. This can be seen by rewriting the definition of $\psi_{\rm D}^c$: $\overline{\psi_{\rm D}} = -(\psi_{\rm D}^c)^T C^{-1}$ and $\overline{\psi_{\rm D}^c} = -(\psi_{\rm D})^T C^{-1}$. Note that $\tilde{B}_{\rm D} = \tilde{B}_{\rm D}^c$ and $\tilde{W}_{\rm D}^3 = \tilde{W}_{\rm D}^{3c}$ are Majorana, whereas $\tilde{W}_{\rm D}^{\pm} = \tilde{W}_{\rm D}^{\mp c}$ are Dirac spinors.

7.3 Classification of Allowed Loops

The possibility of combining two vertices to one diagram like Fig.7.3 is governed by the mixing of the particles in the loop. Thus, the λ' couplings can only form loops with quarks and squarks in it. Similarly, charge conservation also provides a restriction. Taking into account these requirements, it is easy to show that there are five different types of loop diagrams contributing to the Majorana mass of the neutrino.

Fig.7.4 shows the process with a gauge boson and a neutrino in the loop. The clashing arrows on the inner fermion line correspond to a tree level Majorana mass. If there was no such mass term, this graph would be zero. However, there exists a tree level mass for one neutrino due to the mixing with the neutralinos. Still, considering two special cases, it is possible to neglect this contribution to one loop Majorana masses. Either the loop corrections are only applied to the two massless neutrinos, or one assumes the tree level mass to be smaller than the loop contributions; then one loop masses can be calculated for three neutrinos [60]. Therefore, I will not regard the possibility of a neutral gauge boson in the loop any longer.

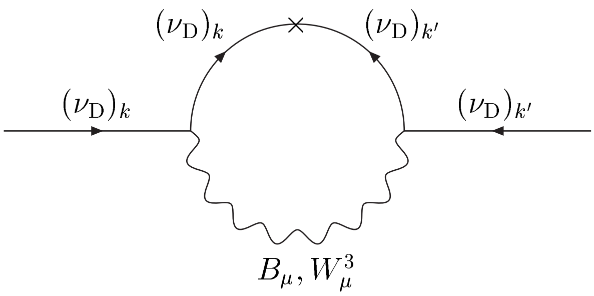


Figure 7.4: Neutral gauge boson in loop

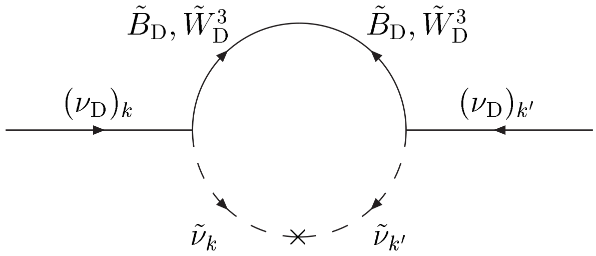
The remaining four fermion-scalar loops give finite contributions to the neutrino mass. In Fig.7.5(a) there are two gauge couplings ("gg-loop") with a neutral loop containing gauginos and sneutrinos. Note that one arrow of the fermionic propagator can be reversed as neutral gauginos are Majorana particles. The clashing arrows on the dashed scalar line signify a mixing of sneutrinos and antisneutrinos. The process depicted in Fig.7.5(b) comprises a gauge and a Yukawa coupling ("gλ-loop"). Evidently, lepton number violation occurs at the $\not{R}_P$ vertex and during the propagation of the fermion. Here the charginos and the charged leptons mix to mass eigenstates. Finally, Fig.7.5(c) and Fig.7.5(d) show loops that include only R-parity violating Yukawa couplings ("$\lambda\lambda$-" and "$\lambda'\lambda'$-loop"). They contribute to the neutrino mass provided there is a mixing between sfermions of left and right chiral origin.

All these loop corrections to the neutrino mass will be discussed and calculated in detail in the following chapters. Before continuing with this, I will briefly turn the attention to the connection between the matrix elements of a graph like Fig.7.3 and a Majorana mass term.

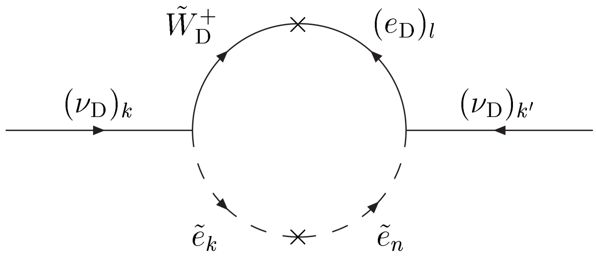
7.4 Loops and Masses

A mass term in the Lagrangian density can be regarded as an interaction of two particles. Applying the usual technique of deriving the Feynman rules [51], the Majorana mass term of the neutrino, given in Eq.(6.5), with $L_\mathrm{D} = \nu_\mathrm{D}$, leads to the vertex factor $-\mathrm{i}m$. It should be pointed out, that the factor of $\frac{1}{2}$ appearing in the Lagrangian density is omitted due to symmetry considerations[1] [26]. As there exist three generations of neutrinos, m must be replaced by the

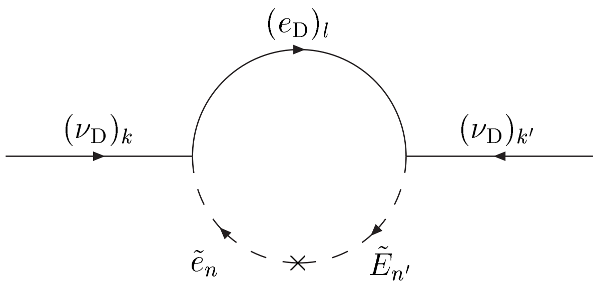
[1] Compare with Φ^4 theory, where a factor of $\frac{1}{4!}$ appears in the Lagrangian density.



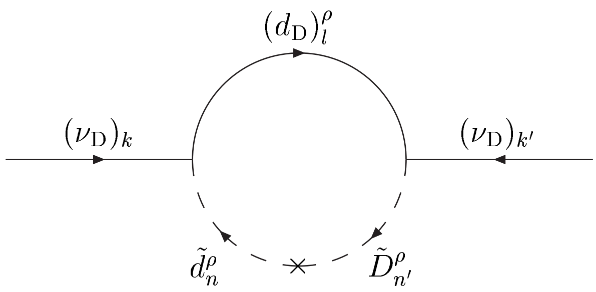
(a) gg-loop



(b) gλ-loop



(c) $\lambda\lambda$-loop



(d) $\lambda'\lambda'$-loop

Figure 7.5: Different types of fermion-scalar loops

mass matrix $m_{k'k}$, with $k, k' = 1, 2, 3$. The corresponding graph is given in Fig.7.6. Here p and p' denote the 4-momentum of the incoming and outgoing neutrino, respectively.

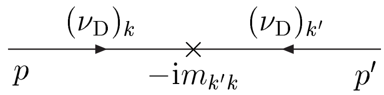


Figure 7.6: Diagram of a Majorana mass term

The calculation of the matrix element $\mathcal{M}_{k'k}$ for this process yields

$$-\mathrm{i}\mathcal{M}_{k'k} \quad = \quad \mathrm{i}\,(2\pi)^4\,\delta(p-p')\;(\nu_{\mathrm{D}})^T_{k'}\;C^{-1}\;m_{k'k}\;(\nu_{\mathrm{D}})_k. \tag{7.7}$$

Taking this expression of a formal Majorana mass term in the Lagrangian density, we can translate the matrix element $\mathcal{M}^{\mathrm{loop}}_{k'k}$ of the loop diagrams in Fig.7.5 into a one loop correction to the neutrino mass. Sometimes this relation is abbreviated by

$$m^{\mathrm{loop}}_{k'k} \quad = \quad -\mathcal{M}^{\mathrm{loop}}_{k'k}. \tag{7.8}$$

In general, the Lorentz invariant matrix element depends on p^2. In the case of neutrinos this can be set to zero as a good approximation.

One might wonder what has happened to the other mass term in Eq.(6.5), i.e. the hermitian conjugate. The answer is: Nothing, this term has not been considered so far. But it describes the process, where the arrows in Fig.7.6 are reversed. Since neutrinos with Majorana mass can be thought of as Majorana particles[2], this does not include any new information. Therefore, I will ignore the hermitian conjugates in the following.

[2]See the discussion below Eq.(6.7).

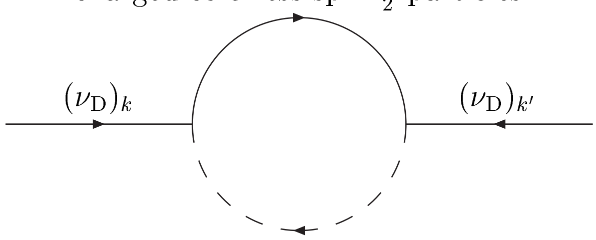
Chapter 8

The $\lambda\lambda$- and $\lambda'\lambda'$-Loops

In this chapter, the contributions of the $\lambda\lambda$- and $\lambda'\lambda'$-loops to the neutrino mass will be computed.

8.1 $\lambda\lambda$-Loop

Considering the possibility of mixing between different particles, the diagram in Fig.7.5(c) must be regarded in a more general form. After a discussion of fermionic and bosonic mixing the calculation of the loop can be tackled.



8.1.1 Charginos and Charged Leptons

In analogy to the neutralino-neutrino mixing dealt with in Chapter 6, all colorless fermions of electromagnetic charge ± 1 can be put together into two vectors $\psi^{\pm}$ of positive and negative charge, respectively. With the definition of $\tilde{W}^{\pm}$ introduced in (7.4) these are given by

$$\psi_K^+ \equiv \left(\tilde{W}^+ \; , \quad \tilde{h}_2^+ \; , \quad \bar{E}_k\right)^T \quad , \quad \psi_K^- \equiv \left(\tilde{W}^- \; , \quad \tilde{h}_1^- \; , \quad e_k\right)^T , \tag{8.1}$$

where $k = 1, 2, 3$ and $K = W, h, k$; as in the case of the neutral fermions, the capital letter K denotes an extension of the generation index k. Collecting all tree level mass terms for these fields leads to

$$
\begin{aligned}
-\mathcal{L}_{\text{mass}} &= \frac{g}{\sqrt{2}}(v_d \tilde{h}_1^- \tilde{W}^+ + v_u \tilde{W}^- \tilde{h}_2^+) + \text{h.c.} \\
&\quad + \Big(\frac{v_d}{\sqrt{2}} \lambda_{0\alpha l} e_\alpha \bar{E}_l + \mu_\alpha e_\alpha \tilde{h}_2^+ + M_2 \tilde{W}^- \tilde{W}^+ \Big) + \text{h.c..} \qquad (8.2)
\end{aligned}
$$

This can be written in the compact form

$$
-\mathcal{L}_{\text{mass}} = (\psi^-)^T m_e \psi^+ + \text{h.c.}, \qquad \text{with} \qquad m_e \equiv \begin{pmatrix} M_2 & \frac{g}{\sqrt{2}} v_u & \\ \frac{g}{\sqrt{2}} v_d & \mu_0 & \\ & \mu_k & \frac{v_d}{\sqrt{2}} \lambda_{0kl} \end{pmatrix}, \qquad (8.3)
$$

being the mass matrix for the charged leptons, gauginos and higgsinos. Again, the 2-component Weyl spinor formalism has been applied. But compared to Eq.(6.8), the factor of $\frac{1}{2}$ is missing. The reason for this is quite simple: Charged particles cannot have Majorana but only Dirac mass terms[1], which, in turn, do not carry this factor, as can be seen in Eq.(2.2).

Some comments on the structure of the mass matrix should be made. In the MSSM without R-parity violation the charged gauginos and higgsinos mix to the so-called charginos. Eq.(8.3) shows this mixing explicitely in the upper 2×2 block. Furthermore, the charged leptons mix with the charginos due to the $\not{R}_P$ parameters μ_k. As these are small compared to the chargino mass scale[2], the admixture of charginos to charged leptons and vice versa is weak. Because of this and the fact that only the leptonic part of the propagating fermion couples to both vertices of the $\lambda\lambda$-loop, mixing effects can be neglected at this stage. However, when coming to the gλ-loop, the mixing of charginos and charged leptons will play an essential role. Before turning to the scalar sector, it should be mentioned that the choice of basis made in Chapter 5 allows the interpretation of $\frac{v_d}{\sqrt{2}}\lambda_{0kl}$ as the mass of the charged leptons in the R-parity conserving theory. Thus, one can define

$$
(m_e)_k \equiv (m_e)_{kl} \equiv \frac{v_d}{\sqrt{2}} \lambda_{0kl}, \qquad (8.4)
$$

where $(m_e)_{kl}$ is diagonal and non-negative. Note that k, l are generation indices, i.e. $k, l = 1, 2, 3$.

[1]One might wonder how this statement can be in line with the decomposition of one Dirac particle into two Majorana ones presented in Chapter 2. There the proposed transformation was allowed, whereas in the gauge field theory considered now, electromagnetic charge is an additional symmetry that forbids such a transformation.

[2]See the footnote in Chapter 6 concerning the mass scale of the neutralinos. The lower bound for the mass of the charginos is also at the GeV scale [55].

8.1.2 Charged Higgs Bosons and Sleptons

In the scalar sector, mixing between charged Higgses and sleptons occurs. This is due to R-parity violating terms. The corresponding tree level masses are given by[3]

$$\begin{aligned}
-\mathcal{L}_{\rm mass} &= |\mu_\alpha|^2|h_2^+|^2 + \frac{v_d^2}{2}\lambda^*_{0\alpha k'}\lambda_{0\alpha k}\tilde{E}_{k'}\tilde{E}_k^* \\
&+\Big(\frac{v_d}{\sqrt{2}}\mu^*_\alpha\lambda_{0\alpha k}(h_2^+)^*\tilde{E}_k^* - \frac{v_u}{\sqrt{2}}\mu^*_\alpha\lambda_{\alpha\beta k}\tilde{e}_\beta\tilde{E}_k^*\Big) + \text{h.c.} \\
&+\mu_\alpha\mu^*_\beta\tilde{e}_\alpha\tilde{e}^*_\beta + \frac{v_d^2}{2}\lambda_{0\beta k}\lambda^*_{0\beta' k}\tilde{e}_\beta\tilde{e}^*_{\beta'} \\
&+\frac{1}{8}g'^2\Big((v_d^2-v_u^2)|\tilde{e}_\alpha|^2 + (v_u^2-v_d^2)|h_2^+|^2 + 2(v_u^2-v_d^2)|\tilde{E}_k|^2\Big) \\
&+\frac{1}{8}g^2\Big(2v_d^2|\tilde{e}_0|^2 + (v_u^2-v_d^2)|\tilde{e}_\alpha|^2 + (v_u^2+v_d^2)|h_2^+|^2 + 2v_uv_d\tilde{e}_0h_2^+ + 2v_uv_d(h_2^+)^*\tilde{e}_0^*\Big) \\
&+(M^2_{\tilde{L}})_{\alpha\beta}\tilde{e}_\beta\tilde{e}^*_\alpha + (M^2_{\tilde{E}})_{kl}\tilde{E}_k\tilde{E}^*_l + (M^2_{H_2})|h_2^+|^2 \\
&+\Big(\mu_\alpha(B)_\alpha\tilde{e}_\alpha h_2^+ + \frac{v_d}{\sqrt{2}}\lambda_{0\beta k}(A_E)_{0\beta k}\tilde{e}_\beta\tilde{E}_k^*\Big) + \text{h.c.}.
\end{aligned} \tag{8.5}$$

Introducing the 8-dimensional vector[4]

$$\phi_K^+ \equiv \left(h_2^+\,,\ \ (h_1^-)^*\,,\ \ \tilde{e}_k^*\,,\ \ \tilde{E}^*_{k'}\right)^T, \tag{8.6}$$

with $k, k' = 1, 2, 3$ and $K = h_2, h_1, k, k'$, one can write the mass terms of the colorless charged scalar particles as

$$-\mathcal{L}_{\rm mass} = (\phi^+)^\dagger M_e^2\,\phi^+, \tag{8.7}$$

with

$$M_e^2 \equiv \begin{pmatrix} M_{22}+D_{22} & b_0^*+\frac{1}{4}g^2v_uv_d & b_l^* & \mu_i^*(m_e)_{il'} \\ b_0+\frac{1}{4}g^2v_uv_d & M_{11}+D_{11} & (M^2_{\tilde{L}})_{l0}+\mu_0\mu_l^* & \frac{v_u}{v_d}\mu_i^*(m_e)_{il'} \\ b_k & (M^2_{\tilde{L}})_{0k}+\mu_k\mu_0^* & (M^2_{\tilde{L}})_{lk}+\mu_k\mu_l^*+M_{kl}+D_{kl} & -\frac{v_u}{\sqrt{2}}\mu^*_\alpha\lambda_{\alpha kl'}+\frac{v_d}{\sqrt{2}}a_{0kl'} \\ \mu_i(m_e)_{ik'} & \frac{v_u}{v_d}\mu_i(m_e)_{ik'} & -\frac{v_u}{\sqrt{2}}\mu_\alpha\lambda^*_{\alpha lk'}+\frac{v_d}{\sqrt{2}}a^*_{0lk'} & (M^2_{\tilde{E}})_{k'l'}+M_{k'l'}+D_{k'l'} \end{pmatrix}. \tag{8.8}$$

---

[3]Here, $|\mu_\alpha|^2$ is an abbreviation for $\mu^*_\alpha\mu_\alpha$, where the sum over $\alpha = 0, 1, 2, 3$ has to be taken.

[4]Again, a capital letter denotes the extension of the slepton generation indices k and k'. Actually, the anti-sleptons appear in the definition of ϕ^+.

Here, I have made use of Eq.(8.4) and the definitions of the supersymmetry breaking parameters in (4.8). In addition, some entries on the diagonal have been abbreviated by

$$\begin{aligned}
D_{22} &\equiv \frac{1}{8}(g'^2+g^2)(v_u^2-v_d^2)+\frac{1}{4}g^2v_d^2, & M_{22} &\equiv M_{H_2}^2+|\mu_\alpha|^2,\\
D_{11} &\equiv -\frac{1}{8}(g'^2-g^2)(v_u^2-v_d^2)+\frac{1}{4}g^2v_d^2, & M_{11} &\equiv (M_{\tilde{L}}^2)_{00}+|\mu_0|^2,\\
D_{kl} &\equiv -\frac{1}{8}(g'^2-g^2)(v_u^2-v_d^2)\ \delta_{kl}, & M_{kl} &\equiv (m_e)_k^2\ \delta_{kl},\\
D_{k'l'} &\equiv \frac{1}{4}g'^2(v_u^2-v_d^2)\ \delta_{k'l'}, & M_{k'l'} &\equiv (m_e)_{k'}^2\ \delta_{k'l'}.
\end{aligned} \tag{8.9}$$

In order to simplify the analysis, it is possible to neglect the R-parity violating parameters in the charged scalar squared-mass matrix M_e^2 as these are small compared to the weak and SUSY scale parameters[5]. Thus, one is left with mixing in the Higgs sector on the one hand[6], which need not be considered for calculating the $\lambda\lambda$-loop, and mixing between the sleptons on the other hand. In this approximation, generation mixing is not excluded since, a priori, $(M_{\tilde{L}}^2)_{lk}$ and $a_{0kl'}$ are not diagonal. However, a large mixing between the generations of the charged sleptons would induce mixing in the corresponding leptonic sector which can be seen in Fig.8.1.

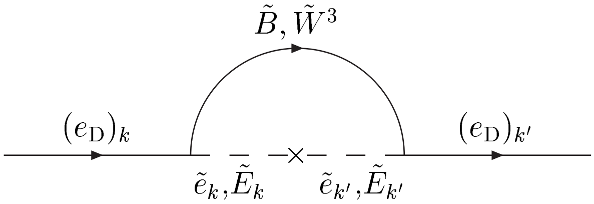


Figure 8.1: Generation mixing for sleptons

Experimentally, lepton generation mixing processes have not been observed; e.g. the limit on $\mu\to e\gamma$ is very stringent [61]. Therefore, it is reasonable to ignore generation mixing in the sleptonic sector. Then, the charged slepton mass matrix necessary for the loop calculation decomposes into three 2×2 matrices $M_{e_k}^2$, mixing $\tilde{e}_k$ and $\tilde{E}_k$ of one generation k

$$M_{e_k}^2 \equiv \begin{pmatrix} (M_{\tilde{L}}^2)_{kk}+(m_e)_k^2-\frac{1}{8}(g'^2-g^2)(v_u^2-v_d^2) & \left((A_E)_{0kk}-\frac{v_u}{v_d}\mu_0^*\right)(m_e)_k \\ \left((A_E)_{0kk}^*-\frac{v_u}{v_d}\mu_0\right)(m_e)_k & (M_{\tilde{E}}^2)_{kk}+(m_e)_k^2+\frac{1}{4}g'^2(v_u^2-v_d^2) \end{pmatrix}. \tag{8.10}$$

Notice that $(A_E)_{0kk}$ and μ_0 are complex valued. Therefore, the matrix cannot be diagonalized by an orthogonal transformation alone. In addition, one has to apply a phase rotation[7]. Then,

[5] See the footnote in Chapter 6.
[6] Note that $(M_{\tilde{L}}^2)_{0k}$ also violates R-parity.
[7] The detailed discussion of this procedure can be found in Appendix D.

the mass eigenstates $\tilde{e}^*_{k_i}$ are given by

$$\tilde{e}^*_{k_i} = (\tilde{V}^e_k)_{i1}\tilde{e}^*_k + (\tilde{V}^e_k)_{i2}\tilde{E}^*_k, \tag{8.11}$$

with $i = 1, 2$. Note that for each generation k there are two mass eigenstates denoted by $\tilde{e}^*_{k_1}$ and $\tilde{e}^*_{k_2}$. $\tilde{V}^e_k$ is the charged slepton mixing matrix

$$\tilde{V}^e_k \equiv \begin{pmatrix} \cos\tilde{\varphi}^e_k & \sin\tilde{\varphi}^e_k \\ -\sin\tilde{\varphi}^e_k & \cos\tilde{\varphi}^e_k \end{pmatrix} \cdot \begin{pmatrix} e^{-\mathrm{i}\tilde{\delta}^e_k} & \\ & e^{\mathrm{i}\tilde{\delta}^e_k} \end{pmatrix}. \tag{8.12}$$

The phase $\tilde{\delta}^e_k$ and the mixing angle $\tilde{\varphi}^e_k$ are given by

$$\begin{aligned} e^{2\mathrm{i}\tilde{\delta}^e_k} &\equiv \frac{(A_E)_{0kk} - \mu_0^*\tan\beta}{|(A_E)_{0kk} - \mu_0^*\tan\beta|}, \\ \tan 2\tilde{\varphi}^e_k &\equiv \frac{2|(A_E)_{0kk} - \mu_0^*\tan\beta|}{(M^2_{\tilde{L}})_{kk} - (M^2_{\tilde{E}})_{kk} - \frac{1}{8}(3g'^2 - g^2)(v_u^2 - v_d^2)}. \end{aligned} \tag{8.13}$$

The corresponding masses for the eigenstates of (8.11) can be calculated to

$$(M^2_{e_k})_i \equiv \frac{(M^2_{e_k})_{11} + (M^2_{e_k})_{22}}{2} \pm \frac{(M^2_{e_k})_{11} - (M^2_{e_k})_{22}}{2\cos 2\tilde{\varphi}^e_k}, \tag{8.14}$$

where the plus is associated with $i = 1$ and the minus sign with $i = 2$.

Some comments on the notation might be helpful here. The index k denotes the generation, whereas e stands for the leptonic sector. The latter will be useful when considering the down quark sector, because the results take a similar form and one need not introduce new symbols. The tilde on V, δ and φ signifies that the mixing refers to scalar particles. Since many masses appear in this thesis it is advantageous to have a systematic notation. Fermionic mass matrices are denoted by a lower case letter m, scalar matrices by an upper case one[8] M. The individual elements of a matrix are written with two indices outside parenthesis. If there is only one index this signifies an eigenvalue of the matrix.

8.1.3 λλ-Loop Mass

After all the preparation it is now straightforward to calculate the λλ-loop correction to the neutrino mass. Applying the Feynman rules derived in Chapter 7 leads to the matrix element

$$\begin{aligned} -\mathrm{i}\mathcal{M}^{\lambda\lambda}_{k'k} &= (2\pi)^4\delta(p-p')\sum_{l,n=1}^{3}\sum_{i=1}^{2}\int\frac{d^4q}{(2\pi)^4}(\nu_{\mathrm{D}})^T_{k'}\Big(\mathrm{i}\lambda_{k'ln}C^{-1}P_L(\tilde{V}^e_n)^*_{i2}\Big)\frac{\mathrm{i}(\not{q} + (m_e)_l)}{q^2 - (m_e)^2_l} \\ &\quad\times\Big(-\mathrm{i}\lambda_{knl}P_L(\tilde{V}^e_n)_{i1}\Big)(\nu_{\mathrm{D}})_k\frac{\mathrm{i}}{(p-q)^2 - (M^2_{e_n})_i} \;+\; (k \longleftrightarrow k'), \end{aligned} \tag{8.15}$$

[8]The only exception to this convention is made for the soft breaking gaugino masses.

where the notation of Fig.7.5(c) has been applied. Special care must be taken in order to get the correct mixing factors. As an example, I will have a look at the vertex with the incoming slepton $\tilde{e}_n$. The associated Feynman rule has been read off from a term in the Lagrangian density containing the flavor state $\tilde{e}_n$. Since the propagating particles are eigenstates of mass, denoted by $\tilde{e}_{n_i}$, it is necessary to rewrite $\tilde{e}_n$ in terms of the mass eigenstates. With Eq.(8.11) this then reads as

$$\tilde{e}_n = \sum_{i=1}^{2} (\tilde{V}_n^e)_{i1} \tilde{e}_{n_i}. \tag{8.16}$$

Notice the order of the indices. Similarly, the vertex for the outgoing slepton carries the factor $(\tilde{V}_n^e)_{i2}^*$, where the star must not be omitted.

It has been pointed out in [62] that the diagram with $k \longleftrightarrow k'$ is physically different from Fig.7.5(c). Therefore it must be added to the matrix element, whereby the loop correction to the mass matrix is symmetrized.

Using the relation of Eq.7.7 and inserting the mixing parameters $\tilde{\varphi}_n^e$, $\tilde{\delta}_n^e$ yields the $\lambda\lambda$-loop contribution to the neutrino mass

$$\begin{aligned}(m_\nu)_{k'k}^{\lambda\lambda} &= \frac{-1}{32\pi^2} \sum_{l,n=1}^{3} \Big(\lambda_{k'ln}\lambda_{knl} + \lambda_{kln}\lambda_{k'nl}\Big)(m_e)_l \, e^{-2\mathrm{i}\tilde{\delta}_n^e} \sin 2\tilde{\varphi}_n^e \\ &\quad \times \Big(B_0(p^2, (M_{e_n}^2)_1, (m_e)_l^2) - B_0(p^2, (M_{e_n}^2)_2, (m_e)_l^2)\Big),\end{aligned} \tag{8.17}$$

where B_0 is a Passarino-Veltman function defined in Appendix E. With $p^2 = 0$ and under the assumption that $(m_e)_l^2 \ll (M_{e_n}^2)_1, (M_{e_n}^2)_2$ the expression can be simplified to

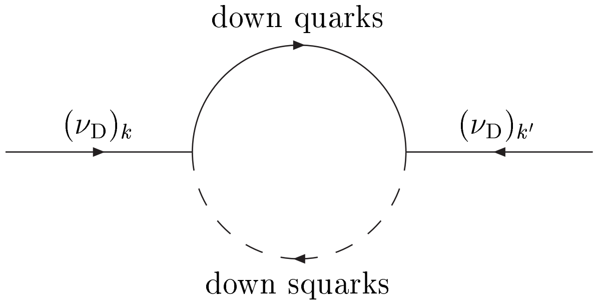
$$(m_\nu)_{k'k}^{\lambda\lambda} \simeq \frac{1}{32\pi^2} \sum_{l,n=1}^{3} \Big(\lambda_{k'ln}\lambda_{knl} + \lambda_{kln}\lambda_{k'nl}\Big)(m_e)_l \, e^{-2\mathrm{i}\tilde{\delta}_n^e} \sin 2\tilde{\varphi}_n^e \, \ln \frac{(M_{e_n}^2)_1}{(M_{e_n}^2)_2}. \tag{8.18}$$

The phase factor $e^{-2\mathrm{i}\tilde{\delta}_n^e}$ takes account of the possibility that $(A_E)_{0kk}$ and μ_0 can be complex[9].

8.2 $\lambda'\lambda'$-Loop

The calculation for the quark loop in Fig.7.5(d) can be performed in much the same way as was done in the previous section. Actually, the procedure is less complex since down quarks and squarks cannot mix with other particles due to conservation of color and charge. The corresponding diagram is depicted below.

[9]Grossman and Haber [53, 63] obtain a similar result, but they do not consider complex values. In addition, they omitted the diagram with $k \longleftrightarrow k'$. Furthermore, their result is incorrect in index structure, as they label the slepton mixing angle with the index of the lepton.



8.2.1 Down Quarks

The down quark mass terms in the Lagrangian density are given by

$$-\mathcal{L}_{\mathrm{mass}} = \frac{v_d}{\sqrt{2}}\lambda'_{0ij} d_i^\rho \bar{D}_j^\rho + \mathrm{h.c.}. \tag{8.19}$$

Recalling the choice of basis from Chapter 5, it is possible to define the diagonal down quark mass matrix in analogy to Eq.(8.4)

$$(m_d)_k \equiv (m_d)_{kl} \equiv \frac{v_d}{\sqrt{2}}\lambda'_{0kl}, \tag{8.20}$$

where k, l are generation indices.

8.2.2 Down Squarks

The mass matrix of the down type scalar quarks is obtained by rearranging the following mass terms in matrix form.

$$\begin{aligned}
-\mathcal{L}_{\mathrm{mass}} = & \frac{v_d^2}{2}\lambda'_{0kj}\lambda'^*_{0kj'}\tilde{D}_{j'}^\rho \tilde{D}_j^{\rho *} + \frac{v_d^2}{2}\lambda'_{0ik}\lambda'^*_{0i'k}\tilde{d}_i^\rho \tilde{d}_{i'}^{\rho *} \\
& -\Big(\frac{v_u}{\sqrt{2}}\mu_\alpha^* \lambda'_{\alpha ij}\tilde{d}_i^\rho \tilde{D}_j^{\rho *} - \frac{v_d}{\sqrt{2}}\lambda'_{0ij}(A_D)_{0ij}\tilde{d}_i^\rho \tilde{D}_j^{\rho *}\Big) + \mathrm{h.c.} \\
& +\frac{1}{8}g'^2\Big(\frac{1}{3}(v_u^2 - v_d^2)|\tilde{d}_k^\rho|^2 + \frac{2}{3}(v_u^2 - v_d^2)|\tilde{D}_k^\rho|^2\Big) \\
& +\frac{1}{8}g^2\Big(- v_d^2|\tilde{d}_k^\rho|^2 + v_u^2|\tilde{d}_k^\rho|^2\Big) \\
& +(M_{\tilde{Q}}^2)_{ij}\tilde{d}_j^\rho \tilde{d}_i^{\rho *} + (M_{\tilde{D}}^2)_{ij}\tilde{D}_i^\rho \tilde{D}_j^{\rho *}.
\end{aligned} \tag{8.21}$$

With the definition of the 6-dimensional vector[10]

$$\phi_K^\rho \equiv \big(\tilde{d}_k^{\rho *}\ ,\quad \tilde{D}_{k'}^{\rho *}\big)^T, \tag{8.22}$$

this can be rewritten as

$$-\mathcal{L}_{\mathrm{mass}} = \phi^{\rho\dagger} M_d^2 \phi^\rho, \tag{8.23}$$

[10] Here, the color index ρ is regarded to be fixed.

where the 6×6 mass matrix is given by

$$M_d^2 \equiv \begin{pmatrix} (m_d)^2_{kl} + \frac{1}{24}(g'^2+3g^2)(v_u^2-v_d^2)\delta_{kl} + (M^2_{\tilde{Q}})_{lk} & -\frac{v_u}{\sqrt{2}}\mu^*_\alpha\lambda'_{\alpha kl'} + \frac{v_d}{\sqrt{2}}a'_{0kl'} \\ -\frac{v_u}{\sqrt{2}}\mu_\alpha\lambda'^*_{\alpha lk'} + \frac{v_d}{\sqrt{2}}a'^*_{0lk'} & (m_d)^2_{k'l'} + \frac{1}{12}g'^2(v_u^2-v_d^2)\delta_{k'l'} + (M^2_{\tilde{D}})_{k'l'} \end{pmatrix}. \quad (8.24)$$

In the context of the mixing parameters, it is important to note that the mass matrices are identical and unconnected for each color as $SU(3)_C$ is unbroken. Neglecting generation mixing one ends up with the analog of Eq.(8.10)

$$M^2_{d_k} \equiv \begin{pmatrix} (M^2_{\tilde{Q}})_{kk} + (m_d)^2_k + \frac{1}{24}(g'^2+3g^2)(v_u^2-v_d^2) & \left((A_D)_{0kk} - \frac{v_u}{v_d}\mu_0^*\right)(m_d)_k \\ \left((A_D)^*_{0kk} - \frac{v_u}{v_d}\mu_0\right)(m_d)_k & (M^2_{\tilde{D}})_{kk} + (m_d)^2_k + \frac{1}{12}g'^2(v_u^2-v_d^2) \end{pmatrix}. \quad (8.25)$$

The mass eigenstates $\tilde{d}^*_{k_i}$ of the down type squarks are identified to be

$$\tilde{d}^*_{k_i} \;=\; (\tilde{V}^d_k)_{i1}\tilde{d}^*_k + (\tilde{V}^d_k)_{i2}\tilde{D}^*_k, \quad (8.26)$$

where $\tilde{V}^d_k$ is the down squark mixing matrix

$$\tilde{V}^d_k \;\equiv\; \begin{pmatrix} \cos\tilde{\varphi}^d_k & \sin\tilde{\varphi}^d_k \\ -\sin\tilde{\varphi}^d_k & \cos\tilde{\varphi}^d_k \end{pmatrix} \cdot \begin{pmatrix} e^{-\mathrm{i}\tilde{\delta}^d_k} & \\ & e^{\mathrm{i}\tilde{\delta}^d_k} \end{pmatrix}. \quad (8.27)$$

The phase $\tilde{\delta}^d_k$, the mixing angle $\tilde{\varphi}^d_k$ and the eigenvalues $(M^2_{d_k})_i$ of the mass matrix (8.25) can be calculated to

$$\begin{aligned} e^{2\mathrm{i}\tilde{\delta}^d_k} &\equiv \frac{(A_D)_{0kk} - \mu_0^*\tan\beta}{|(A_D)_{0kk} - \mu_0^*\tan\beta|}, \\ \tan 2\tilde{\varphi}^d_k &\equiv \frac{2|(A_D)_{0kk} - \mu_0^*\tan\beta|}{(M^2_{\tilde{Q}})_{kk} - (M^2_{\tilde{D}})_{kk} - \frac{1}{24}(g'^2-3g^2)(v_u^2-v_d^2)}, \\ (M^2_{d_k})_i &\equiv \frac{(M^2_{d_k})_{11} + (M^2_{d_k})_{22}}{2} \pm \frac{(M^2_{d_k})_{11} - (M^2_{d_k})_{22}}{2\cos 2\tilde{\varphi}^d_k}. \end{aligned} \quad (8.28)$$

Again, $i = 1, 2$ denotes the two mass eigenstates.

8.2.3 $\lambda'\lambda'$-Loop Mass

For the $\lambda'\lambda'$-loop the vertex factors and the mixing parameters are color independent. Therefore it is sufficient to calculate the diagram for one color and multiply the result by 3. Apart from this, the processes of Fig.7.5(d) and Fig.7.5(c) are identical. Replacing the couplings λ by λ' and inserting the correct masses and mixing parameters, the results of the $\lambda\lambda$-loop can be translated to the case of the $\lambda'\lambda'$-loop. The corresponding contribution to the neutrino mass is then given by

$$\begin{aligned}(m_\nu)^{\lambda'\lambda'}_{k'k} &= \frac{-3}{32\pi^2}\sum_{l,n=1}^{3}\Big(\lambda'_{k'ln}\lambda'_{knl}+\lambda'_{kln}\lambda'_{k'nl}\Big)(m_d)_l\ e^{-2\mathrm{i}\tilde{\delta}^d_n}\sin 2\tilde{\varphi}^d_n \\ &\quad\times\Big(B_0(p^2,(M^2_{d_n})_1,(m_d)^2_l)-B_0(p^2,(M^2_{d_n})_2,(m_d)^2_l)\Big). \qquad (8.29)\end{aligned}$$

As in the case of the lepton-slepton loop, this expression can be simplified by taking the limit $p^2 \longrightarrow 0$ and assuming $(m_d)^2_l \ll (M^2_{d_n})_1, (M^2_{d_n})_2$

$$(m_\nu)^{\lambda'\lambda'}_{k'k} \simeq \frac{3}{32\pi^2}\sum_{l,n=1}^{3}\Big(\lambda'_{k'ln}\lambda'_{knl}+\lambda'_{kln}\lambda'_{k'nl}\Big)(m_d)_l\ e^{-2\mathrm{i}\tilde{\delta}^d_n}\sin 2\tilde{\varphi}^d_n\ \ln\frac{(M^2_{d_n})_1}{(M^2_{d_n})_2}. \qquad (8.30)$$

This result, which has also been derived in [53], is the analog of Eq.(8.18).

Chapter 9

gλ-Loop

In contrast to the $\lambda\lambda$-loop considered in Chapter 8, the mixing of charginos and charged leptons is essential for a non-vanishing contribution of the gauge-Yukawa loop sketched in Fig.7.5(b) to the neutrino mass. Having found the fermionic mass eigenstates, the gλ-loop correction will be calculated directly, since the mixing for the propagating scalar particles has already been treated.

9.1 Mixing of Charginos and Charged Leptons

The mass matrix m_e of the charged colorless fermions is given in Eq.(8.3). As it is not hermitian, it is necessary to apply a biunitary transformation in order to diagonalize it. Then, the corresponding mass eigenstates are denoted by

$$\chi^- = U\psi^- \quad , \quad \chi^+ = V\psi^+. \tag{9.1}$$

U and V are unitary 5×5 matrices chosen such that $U^* m_e V^\dagger$ becomes diagonal with non-negative entries. The general procedure of finding the correct transformation is described in Appendix C. Translating Eqs.(C.5) and (C.6) leads to

$$D^2 = U^*\, m_e m_e^\dagger\, U^{*\dagger} \quad , \quad V = D^{-1} U^*\, m_e, \tag{9.2}$$

where D^2 is the diagonal squared mass matrix. The main task now consists in determining U whereupon the calculation of V is straightforward. The method of matrix perturbation theory will be applied in order to obtain an analytic expression for the gλ-loop correction to the neutrino mass. Expanding $m_e m_e^\dagger$ in the small R-parity violating parameters μ_k yields

$$m_e m_e^\dagger = \begin{pmatrix} |M_2|^2 + \frac{g^2}{2} v_u^2 & \frac{g}{\sqrt{2}}(M_2 v_d + \mu_0^* v_u) & \\ \frac{g}{\sqrt{2}}(M_2^* v_d + \mu_0 v_u) & |\mu_0|^2 + \frac{g^2}{2} v_d^2 & \\ & & (m_e)^2_{kl} \end{pmatrix} + \begin{pmatrix} & & \frac{g}{\sqrt{2}} v_u \mu_l^* \\ & & \mu_0 \mu_l^* \\ \frac{g}{\sqrt{2}} v_u \mu_k & \mu_0^* \mu_k & \mu_k \mu_l^* \end{pmatrix}, \tag{9.3}$$

with $k, l = 1, 2, 3$ being the generation indices. $(m_e)^2_{kl}$ is the square of the diagonal 3×3 matrix defined in Eq.(8.4). As the eigenvectors of a matrix are directly connected to the diagonalization matrix, it is necessary to find the normalized eigenstates. For the unperturbed system these are given in the following list, together with their eigenvalues.

$$\begin{array}{cc|ccc}
\multicolumn{2}{c|}{\text{Charginos}} & \multicolumn{3}{c}{\text{Charged Leptons}} \\
\hline
\begin{pmatrix} e^{i\delta}\cos\varphi \\ e^{-i\delta}\sin\varphi \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} -e^{i\delta}\sin\varphi \\ e^{-i\delta}\cos\varphi \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \\
(m_e)^2_{-1} & (m_e)^2_{-2} & (m_e)^2_1 & (m_e)^2_2 & (m_e)^2_3
\end{array} \tag{9.4}$$

Here the phase δ, the mixing angle φ and the eigenvalues $(m_e)^2_{-1}, (m_e)^2_{-2}$ are defined by

$$\begin{aligned}
e^{2i\delta} &\equiv \frac{M_2 + \mu_0^* \tan\beta}{|M_2 + \mu_0^* \tan\beta|}, \\
\tan 2\varphi &\equiv \frac{\sqrt{2} g |M_2 v_d + \mu_0^* v_u|}{|M_2|^2 - |\mu_0|^2 + \frac{g^2}{2}(v_u^2 - v_d^2)}, \\
(m_e)^2_{-i} &\equiv \frac{|M_2|^2 + |\mu_0|^2 + \frac{g^2}{2} v^2}{2} \pm \frac{|M_2|^2 - |\mu_0|^2 + \frac{g^2}{2}(v_u^2 - v_d^2)}{2\cos 2\varphi},
\end{aligned} \tag{9.5}$$

where the plus sign refers to $i = 1$ and the minus sign to $i = 2$. Note that negative indices have been introduced to denote the chargino sector. Evidently, these five states do not mix charginos and charged leptons. Only through R-parity violation the needed mixing can be achieved. Therefore, one must take into account effects of first order perturbation theory[1]. As the term $\mu_k \mu_l^*$ in Eq.(9.3), describing a leptonic generation mixing, is already second order, it need not be considered. In addition, I assume the chargino masses[2] $(m_e)_{-j}$, $j = 1, 2$, to be non-degenerate and much larger than the masses of the leptons $(m_e)_j$, $j = 1, 2, 3$. It should be recognized that the notation is in line with the convention of denoting an eigenvalue of a matrix by one single index outside parenthesis. Another aspect to keep in mind is that the masses in Eq.(9.4) refer to the unperturbed matrix. A simple but rather tedious calculation yields the first order corrected eigenstates

$$\begin{pmatrix} e^{i\delta}\cos\varphi \\ e^{-i\delta}\sin\varphi \\ \mu_1 \frac{x^*}{(m_e)^2_{-1}} \\ \mu_2 \frac{x^*}{(m_e)^2_{-1}} \\ \mu_3 \frac{x^*}{(m_e)^2_{-1}} \end{pmatrix}, \quad \begin{pmatrix} -e^{i\delta}\sin\varphi \\ e^{-i\delta}\cos\varphi \\ \mu_1 \frac{y^*}{(m_e)^2_{-2}} \\ \mu_2 \frac{y^*}{(m_e)^2_{-2}} \\ \mu_3 \frac{y^*}{(m_e)^2_{-2}} \end{pmatrix}, \quad \begin{pmatrix} -\mu_k^* e^{i\delta} \left(\frac{x\cos\varphi}{(m_e)^2_{-1}} - \frac{y\sin\varphi}{(m_e)^2_{-2}} \right) \\ -\mu_k^* e^{-i\delta} \left(\frac{x\sin\varphi}{(m_e)^2_{-1}} + \frac{y\cos\varphi}{(m_e)^2_{-2}} \right) \\ \delta_{1k} \\ \delta_{2k} \\ \delta_{3k} \end{pmatrix}, \tag{9.6}$$

[1]A brief summary of perturbation theory can be found in Appendix F.

[2]$(m_e)_{-i}$ is defined by the positive square root of $(m_e)^2_{-i}$.

with $x \equiv \frac{v_u g}{\sqrt{2}} e^{-i\delta} \cos\varphi + \mu_0 e^{i\delta} \sin\varphi$ and $y \equiv -\frac{v_u g}{\sqrt{2}} e^{-i\delta} \sin\varphi + \mu_0 e^{i\delta} \cos\varphi$. The last expression in Eq.(9.6) represents three vectors, one for each $k = 1, 2, 3$. It is easy to verify that these five states are still normalized to one and orthogonal up to first order in μ_k. Therefore, the unitary matrix $U^{*\dagger}$ is generated by writing all eigenstates into one array. Taking the hermitian conjugate leads to

$$U^* = \begin{pmatrix} e^{-i\delta}\cos\varphi & e^{i\delta}\sin\varphi & \mu_l^* \frac{x}{(m_e)_{-1}^2} \\ -e^{-i\delta}\sin\varphi & e^{i\delta}\cos\varphi & \mu_l^* \frac{y}{(m_e)_{-2}^2} \\ -\mu_k e^{-i\delta}\left(\frac{x^*\cos\varphi}{(m_e)_{-1}^2} - \frac{y^*\sin\varphi}{(m_e)_{-2}^2}\right) & -\mu_k e^{i\delta}\left(\frac{x^*\sin\varphi}{(m_e)_{-1}^2} + \frac{y^*\cos\varphi}{(m_e)_{-2}^2}\right) & \delta_{kl} \end{pmatrix}. \tag{9.7}$$

The second matrix V of the biunitary transformation is obtained via Eq.(9.2) where D is the diagonal matrix

$$D = \begin{pmatrix} (m_e)_{-1} & & & & \\ & (m_e)_{-2} & & & \\ & & (m_e)_1 & & \\ & & & (m_e)_2 & \\ & & & & (m_e)_3 \end{pmatrix}. \tag{9.8}$$

Notice that although perturbation of first order has been considered, the masses in D are the unperturbed ones. This can be understood by realizing that the terms linear in μ_k appear only off-diagonal in the squared mass matrix of Eq.(9.3). Thus no first order corrections to the eigenvalues of $m_e m_e^\dagger$ exist. Now, almost everything seems to be disposed for calculating the gλ-loop: The vertex factors are known and the mixing of the charged colorless fermions has just been derived. However, so far fermionic mixing has been described in Weyl spinor formalism only. It might be instructive and useful to see how one can construct Dirac spinors. Starting with Eq.(9.1) it is possible to define the positively charged 4-component spinors

$$(\chi_{\mathrm{D}}^+)_J \equiv \begin{pmatrix} \chi_J^+ \\ \overline{\chi_J^-} \end{pmatrix} = \begin{pmatrix} V_{JK}\psi_K^+ \\ U_{JK}^* \overline{\psi_K^-} \end{pmatrix}, \tag{9.9}$$

where $J = -1, -2, 1, 2, 3$ and $K = W, h, 1, 2, 3$. In this chapter I consistently use I, J as indices connected to mass eigenstates, whereas K, L are reserved for the basis of Eq.(8.1) which I will call the flavor basis. The convention concerning small and capital letters has already been established before. One might doubt if the $(\chi_{\mathrm{D}}^+)_J$ really are eigenstates of mass. To see that this is in fact the case, one has to rewrite Eq.(8.3). Thus, one ends up with the usual Dirac mass term

$$-\mathcal{L}_{\text{mass}} = \overline{(\chi_{\mathrm{D}}^+)_I}\, D_{IJ}\, (\chi_{\mathrm{D}}^+)_J. \tag{9.10}$$

Note that, equally well, negatively charged Dirac spinors $(\chi_{\mathrm{D}}^-)_J$ could have been defined. This does not lead to fundamental changes since both possibilities are related by charge conjugation.

I have adopted the choice for the chargino type Dirac spinors made in [43] and extended it to the charged leptons. This looks somewhat unnatural as one must now think of anti-leptons that couple to the second vertex in Fig.7.5(b).

Before finishing this section, I want to emphasize that the conventions for distinguishing mass from flavor states and charginos from charged leptons are used extensively in the following. The mass indices are $I, J = -1, -2, 1, 2, 3$. The flavor indices are $K, L = W, h, 1, 2, 3$. Small indices always run from 1 to 3, independently whether they are mass or flavor type.

9.2 gλ-Loop Mass

Having derived the mixing properties of the fermions it is necessary to make some remarks on how to attach the correct mixing factors to the vertices. First one should realize that in the fermionic sector of Fig.7.5(b) there are two cases to be considered. These differ in the direction of the arrows that indicate the flow of charge. The vertex with an outgoing charged fermion corresponds to a term in the Lagrangian density of the form $\overline{\psi_{\mathrm{D}}^{+}}\nu_{\mathrm{D}}$ whereas for an incoming charged fermion this term is given by $(\psi_{\mathrm{D}}^{+})^{T}\nu_{\mathrm{D}}$. Inserting the projection operator which leaves the neutrino unchanged, one can rewrite both terms

$$\begin{aligned} \text{outgoing:} \qquad & \overline{\psi_{\mathrm{D}}^{+}}P_{\mathrm{L}}\nu_{\mathrm{D}} &=& \overline{P_{\mathrm{R}}\psi_{\mathrm{D}}^{+}}\nu_{\mathrm{D}}, \\ \text{incoming:} \qquad & (\psi_{\mathrm{D}}^{+})^{T}P_{\mathrm{L}}\nu_{\mathrm{D}} &=& (P_{\mathrm{L}}\psi_{\mathrm{D}}^{+})^{T}\nu_{\mathrm{D}}. \end{aligned} \tag{9.11}$$

Hence, the transformation matrix U appears with each outgoing charged fermion, V on the other hand is connected to an incoming one. This still holds if the charge conjugation matrix appears in the ordinary vertex factor without mixing phenomena since $P_{\mathrm{L,R}}$ and C commute. As in the case of scalar mixing one has to pay attention to the complex conjugation of the mixing coefficients. The complete vertex factors are added to the diagram of Fig.9.1.

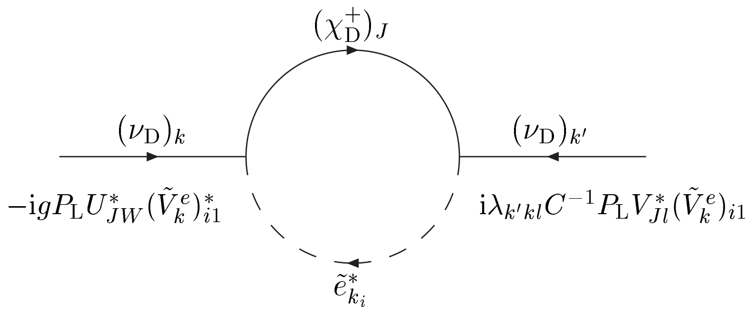


Figure 9.1: gλ-loop with mass eigenstates propagating

The index $i = 1, 2$ denotes the mass eigenstates of the scalar particles[3]. $J = -1, -2, 1, 2, 3$ labels the five fermionic mass eigenstates. W is the first component of the flavor index K.

[3]The mass indices of scalar particles run from 1 to 2. For example, $\tilde{e}_k$ mixes with $\tilde{E}_k$ in one generation k.

Finally, $l = 1, 2, 3$ represents the generation of the (flavor) lepton that couples to the vertex on the right side. With these preliminary remarks the matrix element of the gλ-loop can easily be calculated:

$$\begin{aligned} -\mathrm{i}\mathcal{M}^{\mathrm{g}\lambda}_{k'k} &= (2\pi)^4\delta(p-p')\sum_{i,J,l}\int\frac{d^4q}{(2\pi)^4}(\nu_{\mathrm{D}})^T_{k'}\Big(\mathrm{i}\lambda_{k'kl}C^{-1}P_{\mathrm{L}}V^*_{Jl}(\tilde{V}^e_k)_{i1}\Big)\frac{\mathrm{i}(\not{q}+(m_e)_J)}{(q^2-(m_e)^2_J)} \\ &\times\Big(-\mathrm{i}gP_{\mathrm{L}}U^*_{JW}(\tilde{V}^e_k)^*_{i1}\Big)(\nu_{\mathrm{D}})_k\frac{\mathrm{i}}{(p-q)^2-(M^2_{e_k})_i}\;+\;(k\longleftrightarrow k'). \end{aligned} \tag{9.12}$$

Note that it is not possible to take the sum over different generations of the scalar particles in the loop as it is fixed by the generation of the incoming neutrino. As for the $\lambda\lambda$-loop one has to consider another diagram with the gauge coupling on the right and the Yukawa coupling on the left side of the loop. This is equivalent to the exchange of the external neutrino legs. Using the definition of the B_0 function the gλ-loop contribution to the neutrino mass takes the form

$$\begin{aligned} (m_\nu)^{\mathrm{g}\lambda}_{k'k} &= \frac{-g}{16\pi^2}\sum_{i,J,l}\lambda_{k'kl}\Big(|(\tilde{V}^e_k)_{i1}|^2\;B_0(p^2,(M^2_{e_k})_i,(m_e)^2_J)-|(\tilde{V}^e_{k'})_{i1}|^2\;B_0(p^2,(M^2_{e_{k'}})_i,(m_e)^2_J)\Big) \\ &\times U^*_{JW}(m_e)_JV^*_{Jl}, \end{aligned} \tag{9.13}$$

where the second term is due to the other diagram. The minus sign appears because $\lambda_{k'kl}$ is antisymmetric in k, k' and has been factored out. Unfortunately, the infinities of B_0 do not cancel directly because, in general, the three sleptons mix differently. In order to remove the infinities explicitly, further mathematical transformations are required. First one can make use of Eq.(9.2) and replace V by $D^{-1}U^*m_e$. As D is the diagonal charged fermion mass matrix, the second line of Eq.(9.13) simplifies to

$$U^*_{JW}(m_e)_JV^*_{Jl} = U^*_{JW}(m_e)_J\Big(D^{-1}U(m_e)^*\Big)_{Jl} = \sum_K U^*_{JW}U_{JK}(m_e)^*_{Kl} = U^*_{JW}U_{Jl}(m_e)_l. \tag{9.14}$$

In the last step I have applied the explicit form of the mass matrix m_e given in Eq.(8.3). Before proceeding with the evaluation of the gλ-loop contribution to the neutrino mass I shall introduce a shorter notation for the B_0 function

$$B_0(k_i,J)\;\equiv\;B_0(p^2,(M^2_{e_k})_i,(m_e)^2_J), \tag{9.15}$$

where the first argument refers to the scalar particle and the second to the fermion. The dependence of B_0 on p^2 has been omitted because it is set to zero for neutrinos. Inserting the elements of the matrix U into Eq.(9.13), performing the summation over J and reordering the terms yields

$$\begin{aligned} (m_\nu)^{\mathrm{g}\lambda}_{k'k} &= \frac{-g}{16\pi^2}\sum_{i,l}\lambda_{k'kl}\;\mu_l\;(m_e)_l\;e^{-\mathrm{i}\delta} \\ &\times\Bigg(\frac{x^*\cos\varphi}{(m_e)^2_{-1}}\Big\{|(\tilde{V}^e_k)_{i1}|^2\Big(B_0(k_i,-1)-B_0(k_i,l)\Big)-|(\tilde{V}^e_{k'})_{i1}|^2\Big(B_0(k'_i,-1)-B_0(k'_i,l)\Big)\Big\} \\ &-\frac{y^*\sin\varphi}{(m_e)^2_{-2}}\Big\{|(\tilde{V}^e_k)_{i1}|^2\Big(B_0(k_i,-2)-B_0(k_i,l)\Big)-|(\tilde{V}^e_{k'})_{i1}|^2\Big(B_0(k'_i,-2)-B_0(k'_i,l)\Big)\Big\}\Bigg). \end{aligned} \tag{9.16}$$

Note that in this form the infinities of the Passarino-Veltman function cancel explicitly. Without additional assumptions, Eq.(9.16) cannot be simplified. The approximations made for the difference of two B_0 functions[4] are applicable if either the squared chargino masses $(m_e)^2_{-i}$, $i = 1, 2$, are small compared to the slepton masses $(M^2_{e_k})_i$ or if the charged lepton mass scale was near the chargino masses. Evidently, the latter is not realized in nature [55]. The possibility of $(m_e)^2_{-i} \ll (M^2_{e_k})_i$ cannot be assumed or ruled out as both masses are unknown. Moreover, the lower bounds for the masses of charginos and charged sleptons are similar and of the order 100 GeV [55]. Since the supersymmetry breaking scale must not exceed 1 TeV very much if one wants supersymmetry to solve the hierarchy problem, the mass range for the charginos and sparticles is not very wide. However, it could still be possible that charginos are somewhat lighter than charged sleptons. Since it is the square of the masses that enters the B_0 function, the approximation could still be reasonable. But without knowing further details one should not restrict Eq.(9.16) to such a special case.

Like the $\lambda\lambda$-loop the radiative correction to the neutrino mass is here proportional to two R-parity violating parameters. The $\lambda_{k'kl}$ coefficient is originated in the Yukawa coupling whereas the factor of μ_l stems from the mixing of charginos and charged leptons. Furthermore, it is interesting to note that the gλ-loop does not give contributions to the diagonal of the neutrino mass matrix due to the antisymmetry of $\lambda_{k'kl}$ in the first two indices.

[4]See Appendix E.

Chapter 10

gg-Loop

A third type of radiative corrections to the neutrino mass is provided by the gauge-gauge-loop of Fig.7.5(a). Due to the clashing arrows for the propagating scalar particle, it is necessary to consider sneutrino-antisneutrino mixing in detail. In addition to the result of Grossman and Haber [64], I will show that there is another contribution coming from the mass eigenstates of the neutral Higgses which also contain small admixtures of sneutrinos.

10.1 Neutralinos and Neutrinos

In order to find the neutral fermion mass eigenstates it is necessary to diagonalize the matrix $m_\nu^{\rm tree}$ given in Eq.(6.10). This can be achieved by a unitary matrix N, which transforms the neutral fermions of Eq.(6.2) into their mass eigenstates χ^0

$$\chi_J^0 \equiv N_{JK}\psi_K^0, \tag{10.1}$$

with $K = B, W, h_2, h_1, 1, 2, 3$ and $J = -1, -2, -3, -4, 1, 2, 3$. Note that the conventions for distinguishing mass and flavor indices are analogous to the case of charginos and charged leptons. The corresponding mass term in the Lagrangian density then changes to

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}(\psi^0)^T m_\nu^{\rm tree}\psi^0 + {\rm h.c.} = \frac{1}{2}(\chi^0)^T\ N^* m_\nu^{\rm tree} N^\dagger\ \chi^0 + {\rm h.c.}, \tag{10.2}$$

where $N^* m_\nu^{\rm tree} N^\dagger$ is diagonal with non-negative entries. The existence of such a matrix N can be proven easily by considering the biunitary transformation discussed in Appendix D for a symmetric[1] matrix X.

Having obtained the seven mass eigenstates, it becomes obvious that four are dominantly neutralino type with only small admixtures of neutrinos, whereas the other three are primarily composed of flavor neutrinos. This is due to the fact that the small R-parity violating parameters μ_k govern the mixing between neutralinos and neutrinos. Since the fermions propagating in the loop couple to the vertices only via the neutral gauginos $\tilde{B}$ and $\tilde{W}^3$, it is possible to ignore this mixing completely and restrict the calculations to neutralinos traveling in the loop.

[1]In general, $m_\nu^{\rm tree}$ is symmetric but not real. Therefore it cannot be diagonalized by an orthogonal matrix.

Thus, I will only make use of the upper 4×4 block of the mixing matrix N. As its analytic expression [65] derived from the neutralino mass matrix becomes very messy, the explicit form of N will be calculated numerically when necessary.

In analogy to the charged fermions one can define four component Majorana spinors

$$(\chi^0_{\rm D})_J \;\equiv\; \begin{pmatrix} \chi^0_J \\ \overline{\chi^0_J} \end{pmatrix} = \begin{pmatrix} N_{JK}\psi^0_K \\ N^*_{JK}\overline{\psi^0_K} \end{pmatrix}, \tag{10.3}$$

which carry no conserved charge. I should mention that the introduction of $\chi^0_{\rm D}$ only makes sense because the entries of the diagonal matrix $N^* m_\nu^{\rm tree} N^\dagger$ are real, which allows one to include the hermitian conjugate in Eq.(10.2) to one single mass term $\frac{1}{2}(m_\nu^{\rm tree})_J \overline{(\chi^0_{\rm D})_J}(\chi^0_{\rm D})_J$. According to the conventions in this thesis, the eigenvalue of a matrix is denoted by one index outside the parentheses.

10.2 Neutral Higgses and Sneutrinos

Before deriving the mass matrix for the neutral Higgs bosons and sneutrinos it is advisable to have a closer look at the neutral scalar potential, $V_{\rm neutral}$, whose minimum determines the five vacuum expectation values. Since the VEVs of the sneutrinos have been fixed[2] by the choice of basis made in Chapter 5, the minimization conditions set some restrictions on the parameter space. Gathering all relevant terms of the Lagrangian density yields

$$\begin{aligned} V_{\rm neutral} \;=\; & \Big(M^2_{H_2} + |\mu_\alpha|^2\Big)|h^0_2|^2 + \Big((M^2_{\tilde L})_{\beta\alpha} + \mu_\alpha\mu^*_\beta\Big)\tilde\nu_\alpha\tilde\nu^*_\beta - b_\alpha\tilde\nu_\alpha h^0_2 - b^*_\alpha\tilde\nu^*_\alpha h^{0*}_2 \\ & + \frac{1}{8}(g'^2+g^2)\Big(|h^0_2|^4 + |\tilde\nu_\alpha|^2|\tilde\nu_\beta|^2 - 2|h^0_2|^2|\tilde\nu_\alpha|^2\Big). \end{aligned} \tag{10.4}$$

The minimization conditions are

$$M^2_{H_2} + |\mu_\alpha|^2 + \frac{1}{8}(g'^2+g^2)(v_u^2 - v_d^2) \;=\; b_0 \cot\beta, \tag{10.5}$$

$$(M^2_{\tilde L})_{0\alpha} + \mu_\alpha\mu^*_0 - \frac{1}{8}(g'^2+g^2)(v_u^2 - v_d^2)\delta_{\alpha 0} \;=\; b_\alpha \tan\beta, \tag{10.6}$$

where the choice of basis has already been applied. From Eq.(10.5) it is evident that b_0, which generally is complex, has to be real now. Turning to the tree level mass terms, one must pay attention to the quadratic terms of the neutral scalar potential. As all fields appearing in Eq.(10.4) can principally acquire a VEV, also the terms in the second line, which arise from the elimination of the auxiliary D-fields, contribute to the mass matrix. One consequence of this is, that it is not sufficient to consider a 5×5 matrix, but one is forced to deal with a 10×10 matrix. This can be understood by noticing that mass terms of the general form $\phi^*_1\phi_2$ as well as $\phi_1\phi_2$ are generated, where $\phi_{1,2}$ symbolizes one of the five neutral scalar fields. Assuming CP-conservation in the neutral scalar sector it is possible to consider CP-even and CP-odd states separately. However, if for example b_k becomes complex, CP is violated and the full

[2]Remember $v_k = 0$ for $k = 1, 2, 3$.

analysis is required. Grossman and Haber [66] have suggested to investigate the general case. This is done in the following.

Starting with the definition of the 10-dimensional vector

$$\phi^0_K \;\equiv\; \left(h^0_2\,,\;\; h^{0*}_2\,,\;\; h^0_1\,,\;\; h^{0*}_1\,,\;\; \tilde{\nu}_k\,,\;\; \tilde{\nu}^*_{\bar{k}}\right), \tag{10.7}$$

with $k, \bar{k} = 1, 2, 3$ and $K = h_2, \bar{h}_2, h_1, \bar{h}_1, k, \bar{k}$, the mass terms of the neutral scalar particles can be written as

$$-\mathcal{L}_{\text{mass}} \;=\; \frac{1}{2}(\phi^0)^\dagger M^2_\nu\,\phi^0, \tag{10.8}$$

where M^2_ν is the hermitian squared mass matrix. Its explicit form is calculated to

$$M^2_\nu \;\equiv\; \begin{pmatrix} b_0\frac{v_d}{v_u}+\check{v}^2_u & \check{v}^2_u & -\check{v}_u\check{v}_d & -b_0-\check{v}_u\check{v}_d & & -b^*_{\bar{l}} \\ \check{v}^2_u & b_0\frac{v_d}{v_u}+\check{v}^2_u & -b_0-\check{v}_u\check{v}_d & -\check{v}_u\check{v}_d & -b_l & \\ -\check{v}_u\check{v}_d & -b_0-\check{v}_u\check{v}_d & b_0\frac{v_u}{v_d}+\check{v}^2_d & \check{v}^2_d & b_l\frac{v_u}{v_d} & \\ -b_0-\check{v}_u\check{v}_d & -\check{v}_u\check{v}_d & \check{v}^2_d & b_0\frac{v_u}{v_d}+\check{v}^2_d & & b^*_{\bar{l}}\frac{v_u}{v_d} \\ & -b^*_k & b^*_k\frac{v_u}{v_d} & & M^2_{lk} & \\ -b_{\bar{k}} & & & b_{\bar{k}}\frac{v_u}{v_d} & & M^2_{\bar{k}\bar{l}} \end{pmatrix}. \tag{10.9}$$

For notational convenience I have introduced $\check{v}^2_{u,d} \equiv \frac{1}{8}(g'^2+g^2)v^2_{u,d,}$ and $M^2_{kl} \equiv (M^2_{\tilde{L}})_{lk} + \mu_k\mu^*_l - (\check{v}^2_u - \check{v}^2_d)\delta_{kl}$. Furthermore, the minimization conditions of Eqs.(10.5) and (10.6) have been used. In the absence of R-parity violating parameters, M^2_ν decomposes into the neutral Higgs mass matrix and two 3×3 matrices for the sneutrinos. In particular, there is no lepton number violating mixing between sneutrinos and antisneutrinos, which however is necessary for the gg-loop to give a non-zero contribution to the neutrino mass. In order to diagonalize the hermitian matrix M^2_ν, again matrix perturbation theory is applied, where the R-parity violating parameters b_k and the flavor changing off-diagonals of M^2_{kl} are treated as a perturbation. To zeroth order, the eigenstates and eigenvalues of the Higgses are given by

$$\varphi_G \sim \begin{pmatrix} \tan\beta \\ -\tan\beta \\ -1 \\ 1 \end{pmatrix},\; (M^2_\nu)_{\text{Goldstone}} = 0 \;\;;\;\; \varphi_A \sim \begin{pmatrix} -1 \\ 1 \\ -\tan\beta \\ \tan\beta \end{pmatrix},\; (M^2_\nu)_A = \frac{2b_0}{\sin 2\beta}\;\;;$$

$$\varphi_{h,H} \sim \begin{pmatrix} (m^2_Z - (M^2_\nu)_A)\cos 2\beta \pm r \\ (m^2_Z - (M^2_\nu)_A)\cos 2\beta \pm r \\ (m^2_Z + (M^2_\nu)_A)\sin 2\beta \\ (m^2_Z + (M^2_\nu)_A)\sin 2\beta \end{pmatrix},\; (M^2_\nu)_{h,H} = \frac{1}{2}\Big(m^2_Z + (M^2_\nu)_A \mp r\Big), \tag{10.10}$$

with $r \equiv \sqrt{(m^2_Z + (M^2_\nu)_A)^2 - 4m^2_Z(M^2_\nu)_A\cos^2 2\beta}$. The upper sign in front of r refers to the index h, the lower one to H. m_Z is the standard model Z boson mass. One can easily check that these four vectors are orthogonal to each other. Normalization factors have been omitted.

Concerning the sneutrino sector, one can reduce the considerations to one generation k in zeroth order. Then the six dimensional vectors are

$$\varphi_{k_1} = \begin{pmatrix} e^{-\mathrm{i}\gamma_k} \cos\vartheta_k \; \delta_{kl} \\ -e^{\mathrm{i}\gamma_k} \sin\vartheta_k \; \delta_{k\bar{l}} \end{pmatrix} \quad , \quad \varphi_{k_2} = \begin{pmatrix} e^{-\mathrm{i}\gamma_k} \sin\vartheta_k \; \delta_{kl} \\ e^{\mathrm{i}\gamma_k} \; \cos\vartheta_k \; \delta_{k\bar{l}} \end{pmatrix}, \tag{10.11}$$

where the index k is not summed over and $l, \bar{l} = 1, 2, 3$. The label at k distinguishes between the two sneutrino eigenvectors for generation k. Evidently, these vectors build an orthonormal basis for the sneutrino-antisneutrino space. The angle ϑ_k is not determined in zeroth order as the sneutrinos and the antisneutrinos are degenerate in one generation with the eigenvalue $M^2_{kk} = (M^2_{\tilde{L}})_{kk} - (\check{v}^2_u - \check{v}^2_d) + \mu_k \mu^*_k$. The phase γ_k, defined by

$$e^{\mathrm{i}\gamma_k} \;\; \equiv \;\; \frac{b_k}{|b_k|}, \tag{10.12}$$

is introduced in order to simplify the following analysis. At this stage however, it appears somewhat arbitrary. For later convenience, it is useful to choose a notation which distinguishes between the normalized zeroth order states φ_J, φ_{k_i} and their corresponding first order corrections $\varphi'_J, \varphi'_{k_i}$, with[3] $J = G, A, h, H$; $k = 1, 2, 3$ and $i = 1, 2$. Furthermore, I do not distinguish between the four/six component vectors of Eqs.(10.10) and (10.11) and their embedded 10-dimensional version.

Switching on the perturbation, I first focus on the sneutrino states. To first order, there is no correction to the eigenvalues, so ϑ_k stays undetermined[4]. The calculation of the perturbed eigenvectors yields an additional contribution to each of the two eigenstates of Eq.(10.11)

$$\varphi'_{k_1} = \sum_J \frac{|b_k|}{M^2_{kk} - (M^2_\nu)_J} \left(\varphi^\dagger_J \cdot \begin{pmatrix} \sin\vartheta_k \\ -\cos\vartheta_k \\ \frac{v_u}{v_d}\cos\vartheta_k \\ -\frac{v_u}{v_d}\sin\vartheta_k \end{pmatrix} \right) \varphi_J + \sum_{k' \neq k} \frac{1}{M^2_{kk} - M^2_{k'k'}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ M^{2*}_{k'k} e^{-\mathrm{i}\gamma_k} \cos\vartheta_k \delta_{k'l} \\ -M^2_{k'k} e^{\mathrm{i}\gamma_k} \sin\vartheta_k \delta_{k'\bar{l}} \end{pmatrix},$$

$$\varphi'_{k_2} = \sum_J \frac{|b_k|}{M^2_{kk} - (M^2_\nu)_J} \left(\varphi^\dagger_J \cdot \begin{pmatrix} -\cos\vartheta_k \\ -\sin\vartheta_k \\ \frac{v_u}{v_d}\sin\vartheta_k \\ \frac{v_u}{v_d}\cos\vartheta_k \end{pmatrix} \right) \varphi_J + \sum_{k' \neq k} \frac{1}{M^2_{kk} - M^2_{k'k'}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ M^{2*}_{k'k} e^{-\mathrm{i}\gamma_k} \sin\vartheta_k \delta_{k'l} \\ M^2_{k'k} e^{\mathrm{i}\gamma_k} \cos\vartheta_k \delta_{k'\bar{l}} \end{pmatrix}. \tag{10.13}$$

The sum over J gives the admixture of Higgses to the sneutrinos, whereas the index k' in the second term runs from 1 to 3, with $k' = k$ excluded. Therefore this part of the first order correction to the sneutrino mass eigenstates provides for generation mixing.

[3]Here, J labels the neutral Higgses.

[4]Appendix F gives an explicit example for this feature.

As is pointed out in Appendix F, the requirement of orthogonality determines the angle ϑ_k. Since the two zeroth order eigenstates of Eq.(10.11) are automatically orthogonal to each other and to the first order corrections of Eq.(10.13), one merely has to consider the product of φ'_{k_1} and φ'_{k_2}. Thus, the orthogonality condition is reduced to

$$\sum_J \frac{|b_k|^2}{(M_{kk}^2-(M_\nu^2)_J)^2}\left(\varphi_J^\dagger\cdot\begin{pmatrix}\sin\vartheta_k\\ -\cos\vartheta_k\\ \frac{v_u}{v_d}\cos\vartheta_k\\ -\frac{v_u}{v_d}\sin\vartheta_k\end{pmatrix}\right)\cdot\left(\varphi_J^\dagger\cdot\begin{pmatrix}-\cos\vartheta_k\\ -\sin\vartheta_k\\ \frac{v_u}{v_d}\sin\vartheta_k\\ \frac{v_u}{v_d}\cos\vartheta_k\end{pmatrix}\right)=0, \tag{10.14}$$

which is satisfied by $\vartheta_k = \frac{\pi}{4}$. This can be seen by looking at the symmetries of the Higgs eigenvectors: For $\varphi_{h,H}$ the first factor becomes zero, and for $\varphi_{G,A}$ the second one vanishes. At first sight, it might appear problematic that the above condition is of second order. Actually, it is necessary to include the second order corrections to the eigenvectors as well. But doing so, one immediately realizes that these additional terms are trivially orthogonal to the unperturbed vectors, therefore they do not need to be taken into account here. Note that the choice of $\vartheta_k = \frac{\pi}{4}$ is independent of the generation k. It orthogonalizes the two perturbed eigenvectors in second order. With respect to normalization, one should notice that the states $\varphi_{k_i}+\varphi'_{k_i}$, $i=1,2$, are normalized in first, but not in second order. Finally, I present the corrections to the difference of the eigenvalues in second order, which is nothing but the sneutrino squared mass splitting

$$(M_\nu^2)_{k_2}-(M_\nu^2)_{k_1} \;=\; \frac{4|b_k|^2 M_{kk}^2 m_Z^2\sin^2\beta}{(M_{kk}^2-(M_\nu^2)_A)(M_{kk}^2-(M_\nu^2)_h)(M_{kk}^2-(M_\nu^2)_H)}, \tag{10.15}$$

where $(M_\nu^2)_{k_i}$ are the squared masses up to second order. This result is a generalization of the one derived for the CP-conserving scenario, given in [53]. However, the difference is only marginal: Instead of b_k one has to insert the absolute values of the R-parity violating parameters into Eq.(10.15).

Coming to the perturbation of the Higgs mass eigenstates φ_J, one notices that again there is no first order contribution to the eigenvalues. The corrections to the eigenstates are obtained by a straightforward calculation

$$\begin{aligned}
\varphi'_G &= 0,\\
\varphi'_A &= \sum_{k=1}^{3}\frac{|b_k|}{M_{kk}^2-(M_\nu^2)_A}\,\frac{1}{\cos\beta}\;\varphi_{k_1},\\
\varphi'_{h,H} &= \sum_{k=1}^{3}\frac{|b_k|}{M_{kk}^2-(M_\nu^2)_{h,H}}\,\frac{m_Z^2(1-4\sin^2\beta)-(M_\nu^2)_A\pm r}{\sqrt{((m_Z^2-(M_\nu^2)_A)\cos 2\beta\pm r)^2+((m_Z^2+(M_\nu^2)_A)\sin 2\beta)^2}}\;\varphi_{k_2}.
\end{aligned} \tag{10.16}$$

These are the crucial expressions that have been left out in the papers by Grossman and Haber [64, 63]. Since the mass eigenstates of the neutral Higgses contain an admixture of sneutrinos, a contribution to the gg-loop correction to the neutrino mass also arises from propagating Higgs type particles. Note that the neutral Goldstone boson which gives mass to the Z boson does not enter the gg-loop calculation because there is no sneutrino component added to φ_G. This can be regarded as a sort of consistency check since a Goldstone boson must not emerge in physical results.

The transformation from the flavor states ϕ^0 defined in Eq.(10.7) to the mass eigenvectors is expressed by a unitary matrix. In the first order approximation presented above one encounters this feature in the fact that all mass eigenstates are normalized and orthogonal to each other up to first order.

10.3 gg-Loop Mass

The calculation of the gg-loop contribution to the neutrino mass is divided into two parts. After the derivation of the complete formula, I will first concentrate on sneutrino type mass eigenstates propagating in the loop, which has been discussed by Grossman and Haber. Then, the additional diagram with the Higgs type eigenstates will be considered. For both parts, the loop graph is sketched in Fig.10.1 where the vertex factors have been included.

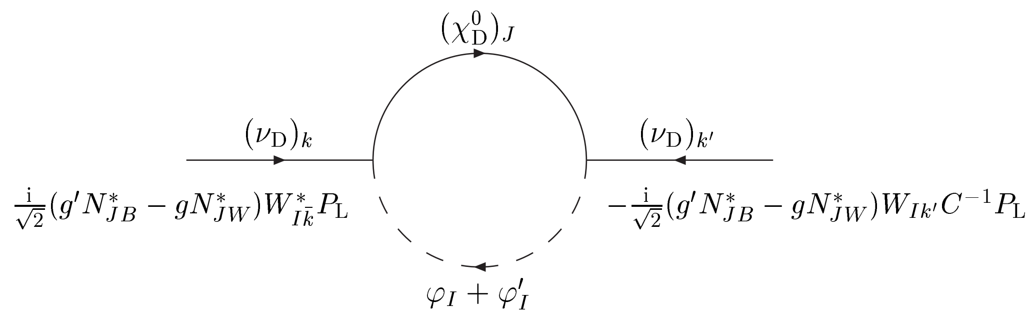


Figure 10.1: gg-loop with mass eigenstates propagating

Here, the notation of the preceding sections has been applied. $(\chi^0_{\rm D})_J$, $J = -1, -2, -3, -4, 1, 2, 3$, are the neutral fermion mass eigenstates. As argued before one can ignore the neutralino-neutrino mixing completely and restrict the calculation to neutralinos traveling in the loop. Thus, J only takes negative values in the following. In first order approximation, the mass eigenstates of the neutral scalars are denoted by $\varphi_I + \varphi'_I$, $I = G, A, h, H, l_1, l_2$, with $l_i = 1, 2, 3$. The connection between flavor and mass eigenstates is given by the first order transformation matrix W which is obtained from[5]

$$\varphi_I + \varphi'_I \;\equiv\; W_{IK}\,\phi^0_K, \tag{10.17}$$

where $K = h_2, \bar{h}_2, h_1, \bar{h}_1, k, \bar{k}$ like in Eq.(10.7). Note that $\bar{k}$ appearing in the left vertex factor is numerically equal to k because the vertex requires an incoming $\tilde{\nu}^*_k = \phi^0_{\bar{k}}$, so if for example $k = 1$ then $\bar{k} = 1$. This is not true for k', which can be different from k. One remark on the directions of the arrows in the loop should be made. It has already been stated that the neutral

[5] A summary of the notation for interaction and mass eigenstates of the spin zero and spin one-half fields introduced up to this point can be found in Appendix G.

fermions are Majorana particles. Therefore the arrow's direction can be chosen arbitrarily, but after having settled one choice, it is essential to keep to it [43]. In particular, switching the arrows does not give new physics. Similar considerations hold for the scalar propagator. The convention applied in the following is that of Fig.10.1. In analogy to the gλ-loop one has to pay attention to possible complex conjugation of the mixing factors, depending on whether the particle goes into the vertex or comes out of it. Taking this into account, one ends up with the following matrix element for the diagram of Fig.10.1

$$\begin{aligned}-\mathrm{i}\mathcal{M}^{\mathrm{gg}}_{k'k} \;=\; & (2\pi)^4\delta(p-p')\sum_{I,J}\int\frac{d^4q}{(2\pi)^4}(\nu_\mathrm{D})^T_{k'}\Big(-\frac{\mathrm{i}}{\sqrt{2}}(g'N^*_{JB}-gN^*_{JW})W_{Ik'}C^{-1}P_\mathrm{L}\Big)\\ & \times\frac{\mathrm{i}(\not{q}+(m^{\mathrm{tree}}_\nu)_J)}{(q^2-(m^{\mathrm{tree}}_\nu)^2_J)}\Big(\frac{\mathrm{i}}{\sqrt{2}}(g'N^*_{JB}-gN^*_{JW})W^*_{I\bar{k}}P_\mathrm{L}\Big)(\nu_\mathrm{D})_k\frac{\mathrm{i}}{(p-q)^2-(M^2_\nu)_I}.\end{aligned} \tag{10.18}$$

The symmetrizing term ($k \longleftrightarrow k'$) must not be added here, as this would signify nothing but the reversal of the arrows in the loop. By application of Eq.(7.7) one obtains the corresponding mass contribution

$$(m_\nu)^{\mathrm{gg}}_{k'k} = -\frac{1}{32\pi^2}\sum_{I,J}(g'N^*_{JB}-gN^*_{JW})^2\;\; W_{Ik'}W^*_{I\bar{k}}\;(m^{\mathrm{tree}}_\nu)_J\;B_0(p^2,(M^2_\nu)_I,(m^{\mathrm{tree}}_\nu)^2_J). \tag{10.19}$$

In contrast to the results of Chapter 8 and 9 there is no difference of two B_0 functions, so one has to find another way to get rid of unwanted infinities. Unitarity of the scalar transformation matrix W provides a solution to this problem [67]: Since the infinite part of B_0 can be separated as in Eq.(E.2), it is independent of $(M^2_\nu)_I$. Thus, the summation over I leads to a factor of $(W^\dagger W)_{\bar{k}k'} = 0$ in front of the infinity. It is therefore reasonable to introduce a modification of the Passarino-Veltman function, where the constant term is subtracted

$$B_0^{\not\infty}(p^2,M^2,m^2) \;\equiv\; B_0(p^2,M^2,m^2) - \Big(\frac{2}{\epsilon}-\gamma+\ln 4\pi+\mathcal{O}(\epsilon)\Big). \tag{10.20}$$

Now everything is prepared to finally calculate both types of neutral loop diagrams. The relevant mixing elements for the mass eigenstates of the sneutrinos are read off from Eqs.(10.11) and (10.13). One finds

$$\begin{aligned} W_{l_ik'} &= \frac{1}{\sqrt{2}}\,e^{-\mathrm{i}\gamma_l}\Big(\delta_{lk'}+\frac{M^{2*}_{k'l}}{M^2_{ll}-M^2_{k'k'}}\sum_{k''\neq l}\delta_{k''k'}\Big),\\ W^*_{l_i\bar{k}} &= \mp\frac{1}{\sqrt{2}}\,e^{-\mathrm{i}\gamma_l}\Big(\delta_{l\bar{k}}+\frac{M^{2*}_{\bar{k}l}}{M^2_{ll}-M^2_{\bar{k}\bar{k}}}\sum_{k''\neq l}\delta_{k''\bar{k}}\Big).\end{aligned} \tag{10.21}$$

Here, the upper sign belongs to $i=1$, the lower to $i=2$. Taking the sum over i in Eq.(10.19) leads to a difference of B_0 functions with one depending on $(M^2_\nu)_{l_1}$ and the other on $(M^2_\nu)_{l_2}$. As this mass squared difference is of second order[6] one can safely neglect the first order generation

[6]Compare with Eq.(10.15).

mixing of sneutrinos given in Eq.(10.21). Using the approximation of Appendix E one ends up with

$$(m_\nu)^{\rm gg1}_{k'k} \simeq \frac{1}{64\pi^2}\sum_J (g'N^*_{JB} - gN^*_{JW})^2\, e^{-2{\rm i}\gamma_k}\, \frac{(M^2_\nu)_{k_2} - (M^2_\nu)_{k_1}}{(m^{\rm tree}_\nu)_J}\, \frac{(z^k_J - \ln z^k_J - 1)}{(1-z^k_J)^2}\quad \delta_{k'k}, \qquad (10.22)$$

where $z^k_J \equiv \frac{(M^2_\nu)_{k_1}+(M^2_\nu)_{k_2}}{2\ (m^{\rm tree}_\nu)^2_J}$ is the ratio of the squared sneutrino and neutralino masses, and J runs from -1 to -4. This loop correction to the neutrino mass depends on the sneutrino-antisneutrino mass splitting in one generation. Thus, no new generation mixing in the neutrino sector occurs. Comparing Eq.(10.22) with Grossman and Haber [64], we notice that both results do not agree in regard to complex phases. Whereas I have the complex conjugates of the neutralino mixing coefficients N_{JK} squared, they obtain absolute values. Also, the factor $e^{-2{\rm i}\gamma_k}$ which is due to the phase of the b_k parameter does not appear in [64].

So far, only sneutrinos traveling in the loop have been considered. But there are also the neutral Higgses with small admixtures of sneutrinos which can couple to the vertices in the gg-loop. In analogy to the procedure above the correction to the neutrino mass coming from these parts is calculated to

$$\begin{aligned}(m_\nu)^{\rm gg2}_{k'k} \;=\; & -\frac{|b_{k'}|\,|b_k|}{64\pi^2}\sum_J (g'N^*_{JB} - gN^*_{JW})^2\, e^{-{\rm i}(\gamma_{k'}+\gamma_k)}\, (m^{\rm tree}_\nu)_J \\ & \times\left\{ \frac{w_h^2\ B_0^{\not\infty}(p^2,(M^2_\nu)_h,(m^{\rm tree}_\nu)^2_J)}{(M^2_{k'k'}-(M^2_\nu)_h)(M^2_{kk}-(M^2_\nu)_h)} + \frac{w_H^2\ B_0^{\not\infty}(p^2,(M^2_\nu)_H,(m^{\rm tree}_\nu)^2_J)}{(M^2_{k'k'}-(M^2_\nu)_H)(M^2_{kk}-(M^2_\nu)_H)}\right.\\ & \left. -\frac{w_A^2\ B_0^{\not\infty}(p^2,(M^2_\nu)_A,(m^{\rm tree}_\nu)^2_J)}{(M^2_{k'k'}-(M^2_\nu)_A)(M^2_{kk}-(M^2_\nu)_A)}\right\}, \end{aligned}\qquad (10.23)$$

where $w_{A,h,H}$ are the k-independent factors in Eq.(10.16), i.e.

$$w_A \equiv \frac{1}{\cos\beta}\quad , \quad w_{h,H} \equiv \frac{m^2_Z(1-4\sin^2\beta) - (M^2_\nu)_A \pm r}{\sqrt{((m^2_Z-(M^2_\nu)_A)\cos 2\beta \pm r)^2 + ((m^2_Z+(M^2_\nu)_A)\sin 2\beta)^2}}. \qquad (10.24)$$

For $p^2 = 0$ the modified Passarino-Veltman function can be evaluated analytically

$$B_0^{\not\infty}(0,M^2,m^2) \;=\; -\frac{m^2\ln m^2 - M^2\ln M^2}{m^2-M^2} + 1. \qquad (10.25)$$

Putting all that together seems to be quite a messy business, and it is! But still some general comments on Eq.(10.23) can be made. The propagation of neutral Higgs mass eigenstates does not only give rise to diagonal but also to off-diagonal entries in the 3×3 neutrino mass matrix. Furthermore, the expression in Eq.(10.23) is explicitely symmetric under the exchange of k and k' although the term $(k \longleftrightarrow k')$ has not been added.

Concluding this chapter, it should be emphasized that the gg-loop corrections are quadratic in R-parity violating parameters, just like for all the other loop diagrams. In the case of the sneutrino type scalar propagator, this is based on the sneutrino-antisneutrino mass splitting, which is of second order. The other contribution arises from the first order admixture of sneutrinos to the Higgses. Both have to be taken into account when dealing with the gg-loop.

Chapter 11

Discussion

Having derived the analytic expressions for the loop contributions to the neutrino mass matrix it is possible to investigate the predictions of the R-parity violating theory. Many input parameters have to be chosen, with simplifying assumptions being made. The neutrino mass spectra will be shown and discussed for different scenarios in detail . The results will then be compared to experimental data.

11.1 Physical and Unphysical Parameters

Before plugging in numerical values for the parameters of the theory some comments on the distinction of physical and unphysical degrees of freedom are necessary. In Chapter 5 I have already addressed the possibility of rotating fields in order to diagonalize mass matrices. Doing so does not only simplify things. In addition, one also gets rid of unphysical degrees of freedom. To understand how this happens and what is meant by unphysical parameters it is instructive to analyze the case of the standard model first[1]. The standard model Lagrangian density is given by the kinetic parts which are diagonal in the generation indices, and the Yukawa terms which, in general, mix the generations. The allowed gauge invariant Yukawa coupling matrices, denoted by $h^{\{e,u,d\}}$, enter the Lagrangian density in the following form

$$h^e_{ij}\,\Phi^\dagger\,L_i\,\bar{E}_j \quad , \quad h^u_{ij}\,\Phi'^\dagger\,Q_i\,\bar{U}_j \quad , \quad h^d_{ij}\,\Phi^\dagger\,Q_i\,\bar{D}_j, \tag{11.1}$$

where $\Phi = \begin{pmatrix}\Phi^+\\ \Phi^0\end{pmatrix}$ is the standard model Higgs doublet and $\Phi' \equiv \mathrm{i}\sigma^2\Phi^*$, with the Pauli matrix σ^2 acting on the $SU(2)_L$ doublet. The notation of the fermionic matter fields is adapted to the superfield definitions of Chapter 4. Since the indices enumerate the three generations, each Yukawa matrix includes nine real numbers and nine phases. Now, the pivotal point is the following: Independent global $U(3)$ rotations, where the 3 refers to the generations, can be applied to the fields L_i, E_i, Q_i, U_i and D_i without changing the kinetic terms. Choosing these transformations properly diagonalizes e.g. h^e and h^u. Thus, unphysical parameters are eaten up by rotation angles and phases. But the procedure of diagonalization does not determine all

[1]The discussion here follows that of Ref.[68].

phases of the unitary transformation matrices. This can be seen easily by comparing the number of parameters before and after the transformation. In the case of the leptonic Yukawa coupling h^e, six angles and nine phases are needed from the transformation matrices U_L and U_E so that it adds up, together with the three remaining lepton masses, to 18 parameters. However, two independent unitary 3×3 matrices include six angles and twelve phases. Therefore, three phases are not determined by the diagonalization. They correspond to three global $U(1)$ symmetries connected with lepton number conservation in each generation. Similarly, U_Q and U_U are determined by diagonalizing h^u. As the parameters of U_Q are fixed in that way, except for three phases, the same procedure cannot be applied to h^d any more. Here, one can only remove three angles and $6+2$ phases from the Yukawa matrix. The remaining physical parameters are the down quark masses and the parameters of the CKM matrix, three angles and one phase. Naively one would think that six phases could be removed from the theory by U_D and another three by U_Q. This is not correct because a universal phase factor for all quarks in all generations[2] does not change the coupling constants h^u and h^d. In particular, one could have chosen this factor in a way that $U_Q \in SU(3)$, which reduces the number of phases undetermined by the diagonalization of h^u from three to two. Therefore only $6+2$ phases can be absorbed by a redefinition of fields.

The above discussion can be translated to supersymmetry, where the rotations are employed on superfields. In the R-parity violating model considered in this thesis $L_\alpha = (H_1\,,\,L_i)$ can be redefined by a unitary 4×4 matrix which depends on 16 parameters. This has been used in Chapter 5 to get rid of three complex vacuum expectation values and to render the VEV of H_1 real. Thus, seven unphysical degrees of freedom have been canceled. The remaining nine parameters are collected in a $U(3)$ matrix acting on L_i. Taking into account the other lepton and quark superfields one is left with five $U(3)$ symmetries just as in the standard model. The process of absorbing parameters from the coupling constants takes place at the level of the superpotential given in Eq.(4.4). Partly this has already been done in Chapter 5, where the electron and the down quark mass matrices have been diagonalized. In contrast to the standard model, lepton number is generally violated in $\not{R}_P$ supersymmetry. Therefore it is possible to absorb three more phases. A convenient choice which I will follow below is to choose the mass parameters μ_i from the bilinear term $\mu_i L_i H_2$ to be real and non-negative. Concerning the up quark masses one ends up with the CKM matrix. However, the up quarks are irrelevant for calculating the loop contribution to the neutrino masses. Therefore, the CKM matrix, which includes a CP violating phase, has been shifted from the down to the up quarks[3], compared to the standard model. If baryon number is violated by the term $\lambda'' U^* D^* D^*$, one can in addition remove another phase from λ''. It is not possible to get rid of the phase in the up quark CKM matrix since the λ'- and the h-term of the superpotential are baryon number conserving.

Having removed the unphysical degrees of freedom one could in principle scan over the remaining $\not{R}_P$-MSSM parameter space and find regions which make reasonable predictions for the neutrino masses. This is clearly not practicable since even the MSSM contains more that one hundred physical parameters. A way out is given by assuming certain supersymmetry breaking mechanisms which lead to a drastically reduced number of independent parameters.

[2]This corresponds to the $U(1)$ symmetry which gives baryon number conservation.

[3]This is a matter of convention and does not change physical predictions.

11.2 Input Parameters and Simplifications

At the 2001 "Snowmass Workshop on the Future of Particle Physics" a set of benchmark points and parameter lines in the MSSM parameter space has been agreed upon. There, the most prominent supersymmetry breaking mechanisms, namely minimal supergravity, gauge-mediated and anomaly-mediated supersymmetry breaking, have been applied to obtain the low energy MSSM parameters from a few high energy parameters via renormalization group running. The choice of the high energy input parameters defines different scenarios which, at the weak scale, have to respect currently existing experimental constraints. The Snowmass points and slopes (SPS) are thus motivated both theoretically and experimentally. They "should be regarded as a recommendation for future studies of SUSY phenomenology" [69]. In the following I will only consider the points of the SPS-scenarios suggested in [69] where a detailed discussion of the issue can be found. The corresponding low energy MSSM parameters needed for calculating the neutrino mass matrix are given in Appendix H.

Unfortunately, the procedure described above does not provide values for the R-parity violating parameters which, however, are essential for generating neutrino masses without introducing right chiral neutrinos. Altogether, there are 42 complex $R\!\!\!/_P$ parameters entering the calculation of the loop contributions to the neutrino mass matrix

$$\mu_i \qquad , \qquad b_i \equiv \mu_i(B)_i \qquad , \qquad \lambda_{ijk} \qquad , \qquad \lambda'_{ijk}, \tag{11.2}$$

where the indices run from 1 to 3 and λ_{ijk} is antisymmetric in i and j. Obviously one has to think of an additional way to reduce the number of parameters. Universality for the bilinear soft supersymmetry breaking term only removes three parameters by setting all $(B)_i$ equal to $(B)_0$ which is calculated in the SPS-scenarios. Therefore it is necessary to choose further simplification methods. As a first try, one can consider the crude choice where the parameters of Eq.(11.2) are independent of the attached indices. In the case of λ one has to take care of the antisymmetry mentioned above. Finally, one ends up with three independent parameters μ, λ and λ'

$$\mu \equiv \mu_i \qquad , \qquad \lambda \equiv \lambda_{12k} = \lambda_{13k} = \lambda_{23k} \qquad , \qquad \lambda' \equiv \lambda'_{ijk}. \tag{11.3}$$

Note the difference between the MSSM parameter μ_0 and the R-parity violating one μ. In order to fix the intervals of the remaining three variables to be scanned over one can have a look at existing constraints on $R\!\!\!/_P$ couplings. References [70, 57] give a list of bounds for both bilinear and trilinear couplings. However, it should be emphasized that these bounds depend on common but still selected assumptions, one of which is that the supersymmetry mass scale is around 100 GeV. Therefore I do not only take the weakest bounds as upper limits for the intervals but allow for values going beyond these bounds. Choosing the lower limits of the intervals three orders of magnitude smaller, μ, λ and λ' are taken to be out of

$$\frac{\mu}{\mathrm{GeV}}\,,\ \lambda\,,\ \lambda' \ \in\ [\,10^{-6}, 10^{-3}\,]\,. \tag{11.4}$$

Other constraints concerning products of R-parity violating trilinear couplings have to be taken into account as well, since they are very strict [56]. Consequently, I choose the point of reference as

$$\frac{\mu}{\mathrm{GeV}} = 10^{-4} \qquad , \qquad \lambda = 10^{-4} \qquad , \qquad \lambda' = 10^{-4}, \tag{11.5}$$

which satisfies all constraints except for $|\lambda'_{i12}\lambda'_{i21}| < 10^{-9}$. Luckily, this trilinear coupling does not change the contribution to the neutrino mass matrix very much assuming that all λ'_{ijk} are approximately of the same order. The reason for this is the hierarchy of down quark masses which enter the formula in Eq.(8.30). In the next section, the predictions concerning neutrino masses will be stated and discussed, varying only one of the three parameters: μ, λ and λ'. Furthermore, one can allow for phases being attached to the couplings[4] λ and λ'. Note that due to the choice of basis discussed before, the parameter μ does not carry a phase factor.

11.3 Neutrino Mass Spectra

After all these preliminary remarks, the calculation of the complete 3×3 neutrino mass matrix $(m_\nu)_{k'k}$ given by Eqs.(8.18), (8.30), (9.16), (10.22) and (10.23) can be tackled:

$$(m_\nu)_{k'k} \;\equiv\; (m_\nu)^{\lambda\lambda}_{k'k} + (m_\nu)^{\lambda'\lambda'}_{k'k} + (m_\nu)^{\mathrm{g}\lambda}_{k'k} + (m_\nu)^{\mathrm{gg1}}_{k'k} + (m_\nu)^{\mathrm{gg2}}_{k'k}. \tag{11.6}$$

Here the tree level contributions have been omitted since they can be neglected compared to the one loop masses. Of course it is necessary to verify this assumption for the chosen sets of input parameters. Roughly speaking, the predictions for the lightest neutrino cannot be trusted for this reason. Having calculated the symmetric matrix $(m_\nu)_{k'k}$, one can diagonalize it by application of a unitary 3×3 matrix N, which defines the transformation from the flavor to the mass basis[5]. The resulting real and non-negative eigenvalues are the masses of the three neutrinos[6].

In this diploma thesis, I have considered the evolution of these masses under variation of λ, λ' and μ for all SPS-scenarios. The corresponding graphs are added in Appendix H. It is interesting to note that, with the exception of SPS 2, all Snowmass models show very similar behavior. This can be understood by the fact that, for these scenarios, the neutrino mass matrix is determined almost completely by the $\lambda\lambda$- and $\lambda'\lambda'$-loops, which in turn depend only logarithmically on the ratio of sparticle masses and on the associated sparticle mixing angles. Therefore, the spectrum coming from these loop contributions is dominated by the R-parity violating couplings λ and λ' themselves. In the following, I will pick out SPS 3 as an average case, representing all Snowmass scenarios except for SPS 2. Certainly, this restricts the solutions to the question of neutrino masses, but still general characteristic properties can be examined. As a measure of how much the models subsumed under SPS 3 vary, one can compare the values of the neutrino masses obtained at the beginning and at the end of the scanned intervals. Disregarding SPS 4, which is a large $\tan\beta$ scenario, one finds that the remaining models differ

[4]For convenience, the phases of λ and λ' will be given explicitly if considered complex. In that way one can avoid writing $|\lambda|$ and $|\lambda'|$.

[5]Compare to the discussion at the beginning of Chapter 10.

[6]All numerical calculations have been done with "Mathematica".

from SPS 3 at most by a factor of 1.8. In the case of SPS 4 the largest deviation lies within a factor of 5.

The reason for SPS 2 generating a downright different mass spectrum is provided by the large value of the bilinear soft supersymmetry breaking parameter $(B)_0$. Thus, the neutrino mass matrix is mainly determined by the gg-loop. Actually, it is the new contribution $(m_\nu)^{\rm gg2}_{k'k}$ which dominates the neutrino mass matrix. This term had to be added to the diagonal loop corrections $(m_\nu)^{\rm gg1}_{k'k}$ induced by sneutrino mass splitting.

11.3.1 The SPS 3 Scenario

Focussing on the the case of SPS 3 one first realizes that for $\lambda = 10^{-4}$ and $\lambda' = 10^{-4}$ the mass spectrum does not depend on the value of $\frac{\mu}{\rm GeV} \in [10^{-6}, 10^{-3}]$. Even lowering the trilinear couplings λ and λ' by a factor of 10 does not change this feature. Therefore, one can reduce the considerations to the dependence of the neutrino masses on λ and λ'.

Fig.11.1 shows the λ dependence of the neutrino masses for different values of λ'. The dashed line indicates the tree level mass contribution calculated by Eq.(6.11). Since tree level effects have been neglected one cannot interpret results below and nearby this line physically. Furthermore, I have included the preferred regions for the masses of the two heaviest neutrinos coming from solar and atmospheric neutrino experiments. This issue will be addressed in the next section in more detail.

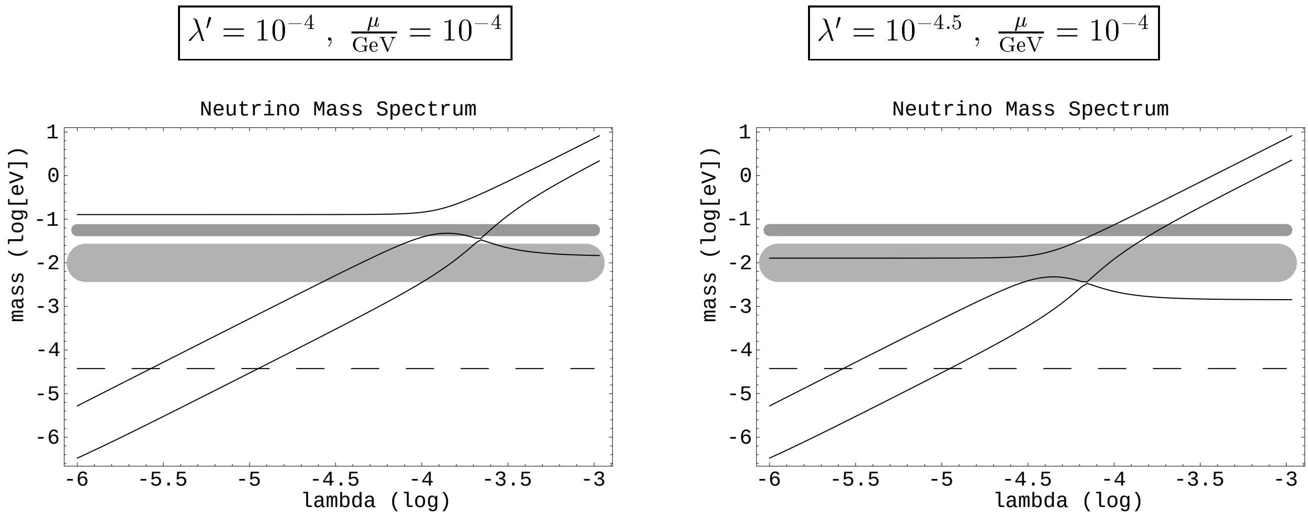


Figure 11.1: Dependence of the neutrino mass spectrum on λ for different values of λ'. The dashed line indicates the tree level contribution to the neutrino mass. The horizontal gray bars show the regions preferred by solar (thick) and atmospheric (thin) neutrino data.

For large values of λ, where the $\lambda\lambda$-loop dominates over the $\lambda'\lambda'$-loop, one obtains two heavy neutrinos which differ in mass only by a factor of approximately 4. In the case of small and decreasing λ, one neutrino stays heavy, not depending on λ any longer, whereas the other two decrease in mass. The region where the line of the heaviest neutrino bends and where the other two curves cross is determined by the relative size of the $\lambda\lambda$- and $\lambda'\lambda'$-loop contributions. Therefore, by lowering λ' as in the right diagram this region is shifted to lower masses and lower values of λ.

Similarly, one can consider the λ' dependence of the neutrino mass spectrum for fixed values of λ. The results are depicted in Fig.11.2.

$\boxed{\lambda = 10^{-4}\ ,\ \frac{\mu}{\mathrm{GeV}} = 10^{-4}}$ $\boxed{\lambda = 10^{-4.5}\ ,\ \frac{\mu}{\mathrm{GeV}} = 10^{-4}}$

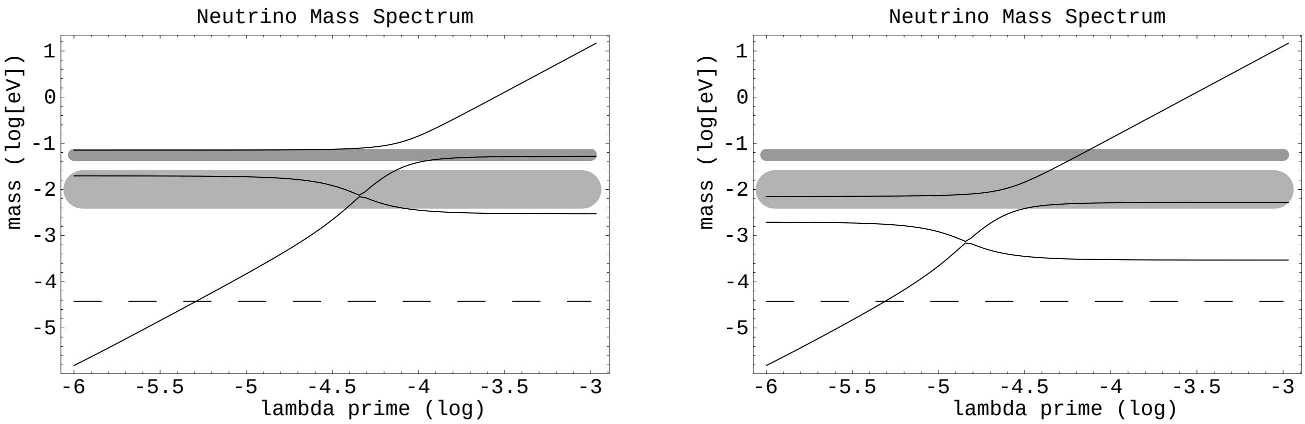


Figure 11.2: Dependence of the neutrino mass spectrum on λ' for different values of λ.

The above diagrams are interpreted in much the same way as before. Domination of the $\lambda\lambda$-loop for small λ' generates two heavy neutrinos with masses being independent of λ' and differing from each other only by a factor of approximately 4. For large and increasing values of λ' the heaviest neutrino increases in mass and the other two stay small and constant.

The neutrino mass spectrum's dependence on the absolute values of λ and λ' in the SPS 3 scenario can be summarized by stating that the $\lambda\lambda$-loop determines the mass of two neutrinos and the $\lambda'\lambda'$-loop that of the third one. Clearly, this result is valid only for the simple model where all λ_{ijk} and λ'_{ijk} are independent of the attached indices.

So far, only the variation of absolute values has been discussed. It is interesting to study the effects on neutrino masses coming from complex phases. For the selected model the spectrum depends only on the relative phase factor between λ and λ'. Therefore, one can restrict the considerations to a phase attached to one trilinear coupling, e.g. λ. Fig.11.3 shows the dependence of the mass spectrum on this phase for three different values of λ'. Obviously, a phase factor changes the neutrino mass spectrum significantly only in the region where the $\lambda\lambda$- and the $\lambda'\lambda'$-loop contributions to the neutrino mass matrix are of the same order. Thus, a complex phase for one of the two terms causes cancellations which alter the neutrino masses. This effect is largest for a phase of $\frac{\pi}{2}$ since λ enters Eq.(8.18) quadratically. Quantitatively, it can change the masses of the two light neutrinos by a factor larger than 2. It is interesting to note that a phase lowers the masses of the heaviest and the lightest neutrino whereas the third neutrino gains mass. However, it is important to keep in mind that the effect of a complex phase is only large for specifically selected values of λ and λ'.

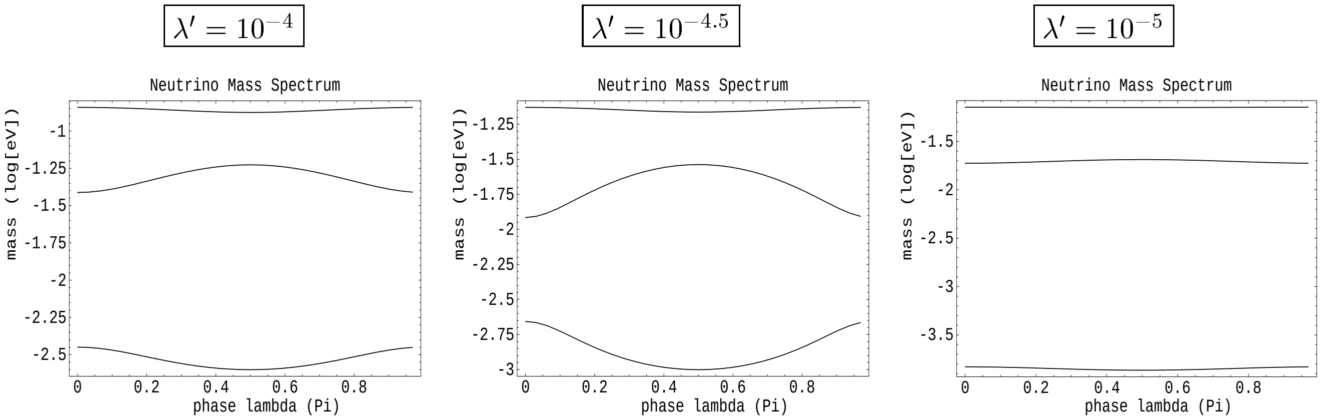


Figure 11.3: Dependence of the neutrino mass spectrum on the phase of λ for different values of λ'. Here $\lambda = 10^{-4}$ and $\frac{\mu}{\text{GeV}} = 10^{-4}$ are fixed.

11.3.2 The SPS 2 Scenario

It has already been mentioned that in the SPS 2 model the neutrino mass matrix is dominated by the gg-loop contribution. Therefore the effects of the trilinear couplings can be neglected for $\frac{\mu}{\text{GeV}} \sim 10^{-4}$ as can be seen in Appendix H. If μ decreases, the $\lambda\lambda$- and the $\lambda'\lambda'$-loop perturb the pure gg-loop mass spectrum, as can be seen by comparing the two mass spectra in Fig.11.4. The diagram on the right hand side is mainly determined by the gg-loop. There the two heavy neutrinos have a constant mass ratio of approximately 50 for almost all values of μ. This changes if the trilinear couplings become more important as shown on the left: For small and decreasing μ one of the heavy neutrinos does not decrease in mass but reaches a constant value. This feature provides the possibility to find solutions in parameter space which generate neutrino masses in agreement with those suggested by solar and atmospheric neutrino experiments, dealt with in the following section.

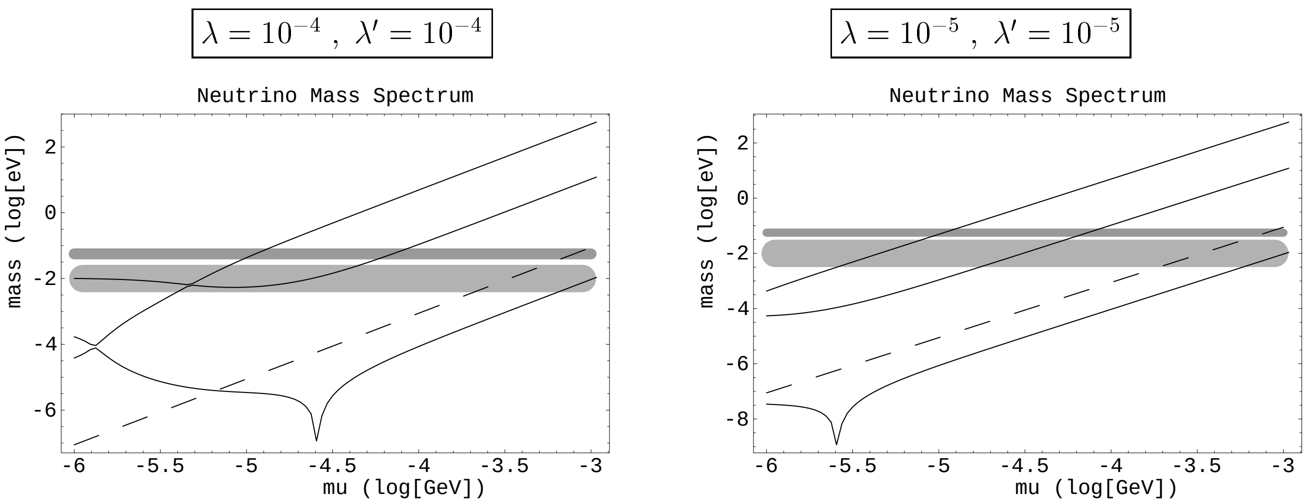


Figure 11.4: Dependence of the neutrino mass spectrum on μ for different values of λ and λ'.

11.4 Comparison with Experimental Data

The crucial question arises now, whether the model can accommodate experimental results coming from direct mass measurements, solar and atmospheric neutrino observation and the measurement of neutrinoless double beta decay. There are other cosmological and astrophysical considerations concerning the size of neutrino masses, some of these will be addressed briefly.

11.4.1 Direct Mass Measurements

Up to now, kinematic tests have only set weak upper limits on the neutrino masses. These experiments carefully examine beta decay spectra. If neutrinos have mass, the energy spectrum of the detected particle is distorted near the endpoint. In the case of tritium beta decay, the energy of an electron is measured. Thus, such an experiment is sensitive to the mass of an electron type neutrino. However, it is necessary to take neutrino mixing into account. Strictly speaking, an electron neutrino does not have a mass in general. It is, in fact, composed of three mass eigenstates which contribute more or less to the flavor neutrino. Therefore, a tritium beta decay experiment actually measures a combination of all three masses, which depends on how much the different mass eigenstates contribute to the electron neutrino. Denoting this value by m_{ν_e}, the Mainz Neutrino Mass Experiment [12] gives an upper bound of

$$m_{\nu_e} \;<\; 2.8\,\mathrm{eV} \;\; (95\%\ \mathrm{C.L.}). \tag{11.7}$$

Similar but less accurate kinematical bounds are obtained by studying $\pi^+ \to \mu^+ \nu_\mu$ [71] and $\tau \to \nu_\tau + 5\,\pi$ [72]

$$\begin{aligned} m_{\nu_\mu} &< 0.17\,\mathrm{MeV} \;\; (90\%\ \mathrm{C.L.}), \\ m_{\nu_\tau} &< 18.2\,\mathrm{MeV} \;\; (95\%\ \mathrm{C.L.}). \end{aligned} \tag{11.8}$$

The neutrino masses calculated in this diploma thesis are in agreement with all these upper limits.

11.4.2 Solar and Atmospheric Neutrino Oscillations

The observed solar and atmospheric neutrino anomalies provide the best evidence for neutrino masses [1]. Applying the concept of neutrino oscillations leads to preferred regions for both mass squared differences and mixing angles. Considering all three neutrinos, the flavor eigenstates ν_α, $\alpha = e, \mu, \tau$, are related to the mass eigenstates ν_i, $i = 1, 2, 3$, through the unitary 3×3 mixing matrix $N^\dagger$

$$\nu_\alpha \;=\; (N^\dagger)_{\alpha i}\, \nu_i, \tag{11.9}$$

which can be parameterized by three angles θ_{ij} and six phases ε_i and ε_{ij} with $i, j = 1, 2, 3$ and $i < j$ [65]:

$$N^\dagger \;=\; \begin{pmatrix} e^{i\varepsilon_1} & & \\ & e^{i\varepsilon_2} & \\ & & e^{i\varepsilon_3} \end{pmatrix} \cdot \begin{pmatrix} 1 & & \\ & c_{23} & s^*_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & & s^*_{13} \\ & 1 & \\ -s_{13} & & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s^*_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}. \tag{11.10}$$

Here, $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij} e^{i\varepsilon_{ij}}$. A global three generation analysis [73] of the solar and atmospheric neutrino data combined with the results coming from CHOOZ [74] favors the following scenario. The atmospheric neutrino anomaly can be explained by a large mixing between ν_μ and ν_τ with a mass squared difference $\Delta m^2_{\text{atm}} \in [1.5\,,\,6] \times 10^{-3}\,(\text{eV})^2$. For solar neutrinos a large $\nu_1 - \nu_2$ mixing is preferred which leads to $\Delta m^2_{\text{solar}} \in [2\,,\,70] \times 10^{-5}\,(\text{eV})^2$. The $(N^\dagger)_{e3}$ mixing is strongly constrained ($(N^\dagger)_{e3} \leq 0.3$ at 99 % C.L.) which corresponds to an upper value for θ_{13} of 17.5°.

Assuming a hierarchy in the neutrino masses m_i, i.e. $m_1 \ll m_2 \ll m_3$, one can interpret the mass squared differences as the mass squares of the the two heavy neutrinos

$$
\begin{aligned}
m_3 &\simeq \sqrt{\Delta m^2_{\text{atm}}} \in [10^{-1.4}\,,\,10^{-1.1}]\ \text{eV}, \\
m_2 &\simeq \sqrt{\Delta m^2_{\text{solar}}} \in [10^{-2.4}\,,\,10^{-1.6}]\ \text{eV}.
\end{aligned}
\tag{11.11}
$$

These two intervals have been included in the neutrino mass spectra of the previous section. In the case of SPS 3 it is clear from Fig.11.1 that λ' has to be smaller than 10^{-4} because otherwise the heaviest neutrino would lie above the atmospheric neutrino mass. One can classify possible solutions in the following way: (a) $\lambda' = 10^{-4.2}$, (b) λ' much smaller, e.g. 10^{-6} and (c) λ' somewhere in between. Fig.11.5 shows the neutrino mass spectrum depending on λ for the first two cases. In addition, the evolution of the three mixing angles is given below

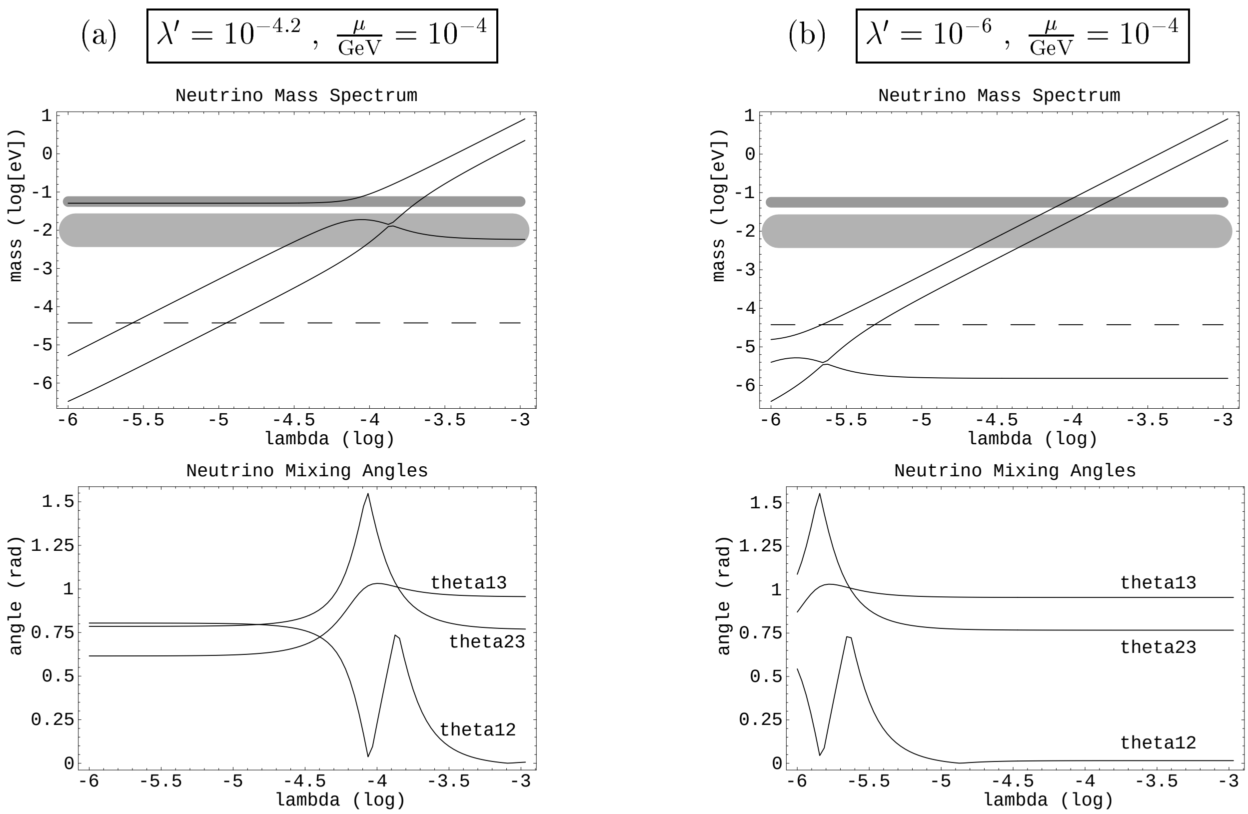


Figure 11.5: Predictions of the SPS 3 scenario with $\lambda' = 10^{-4.2}$ and $\lambda' = 10^{-6}$.

the mass spectra. Mixing angles have to be taken into account as well if searching for solutions compatible with the solar and the atmospheric neutrino anomalies. For (a), i.e. $\lambda' = 10^{-4.2}$, the neutrino mass spectrum suggests $\lambda \in [10^{-4.6}, 10^{-4.0}]$. However, the mixing angles do not agree with the expected values. θ_{13} is expected to be small as mentioned above, but in the model considered here it takes values between 35° and 60°. There are two possible explanations for this disagreement: Either the assumption that the trilinear couplings can be thought of as being independent of the attached indices is too simple, or it is necessary to add tree level effects which may alter θ_{13}. In the latter case, λ has to be small so that the tree level can contribute considerably to the neutrino mass matrix. Case (b) predicts $\lambda \in [10^{-4.2}, 10^{-4}]$ and a large mixing angle θ_{23}. However, θ_{12} is small and θ_{13} large as opposed to the experimentally favored regions. It might be, though, that tree level effects change these values since they are large compared to the smallest mass in the spectrum. For (c) with $\lambda' = 10^{-4.5}$ one obtains large angles θ_{12} and θ_{13} in the preferred λ-region, but the mass of the lightest neutrino is several orders of magnitude larger than the tree level mass scale. Therefore tree level effects cannot shift the large mixing angle θ_{13} to the small values demanded by experiment.

For SPS 2 there exist two different kinds of solutions which agree with the mass squared differences of the solar and atmospheric neutrino measurements. Fig.11.6 shows both cases. For (a′) the preferred region for $\frac{\mu}{\mathrm{GeV}} \in [10^{-5}, 10^{-4.8}]$ lies right from the point where the lines of the heavy neutrinos cross. However, the predicted value for the mixing angle θ_{23} disagrees

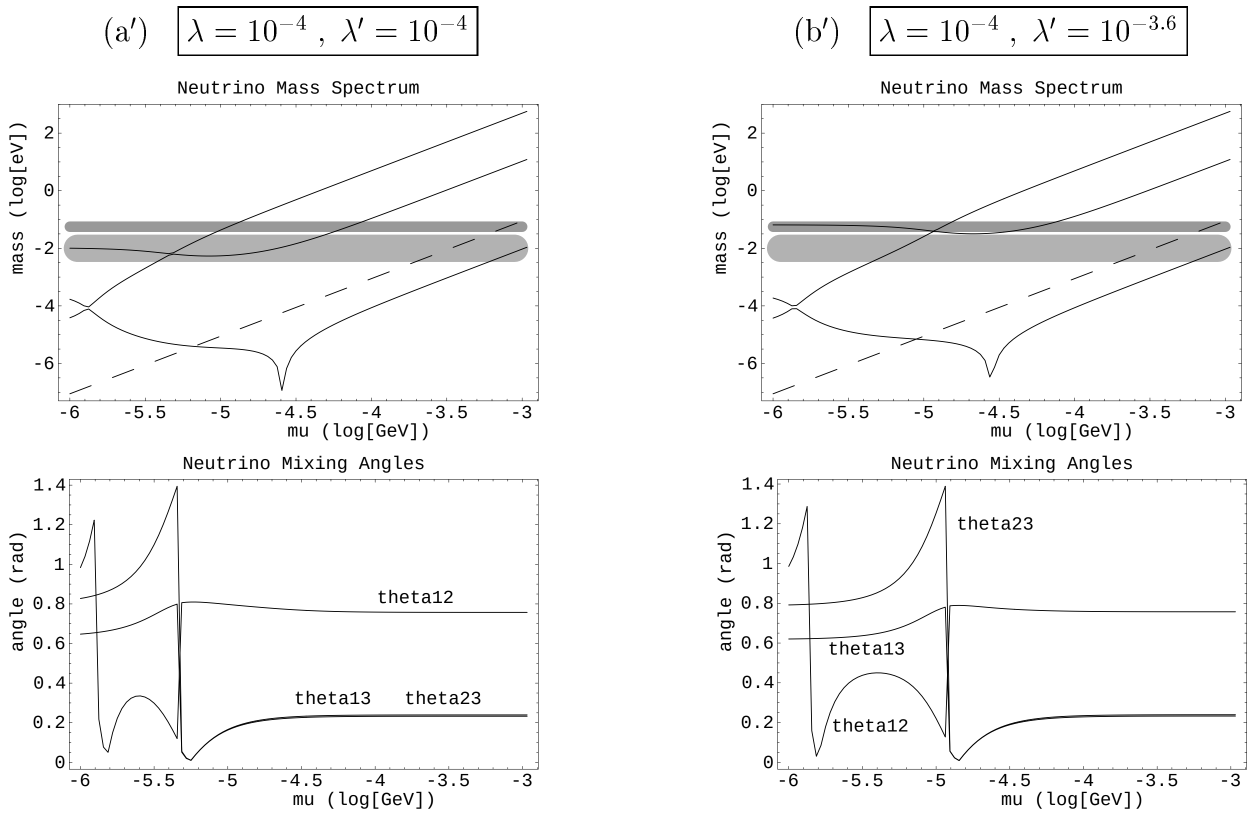


Figure 11.6: Predictions of the SPS 2 scenario with $\lambda' = 10^{-4}$ and $\lambda' = 10^{-3.6}$.

with the large mixing explanation of the atmospheric neutrino problem. In case (b′) the mass spectrum provides solutions for $\frac{\mu}{\mathrm{GeV}} \in [10^{-5}, 10^{-5.4}]$. The mixing angle θ_{12} can be raised up to approximately 25°. As in the SPS 3 scenario, θ_{13} is predicted to be large. Maybe this problem disappears when also tree level effects are taken into account. However, I consider it more likely that the solution will be found by relaxing the simplifying assumption for λ and λ'.

In conclusion, it is possible to find regions in the μ, λ, λ'-parameter space of the simple model considered in this diploma thesis which predict neutrino masses that are in agreement with solar and atmospheric neutrino observations. The calculation of the mixing angles, however, does not yield the experimentally favored values.

11.4.3 Neutrinoless Double Beta Decay

Double beta decay is a very rare process since it requires an arrangement of nuclei of different neighboring Z-values which forbids single beta decays. In 1939, Furry [75] suggested a double beta decay process where one neutrino is emitted by a neutron and absorbed by another one. The corresponding Feynman diagram is shown in Fig.11.7.

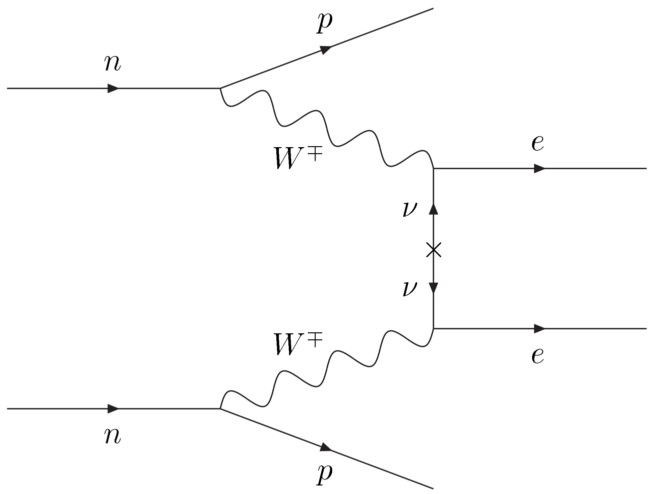


Figure 11.7: Neutrinoless Double Beta Decay

This process is only possible if neutrinos have Majorana mass. Thus the observation of neutrinoless double beta decay ($\beta\beta_{0\nu}$) provides a measurement of the neutrino mass[7]. More precisely, the decay rate is proportional to the square of the "effective mass" m_{eff}, defined by

$$m_{\mathrm{eff}} = |(N^\dagger)^2_{ei}\, m_i|, \tag{11.12}$$

[7]This, however, depends on the assumption that neutrinoless double beta decay is dominated by the diagram of Fig.11.7 [76]. There exist many other non-neutrino processes which might be important for $\beta\beta_{0\nu}$ decay [2].

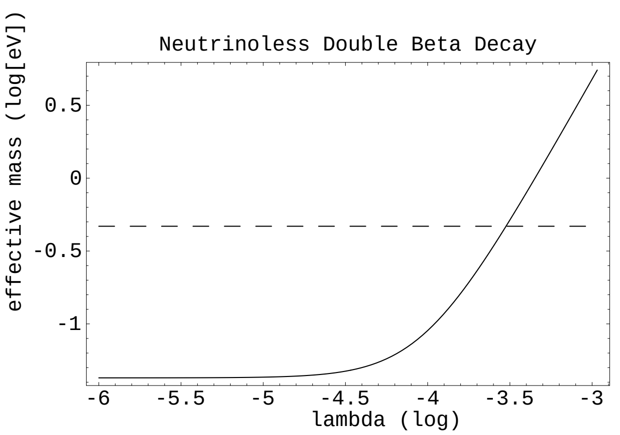
where m_i, $i = 1, 2, 3$, are the three neutrino masses and $N^\dagger$ is the neutrino mixing matrix given in Eq.(11.9). Since the elements of $N^\dagger$ are generally complex, even large neutrino masses can generate small $m_{\rm eff}$ due to cancellations.

The Heidelberg-Moscow Experiment measures the ^{76}Ge decay and puts an upper bound on the effective mass [77]

$$m_{\rm eff} \leq 0.46\,\text{eV} \;\; (90\%\ \text{C.L.}). \tag{11.13}$$

This limit is included in Fig.11.8 where the SPS 3 prediction of the effective mass is depicted for $\lambda' = 10^{-4}$ and $\frac{\mu}{\rm GeV} = 10^{-4}$. In addition, the dependence on the complex phase of λ is given for an absolute value of 10^{-4}.

$\boxed{\lambda' = 10^{-4}\ ,\ \frac{\mu}{\rm GeV} = 10^{-4}}$



$\boxed{\lambda = 10^{-4}\ ,\ \lambda' = 10^{-4}\ ,\ \frac{\mu}{\rm GeV} = 10^{-4}}$

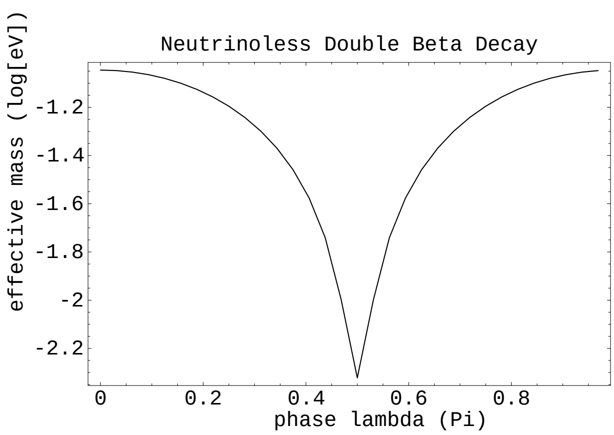


Figure 11.8: Predictions of the SPS 3 scenario with $\lambda' = 10^{-4}$ and $\frac{\mu}{\rm GeV} = 10^{-4}$.

For $\lambda' = 10^{-4}$ the Heidelberg-Moscow Experiment suggests λ to be smaller than $10^{-3.5}$. This value does not change by lowering λ' since the neutrino mass matrix is dominated by the $\lambda\lambda$-loop for large λ. Similarly, one can consider the dependence of the effective mass on λ' for $\lambda = 10^{-4}$. Then, the upper bound for λ' consistent with the experiment is also approximately $10^{-3.5}$. The phase dependence given in the right diagram shows that cancellations in Eq.(11.12) between the three neutrino masses can have a strong impact on the effective mass. However, this effect is only important if at least two neutrinos have masses of similar order and if the contributions to the mass matrix coming from the $\lambda\lambda$- and the $\lambda'\lambda'$-loop are comparable.

The corresponding result for the SPS 2 case is depicted in Fig.11.9. Again, the dashed line gives the upper limit on the effective mass. Therefore, $\frac{\mu}{\rm GeV}$ is constrained by $10^{-3.8}$ in the simple model with $\lambda = \lambda' = 10^{-4}$. The sudden decrease of the effective mass for $\mu \approx 10^{-4.9}$ is due to cancellations. Here the $\lambda\lambda$- and the $\lambda'\lambda'$-loops generate masses of the same order as the gg-loop which dominates the neutrino mass matrix for large μ.

In summary: Neutrinoless double beta decay sets an upper limit on the effective mass. This constrains the values of R-parity violating couplings in the simple model considered in this diploma thesis. I have not included the results of "Evidence for Neutrinoless Double Beta Decay" [78] as they are controversially discussed [79].

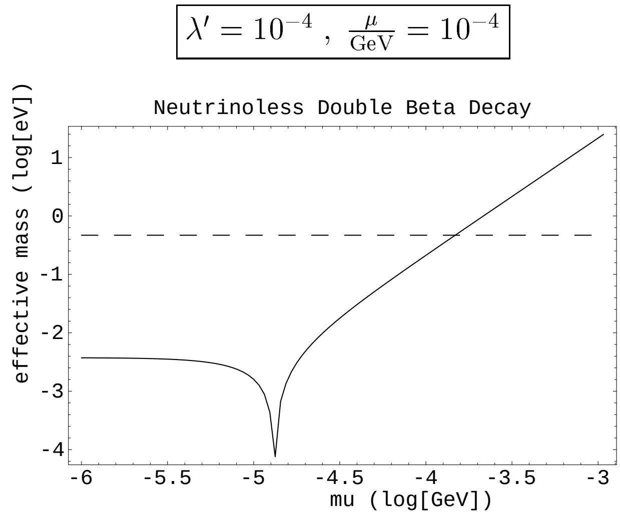


Figure 11.9: Prediction of the SPS 2 scenario with $\lambda = 10^{-4}$ and $\lambda' = 10^{-4}$.

11.4.4 Cosmological and Astrophysical Considerations

Massive neutrinos can play an essential role in cosmology as the standard big bang model predicts the existence of relic background neutrinos whose number density is huge compared to the average number density of baryons in the universe [2]. These neutrinos decouple from equilibrium at a temperature T_d of about 1 MeV. If neutrinos have a mass much smaller than T_d they are relativistic or "hot relics". The number density of hot relics is comparable to that of photons which can be determined by measuring the microwave background radiation. An upper bound on the sum of the neutrino masses is obtained by assuming that the universe is closed due to the mass of neutrinos. The calculation [1] yields

$$\sum_i m_i \simeq 30\ \mathrm{eV}, \tag{11.14}$$

where m_i, $i = 1, 2, 3$, are the neutrino masses. If neutrinos are not the only particles responsible for the closure of the universe, this upper limit declines.

Another way to determine absolute neutrino masses is based on the idea that ultrahigh energy neutrinos scatter on relic neutrinos [80, 81]. In this process Z bosons are produced which in turn decay hadronically, generating a highly boosted "Z-burst". Such Z-bursts occurring within a distance of 50 to 100 Mpc could explain the observed events in the ultrahigh energy cosmic ray spectrum above the Greisen-Kuzmin-Zatsepin [82] cutoff energy $E_{\mathrm{GZK}} \sim 5 \cdot 10^{19}$eV. Following this idea, the mass of the heaviest relic neutrino is predicted to be[8] $0.26^{+0.20}_{-0.14}$ eV [81].

The results of this diploma thesis are compatible with the above considerations.

[8]This result relies on the assumption that the power-like part of the ultrahigh energy cosmic ray spectrum has an extragalactic origin.

Chapter 12

Conclusion

The possibility of generating neutrino masses has been examined in this thesis. Considering a supersymmetric theory, Majorana type neutrino masses can be obtained most naturally by allowing for R-parity violating couplings. Due to the mixing with the neutralinos, one neutrino becomes massive at tree level. The formulae describing the loop corrections to the neutrino mass matrix have been derived using the method of matrix perturbation theory. The entries in the mass matrix are proportional to the product of two R-parity violating parameters. The numerical evaluation of the neutrino mass spectrum yields that all SPS-scenarios are dominated by the $\lambda\lambda$- and $\lambda'\lambda'$-loops, except for SPS 2 which depends predominantly on the gg-loop. Assuming a hierarchy of the neutrino masses, the results of the solar and the atmospheric neutrino oscillation experiments concerning the masses can be reproduced within a simple model. However, the experimentally preferred mixing angles cannot be described very well in this model. A reason for this could be that tree level effects have been neglected. Furthermore, it should be emphasized that the crude simplification in regard to the R-parity violating parameters is ad hoc. In this context it would be desirable to have an underlying theory which motivates the choice of the input parameters in a better way.

Appendix A

Notations and Conventions

This appendix gives a short summary of notations and conventions concerning spinors.

A.1 Weyl Spinors

The Lorentz metric is given by

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \tag{A.1}$$

The two component Weyl spinor representation of the Lorentz group distinguishes between two types: left and right chiral spinors

$$\psi_{\mathrm{L}} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad , \quad \chi_{\mathrm{R}} = \begin{pmatrix} \chi^{\dot{1}} \\ \chi^{\dot{2}} \end{pmatrix}. \tag{A.2}$$

Undotted indices always go with left chiral spinors and dotted with right chiral ones. I do *not* follow the widespread convention of denoting a right chiral Weyl spinor by a bar.
The dual spinors are defined by applying the totally antisymmetric tensor

$$\epsilon^{AB} = \epsilon^{\dot{A}\dot{B}} = \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}, \tag{A.3}$$

and its inverse $\epsilon_{\dot{A}\dot{B}} = \epsilon_{AB}$ to the spinors ψ_B and $\chi^{\dot{B}}$, resulting in

$$\psi^A \equiv \epsilon^{AB}\psi_B \quad , \quad \chi_{\dot{A}} \equiv \epsilon_{\dot{A}\dot{B}}\chi^{\dot{B}}. \tag{A.4}$$

The Lorentz invariant products of two Weyl spinors ψ_{L}, ψ'_{L} and χ_{R}, χ'_{R} are given by

$$\psi_{\mathrm{L}}\psi'_{\mathrm{L}} \equiv \psi^A\psi'_A = \psi'^A\psi_A = \psi'_{\mathrm{L}}\psi_{\mathrm{L}} \quad , \quad \chi_{\mathrm{R}}\chi'_{\mathrm{R}} \equiv \chi_{\dot{A}}\chi'^{\dot{A}} = \chi'_{\dot{A}}\chi^{\dot{A}} = \chi'_{\mathrm{R}}\chi_{\mathrm{R}}. \tag{A.5}$$

It can be shown easily that the transformation properties under the Lorentz group will change undotted into dotted indices and vice versa by complex conjugation

$$\bar{\psi}_{\dot{A}} \equiv (\psi_A)^* \quad , \quad \bar{\chi}^A \equiv (\chi^{\dot{A}})^*. \tag{A.6}$$

A.2 Dirac Spinors

Throughout this thesis I will denote four component Dirac spinors by a lower index D. Working in the chiral basis

$$\psi_{\mathrm{D}} = \begin{pmatrix} \psi_A \\ \chi^{\dot{A}} \end{pmatrix}, \tag{A.7}$$

with the left chiral Weyl spinor ψ_A and the right chiral one $\chi^{\dot{A}}$.
In this representation the Dirac γ matrices take the form

$$\gamma^\mu = \begin{pmatrix} & \sigma^\mu \\ \bar{\sigma}^\mu & \end{pmatrix}, \tag{A.8}$$

where $\sigma^\mu = (\sigma^0, \sigma^k)$ and $\bar{\sigma}^\mu = (\sigma^0, -\sigma^k)$ are the generalized Pauli matrices, with

$$\begin{aligned} \sigma^0 &= \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \quad , \quad \sigma^1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}, \\ \sigma^2 &= \begin{pmatrix} & -\mathrm{i} \\ \mathrm{i} & \end{pmatrix} \quad , \quad \sigma^3 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}. \end{aligned} \tag{A.9}$$

With the definitions

$$\gamma_5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbf{1} & \\ & \mathbf{1} \end{pmatrix}, \tag{A.10}$$

and

$$P_{\mathrm{L}} = \frac{1}{2}(\mathbf{1} - \gamma_5) \quad , \quad P_{\mathrm{R}} = \frac{1}{2}(\mathbf{1} + \gamma_5), \tag{A.11}$$

it is obvious that

$$P_{\mathrm{L}}\psi_{\mathrm{D}} = \begin{pmatrix} \psi_A \\ 0 \end{pmatrix} \quad , \quad P_{\mathrm{R}}\psi_{\mathrm{D}} = \begin{pmatrix} 0 \\ \chi^{\dot{A}} \end{pmatrix}. \tag{A.12}$$

In other words, P_{L} and P_{R} project out left and right chiral spinor components.

Using the Dirac adjoint[1]

$$\overline{\psi_{\mathrm{D}}} \equiv \psi_{\mathrm{D}}^\dagger \gamma^0 = \begin{pmatrix} \bar{\chi}^A & \bar{\psi}_{\dot{A}} \end{pmatrix}, \tag{A.13}$$

the charge conjugation is defined by

$$\psi_{\mathrm{D}}^c \equiv C\overline{\psi_{\mathrm{D}}}^T = \begin{pmatrix} \bar{\chi}_A \\ \bar{\psi}^{\dot{A}} \end{pmatrix}, \tag{A.14}$$

[1]Note that in contrast to the complex conjugation of a Weyl spinor the Dirac spinor here is overlined completely.

where

$$C \equiv \mathrm{i}\gamma^2\gamma^0 = \begin{pmatrix} \epsilon_{AB} & \\ & \epsilon^{\dot{A}\dot{B}} \end{pmatrix}. \tag{A.15}$$

In the context of Majorana mass terms the following relations are useful

$$\psi^A\psi_A = \overline{\psi^c_{\mathrm{D}}} P_{\mathrm{L}} \psi_{\mathrm{D}} \quad , \quad \bar{\psi}_{\dot{A}}\bar{\psi}^{\dot{A}} = \overline{\psi_{\mathrm{D}}} P_{\mathrm{R}} \psi^c_{\mathrm{D}}. \tag{A.16}$$

Appendix B

The R-Parity Violating Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{matter}} \;=\;& (\partial_\mu \tilde{Q}_k^\dagger - \mathrm{i}\frac{g'}{6}B_\mu \tilde{Q}_k^\dagger - \mathrm{i}\frac{g}{2}W_\mu^a \tilde{Q}_k^\dagger \sigma^a - \mathrm{i}\frac{g_3}{2}G_\mu^a \tilde{Q}_k^\dagger \lambda^a)(\partial^\mu \tilde{Q}_k + \mathrm{i}\frac{g'}{6}B^\mu \tilde{Q}_k + \mathrm{i}\frac{g}{2}\sigma^a W^{a\mu}\tilde{Q}_k + \mathrm{i}\frac{g_3}{2}\lambda^a G^{a\mu}\tilde{Q}_k) \\
&+\mathrm{i}\bar{Q}_k^T \bar{\sigma}^\mu (\partial_\mu Q_k + \mathrm{i}\frac{g'}{6}B_\mu Q_k + \mathrm{i}\frac{g}{2}\sigma^a W_\mu^a Q_k + \mathrm{i}\frac{g_3}{2}\lambda^a G_\mu^a Q_k) \\
&+(\partial_\mu \tilde{L}_\alpha^\dagger + \mathrm{i}\frac{g'}{2}B_\mu \tilde{L}_\alpha^\dagger - \mathrm{i}\frac{g}{2}W_\mu^a \tilde{L}_\alpha^\dagger \sigma^a)(\partial^\mu \tilde{L}_\alpha - \mathrm{i}\frac{g'}{2}B^\mu \tilde{L}_\alpha + \mathrm{i}\frac{g}{2}\sigma^a W^{a\mu}\tilde{L}_\alpha) \\
&+\mathrm{i}\bar{L}_\alpha^T \bar{\sigma}^\mu (\partial_\mu L_\alpha - \mathrm{i}\frac{g'}{2}B_\mu L_\alpha + \mathrm{i}\frac{g}{2}\sigma^a W_\mu^a L_\alpha) \\
&+(\partial_\mu H_2^\dagger - \mathrm{i}\frac{g'}{2}B_\mu H_2^\dagger - \mathrm{i}\frac{g}{2}W_\mu^a H_2^\dagger \sigma^a)(\partial^\mu H_2 + \mathrm{i}\frac{g'}{2}B^\mu H_2 + \mathrm{i}\frac{g}{2}\sigma^a W^{a\mu} H_2) \\
&+\mathrm{i}\bar{\tilde{H}}_2^T \bar{\sigma}^\mu (\partial_\mu \tilde{H}_2 + \mathrm{i}\frac{g'}{2}B_\mu \tilde{H}_2 + \mathrm{i}\frac{g}{2}\sigma^a W_\mu^a \tilde{H}_2) \\
&+(\partial_\mu \tilde{E}_k - \mathrm{i}g' B_\mu \tilde{E}_k)(\partial^\mu \tilde{E}_k^* + \mathrm{i}g' B^\mu \tilde{E}_k^*) \\
&+\mathrm{i}E_k \bar{\sigma}^\mu (\partial_\mu \bar{E}_k + \mathrm{i}g' B_\mu \bar{E}_k) \\
&+(\partial_\mu \tilde{U}_k^T + \mathrm{i}\frac{2g'}{3}B_\mu \tilde{U}_k^T + \mathrm{i}\frac{g_3}{2}G_\mu^a \tilde{U}_k^T \lambda^{a*})(\partial^\mu \tilde{U}_k^* - \mathrm{i}\frac{2g'}{3}B^\mu \tilde{U}_k^* - \mathrm{i}\frac{g_3}{2}\lambda^{a*}G^{a\mu}\tilde{U}_k^*) \\
&+\mathrm{i}U_k^T \bar{\sigma}^\mu (\partial_\mu \bar{U}_k - \mathrm{i}\frac{2g'}{3}B_\mu \bar{U}_k - \mathrm{i}\frac{g_3}{2}\lambda^{a*}G_\mu^a \bar{U}_k) \\
&+(\partial_\mu \tilde{D}_k^T - \mathrm{i}\frac{g'}{3}B_\mu \tilde{D}_k^T + \mathrm{i}\frac{g_3}{2}G_\mu^a \tilde{D}_k^T \lambda^{a*})(\partial^\mu \tilde{D}_k^* + \mathrm{i}\frac{g'}{3}B^\mu \tilde{D}_k^* - \mathrm{i}\frac{g_3}{2}\lambda^{a*}G^{a\mu}\tilde{D}_k^*) \\
&+\mathrm{i}D_k^T \bar{\sigma}^\mu (\partial_\mu \bar{D}_k + \mathrm{i}\frac{g'}{3}B_\mu \bar{D}_k - \mathrm{i}\frac{g_3}{2}\lambda^{a*}G_\mu^a \bar{D}_k) \\
&-\frac{g'}{\sqrt{2}}\Big(\frac{1}{3}\tilde{Q}_k^\dagger \tilde{B} Q_k - \tilde{L}_\alpha^\dagger \tilde{B} L_\alpha + H_2^\dagger \tilde{B}\tilde{H}_2 + 2\tilde{E}_k \tilde{B}\bar{E}_k - \frac{4}{3}\tilde{U}_k^T \tilde{B}\bar{U}_k + \frac{2}{3}\tilde{D}_k^T \tilde{B}\bar{D}_k\Big) + \text{h.c.} \\
&-\frac{g}{\sqrt{2}}\Big(\tilde{Q}_k^\dagger \sigma^a \tilde{W}^a Q_k + \tilde{L}_\alpha^\dagger \sigma^a \tilde{W}^a L_\alpha + H_2^\dagger \sigma^a \tilde{W}^a \tilde{H}_2\Big) + \text{h.c.} \\
&-\frac{g_3}{\sqrt{2}}\Big(\tilde{Q}_k^\dagger \lambda^a \tilde{G}^a Q_k - \tilde{U}_k^T \lambda^{a*}\tilde{G}^a \bar{U}_k - \tilde{D}_k^T \lambda^{a*}\tilde{G}^a \bar{D}_k\Big) + \text{h.c.}. \qquad \text{(B.1)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) \\
&+\mathrm{i}\bar{\tilde{B}}\bar{\sigma}^\mu \partial_\mu \tilde{B} \\
&-\frac{1}{4}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^{abc}W^b_\mu W^c_\nu)(\partial^\mu W^{a\nu} - \partial^\nu W^{a\mu} - g\epsilon^{ab'c'}W^{b'\mu}W^{c'\nu}) \\
&+\mathrm{i}\bar{\tilde{W}}^a\bar{\sigma}^\mu(\partial_\mu \tilde{W}^a - g\epsilon^{abc}W^b_\mu \tilde{W}^c) \\
&-\frac{1}{4}(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_3 f^{abc}G^b_\mu G^c_\nu)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu} - g_3 f^{ab'c'}G^{b'\mu}G^{c'\nu}) \\
&+\mathrm{i}\bar{\tilde{G}}^a\bar{\sigma}^\mu(\partial_\mu \tilde{G}^a - g_3 f^{abc}G^b_\mu \tilde{G}^c).
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
-\mathcal{L}_{\text{scalar}} &= (\lambda'_{\alpha kj}\tilde{L}^r_\alpha \tilde{D}^{\rho *}_j - h_{kj}H^r_2\tilde{U}^{\rho *}_j)(\lambda'^*_{\alpha' kj'}\tilde{L}^{r*}_{\alpha'}\tilde{D}^\rho_{j'} - h^*_{kj'}H^{r*}_2\tilde{U}^\rho_{j'}) \\
&+(-\mu_\alpha H^r_2 + \lambda_{\alpha\beta k}\tilde{L}^r_\beta \tilde{E}^*_k + \lambda'_{\alpha ij}\tilde{Q}^{\rho r}_i\tilde{D}^{\rho *}_j)(-\mu^*_\alpha H^{r*}_2 + \lambda^*_{\alpha\beta' k'}\tilde{L}^{r*}_{\beta'}\tilde{E}_{k'} + \lambda'^*_{\alpha i'j'}\tilde{Q}^{\rho' r*}_{i'}\tilde{D}^{\rho'}_{j'}) \\
&+(-\mu_\alpha \tilde{L}^r_\alpha + h_{ij}\tilde{Q}^{\rho r}_i\tilde{U}^{\rho *}_j)(-\mu^*_{\alpha'}\tilde{L}^{r*}_{\alpha'} + h^*_{i'j'}\tilde{Q}^{\rho' r*}_{i'}\tilde{U}^{\rho'}_{j'}) \\
&+(\frac{1}{2}\epsilon_{rs}\lambda_{\alpha\beta k}\tilde{L}^r_\alpha \tilde{L}^s_\beta)(\frac{1}{2}\epsilon_{r's'}\lambda^*_{\alpha'\beta' k}\tilde{L}^{r'*}_{\alpha'}\tilde{L}^{s'*}_{\beta'}) \\
&+(\epsilon_{rs}h_{ik}H^r_2\tilde{Q}^{\rho s}_i - \frac{1}{2}\epsilon^{\rho\sigma\tau}\lambda''_{kij}\tilde{D}^{\sigma *}_i\tilde{D}^{\tau *}_j)(\epsilon_{r's'}h^*_{i'k}H^{r'*}_2\tilde{Q}^{\rho s'*}_{i'} - \frac{1}{2}\epsilon^{\rho\sigma'\tau'}\lambda''^*_{ki'j'}\tilde{D}^{\sigma'}_{i'}\tilde{D}^{\tau'}_{j'}) \\
&+(\epsilon_{rs}\lambda'_{\alpha ik}\tilde{L}^r_\alpha \tilde{Q}^{\rho s}_i + \epsilon^{\sigma\tau\rho}\lambda''_{ijk}\tilde{U}^{\sigma *}_i\tilde{D}^{\tau *}_j)(\epsilon_{r's'}\lambda'^*_{\alpha' i'k}\tilde{L}^{r'*}_{\alpha'}\tilde{Q}^{\rho s'*}_{i'} + \epsilon^{\sigma'\tau'\rho}\lambda''^*_{i'j'k}\tilde{U}^{\sigma'}_{i'}\tilde{D}^{\tau'}_{j'}) \\
&+\frac{1}{8}g'^2\Big(\frac{1}{3}\tilde{Q}^\dagger_k\tilde{Q}_k - \tilde{L}^\dagger_\alpha\tilde{L}_\alpha + H^\dagger_2 H_2 + 2\tilde{E}^*_k\tilde{E}_k - \frac{4}{3}\tilde{U}^\dagger_k\tilde{U}_k + \frac{2}{3}\tilde{D}^\dagger_k\tilde{D}_k\Big)^2 \\
&+\frac{1}{8}g^2\Big(2|\tilde{Q}^\dagger_k\tilde{Q}_l|^2 - (\tilde{Q}^\dagger_k\tilde{Q}_k)(\tilde{Q}^\dagger_l\tilde{Q}_l) + 2|\tilde{L}^\dagger_\alpha\tilde{L}_\beta|^2 - (\tilde{L}^\dagger_\alpha\tilde{L}_\alpha)(\tilde{L}^\dagger_\beta\tilde{L}_\beta) + |H^\dagger_2H_2|^2 \\
&\qquad + 4|\tilde{Q}^\dagger_k\tilde{L}_\alpha|^2 - 2(\tilde{Q}^\dagger_k\tilde{Q}_k)(\tilde{L}^\dagger_\alpha\tilde{L}_\alpha) + 4|\tilde{Q}^\dagger_k H_2|^2 - 2(\tilde{Q}^\dagger_k\tilde{Q}_k)(H^\dagger_2H_2) \\
&\qquad + 4|\tilde{L}^\dagger_\alpha H_2|^2 - 2(\tilde{L}^\dagger_\alpha\tilde{L}_\alpha)(H^\dagger_2H_2)\Big) \\
&+\frac{1}{8}g_3^2\Big(2|\tilde{Q}^\dagger_k\tilde{Q}_l|^2 - \frac{2}{3}(\tilde{Q}^\dagger_k\tilde{Q}_k)(\tilde{Q}^\dagger_l\tilde{Q}_l) + 2|\tilde{U}^\dagger_k\tilde{U}_l|^2 - \frac{2}{3}(\tilde{U}^\dagger_k\tilde{U}_k)(\tilde{U}^\dagger_l\tilde{U}_l) \\
&\qquad + 2|\tilde{D}^\dagger_k\tilde{D}_l|^2 - \frac{2}{3}(\tilde{D}^\dagger_k\tilde{D}_k)(\tilde{D}^\dagger_l\tilde{D}_l) - 4|\tilde{Q}^\dagger_k\tilde{U}_l|^2 + \frac{4}{3}(\tilde{Q}^\dagger_k\tilde{Q}_k)(\tilde{U}^\dagger_l\tilde{U}_l) \\
&\qquad - 4|\tilde{Q}^\dagger_k\tilde{D}_l|^2 + \frac{4}{3}(\tilde{Q}^\dagger_k\tilde{Q}_k)(\tilde{D}^\dagger_l\tilde{D}_l) + 4|\tilde{U}^\dagger_k\tilde{D}_l|^2 - \frac{4}{3}(\tilde{U}^\dagger_k\tilde{U}_k)(\tilde{D}^\dagger_l\tilde{D}_l)\Big).
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} \;=\;& -\frac{1}{2}\lambda_{\alpha\beta k}\epsilon_{rs}\Big((L^r_\alpha L^s_\beta)\tilde{E}^*_k + 2\tilde{L}^r_\alpha(L^s_\beta \bar{E}_k)\Big) + \text{h.c.} \\
& -\lambda'_{\alpha ij}\epsilon_{rs}\Big((L^r_\alpha Q^{\rho s}_i)\tilde{D}^{\rho *}_j + (L^r_\alpha \bar{D}^\rho_j)\tilde{Q}^{\rho s}_i + \tilde{L}^r_\alpha(Q^{\rho s}_i \bar{D}^\rho_j)\Big) + \text{h.c.} \\
& +h_{ij}\epsilon_{rs}\Big((\tilde{H}^r_2 Q^{\rho s}_i)\tilde{U}^{\rho *}_i + (\tilde{H}^r_2 \bar{U}^\rho_j)\tilde{Q}^{\rho s}_i + H^r_2(Q^{\rho s}_i \bar{U}^\rho_j)\Big) + \text{h.c.} \\
& -\frac{1}{2}\lambda''_{ijk}\epsilon^{\rho\sigma\tau}\Big(2(\bar{U}^\rho_i \bar{D}^\sigma_j)\tilde{D}^{\tau *}_k + \tilde{U}^{\rho *}_i(\bar{D}^\sigma_j \bar{D}^\tau_k)\Big) + \text{h.c.} \\
& +\epsilon_{rs}\mu_\alpha(L^r_\alpha \tilde{H}^s_2) + \text{h.c.}.
\end{aligned} \tag{B.4}
$$

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}} \;=\;& (M^2_{\tilde{Q}})_{ij}\tilde{Q}^\dagger_i\tilde{Q}_j + (M^2_{\tilde{U}})_{ij}\tilde{U}^T_i\tilde{U}^*_j + (M^2_{\tilde{D}})_{ij}\tilde{D}^T_i\tilde{D}^*_j + (M^2_{\tilde{L}})_{\alpha\beta}\tilde{L}^\dagger_\alpha\tilde{L}_\beta + (M^2_{\tilde{E}})_{ij}\tilde{E}_i\tilde{E}^*_j + (M^2_{H_2})H^\dagger_2 H_2 \\
& -\epsilon_{rs}\Big(\mu_\alpha(B)_\alpha\tilde{L}^r_\alpha H^s_2\Big) + \text{h.c.} \\
& +\epsilon_{rs}\Big(\frac{1}{2}\lambda_{\alpha\beta k}(A_E)_{\alpha\beta k}\tilde{L}^r_\alpha\tilde{L}^s_\beta\tilde{E}^*_k + \lambda'_{\alpha ij}(A_D)_{\alpha ij}\tilde{L}^r_\alpha\tilde{Q}^{\rho s}_i\tilde{D}^{\rho *}_j - h_{ij}(A_U)_{ij}H^r_2\tilde{Q}^{\rho s}_i\tilde{U}^{\rho *}_j\Big) + \text{h.c.} \\
& +\epsilon^{\rho\sigma\tau}\Big(\frac{1}{2}\lambda''_{ijk}(A_{DU})_{ijk}\tilde{U}^{\rho *}_i\tilde{D}^{\sigma *}_j\tilde{D}^{\tau *}_k\Big) + \text{h.c.} \\
& +\frac{1}{2}\Big(M_1\tilde{B}\tilde{B} + M_2\tilde{W}^a\tilde{W}^a + M_3\tilde{G}^a\tilde{G}^a\Big) + \text{h.c.}.
\end{aligned} \tag{B.5}
$$

Appendix C

Biunitary Transformations

First, I will motivate the use of biunitary transformations at all. Then the technical details of diagonalizing any invertible $n \times n$ matrix X will be shown, leading to non-negative eigenvalues.

C.1 Motivation

Unitary matrices are applied when dealing with basis transformations. In order to diagonalize a hermitian matrix, it is sufficient to consider a unitary transformation, defined by one single matrix U. In the case of a general complex $n \times n$ matrix X, *two* transformation matrices are needed. This so-called biunitary transformation can only be interpreted as a basis transformation if there are two different vectors, called ψ^- and ψ^+ in the following, which do not have to be rotated identically, i.e. they can be subject to different transformations

$$\chi^- \equiv U\psi^- \quad , \quad \chi^+ \equiv V\psi^+, \tag{C.1}$$

with U and V being unitary matrices. Starting with

$$(\psi^-)^T X \psi^+, \tag{C.2}$$

a suitable choice of U and V leads to

$$(\chi^-)^T D \chi^+, \tag{C.3}$$

where

$$D \equiv U^* X V^\dagger \tag{C.4}$$

is a diagonal matrix with only real and non-negative entries.

C.2 Finding the Correct Transformations

Obviously, $XX^\dagger$ is hermitian and can be diagonalized by a unitary matrix[1] U^*

$$D^2 \equiv U^* X X^\dagger U^{*\dagger}. \tag{C.5}$$

[1] Complex conjugation is included in order to work consistently in notation.

Since $XX^\dagger$ and the diagonal matrix D^2 are positive semi-definite we can take the positive square root of D^2, which defines the diagonal matrix D with only non-negative eigenvalues.

Turning to the second transformation matrix V, I will simply quote the result which is discussed afterwards

$$V \equiv D^{-1}U^*X. \tag{C.6}$$

Here V is given as a function of already known matrices. To see that V is unitary, Eq.(C.5) must be applied to $V^\dagger$

$$V^\dagger = X^\dagger U^{*\dagger} D^{-1} = (X^{-1}U^{*\dagger}D^2U^*)U^{*\dagger}D^{-1} = X^{-1}U^{*\dagger}D, \tag{C.7}$$

which makes the unitarity of V evident. Finally, it has to be shown that the notations are compatible with those of Section C.1. This is obvious if we calculate $U^*XV^\dagger$

$$U^*XV^\dagger = U^*X(X^{-1}U^{*\dagger}D) = D, \tag{C.8}$$

which leads back to Eq.(C.4) again. Thus, U and V, defined in Eqs.(C.5) and (C.6), give the correct transformations.

Appendix D

Diagonalization of a Hermitian 2×2 Matrix

The diagonalization of a symmetric 2×2 matrix with real entries is achieved by an orthogonal transformation. This is well known. On the other hand, the corresponding unitary transformation for a hermitian matrix is rarely mentioned. This appendix provides a short survey of the topic and explains the notation used in the thesis.

The most general hermitian 2×2 matrix is given by

$$X = \begin{pmatrix} A & B \\ B^* & D \end{pmatrix}, \tag{D.1}$$

where A, D are real and B is complex. The question of diagonalizing this matrix can be divided into two parts. First, a phase rotation renders the off-diagonals real, then the usual orthogonal transformation must be applied. Defining the phase δ by

$$e^{2\mathrm{i}\delta} \equiv \frac{B}{|B|}, \tag{D.2}$$

it is obvious that

$$\begin{pmatrix} A & |B| \\ |B| & D \end{pmatrix} = \begin{pmatrix} e^{-\mathrm{i}\delta} & \\ & e^{\mathrm{i}\delta} \end{pmatrix} \cdot \begin{pmatrix} A & B \\ B^* & D \end{pmatrix} \cdot \begin{pmatrix} e^{\mathrm{i}\delta} & \\ & e^{-\mathrm{i}\delta} \end{pmatrix}. \tag{D.3}$$

An additional rotation about the angle φ leads to

$$\begin{pmatrix} (X)_1 & \\ & (X)_2 \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \cdot \begin{pmatrix} A & |B| \\ |B| & D \end{pmatrix} \cdot \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}, \tag{D.4}$$

with

$$\tan 2\varphi \equiv \frac{2|B|}{A-D} \quad , \quad (X)_i \equiv \frac{A+D}{2} \pm \frac{A-D}{2\cos 2\varphi}. \tag{D.5}$$

Here, the plus sign corresponds to $i = 1$ and the minus sign to $i = 2$.

The complete unitary transformation V which diagonalizes the hermitian matrix X through $VXV^\dagger$ then takes the form

$$V \equiv \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \cdot \begin{pmatrix} e^{-\mathrm{i}\delta} & \\ & e^{\mathrm{i}\delta} \end{pmatrix} = \begin{pmatrix} e^{-\mathrm{i}\delta}\cos\varphi & e^{\mathrm{i}\delta}\sin\varphi \\ -e^{-\mathrm{i}\delta}\sin\varphi & e^{\mathrm{i}\delta}\cos\varphi \end{pmatrix} . \tag{D.6}$$

Appendix E

The B_0 Function

The characteristic integrals appearing in loop calculations have been studied extensively by 't Hooft, Veltman and Passarino. A systematic overview can be found in the appendix of [83]. For the scalar one-loop diagrams a collection of formulae is given in [84]. In the context of loop induced neutrino masses, it is sufficient to consider the two point function B_0.

The B_0 function is defined by

$$B_0(p^2, M^2, m^2) \;=\; \frac{16\pi^2}{\mathrm{i}} \int \frac{d^{4+\epsilon}q}{(2\pi)^{4+\epsilon}} \, \frac{1}{(q^2 - m^2)((q-p)^2 - M^2)}. \tag{E.1}$$

Evidently, the method of dimensional regularization has been applied, with $\epsilon \ll 1$. It can easily be verified that B_0 is symmetric under interchange of the masses. By the procedure of Feynman parameterization and Wick rotation one can show that

$$B_0(p^2, M^2, m^2) = \frac{2}{\epsilon} - \gamma + \ln 4\pi + \mathcal{O}(\epsilon) - \int_0^1 dz \ln\Big(m^2 z + M^2(1-z) - p^2 z(1-z)\Big), \tag{E.2}$$

where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. In this form the diverging part is separated from the finite terms. As the expressions appearing in the calculations of this thesis contain differences of two such functions, the remaining values are always finite. In the limiting case of $p^2 \longrightarrow 0$ one can evaluate the integral in Eq.(E.2)

$$B_0(0, M_1^2, m^2) - B_0(0, M_2^2, m^2) \;=\; \frac{M_1^2}{m^2 - M_1^2} \ln\Big(\frac{M_1^2}{m^2}\Big) - \frac{M_2^2}{m^2 - M_2^2} \ln\Big(\frac{M_2^2}{m^2}\Big). \tag{E.3}$$

For $m^2 \ll M_1^2, M_2^2$ this simplifies to

$$B_0(0, M_1^2, m^2) - B_0(0, M_2^2, m^2) \;\approx\; -\ln\Big(\frac{M_1^2}{M_2^2}\Big). \tag{E.4}$$

On the other hand, if $|M_1^2 - M_2^2| \ll M_1^2, M_2^2$, Eq.(E.3) gives approximately

$$B_0(0, M_1^2, m^2) - B_0(0, M_2^2, m^2) \;\approx\; \frac{M_1^2 - M_2^2}{m^2} \, \frac{1}{(1-y)^2}\Big(\ln y - y + 1\Big), \tag{E.5}$$

with $y \equiv \frac{M_1^2 + M_2^2}{2\, m^2}$.

Appendix F

Matrix Perturbation Theory

Matrix perturbation theory is discussed in every textbook on quantum mechanics. Still some remarks might be interesting if one considers the degenerate case. First I give all formulae up to second order. Then two typical examples are discussed.

F.1 General Ideas and Results

Starting with the definitions

$$\begin{aligned} H &\equiv H_0 + \lambda H', \\ \psi &\equiv \psi_0 + \lambda\psi_1 + \lambda^2\psi_2 + \cdots, \\ E &\equiv E_0 + \lambda E_1 + \lambda^2 E_2 + \cdots, \end{aligned} \tag{F.1}$$

it is the aim of perturbation theory to solve the eigenvalue equation

$$H\psi = E\psi, \tag{F.2}$$

in orders of the parameter[1] λ. Here, H' is regarded as a perturbation of the simpler system described by the operator H_0. The zeroth order condition

$$(H_0 - E_0)\psi_0 = 0, \tag{F.3}$$

signifies that ψ_0 has to be chosen such that it is an element of the eigenspace $\mathrm{Eig}(H_0, E_0)$. Considering first order then leads to the equation

$$(H_0 - E_0)\psi_1 + (H' - E_1)\psi_0 = 0. \tag{F.4}$$

Obviously, this expression does not determine the component of ψ_1 in the direction of the eigenspace $\mathrm{Eig}(H_0, E_0)$. Therefore, without loss of generality, ψ_1 is taken to be orthogonal to it. Introducing the projector P onto $\mathrm{Eig}(H_0, E_0)$, the first order condition in Eq.(F.4) can be rewritten as

$$(PH'P - E_1)\psi_0 = 0, \tag{F.5}$$

[1] In the end, λ is set to one.

which, again, is an eigenvector equation. Thus, one has to diagonalize H' in the eigenspace $\mathrm{Eig}(H_0, E_0)$. The solution to (F.5) gives first order corrections to the eigenvalue E_0 and the associated eigenvectors of zeroth order

$$(E_1^k, \psi_0^{k_i}), \tag{F.6}$$

with k labeling the different eigenvalues E_1^k of H' and i denoting a possible degeneracy. All following calculations have to be made separately for each of these pairs. If one now defines a basis of the orthogonal complement to $\mathrm{Eig}(H_0, E_0)$ in terms of normalized eigenvectors φ_μ to the operator H_0 with eigenvalues ϵ_μ, the first order correction to the vector takes the form

$$\psi_1^{k_i} = \sum_\mu \frac{\langle \varphi_\mu H' \psi_0^{k_i} \rangle}{E_0 - \epsilon_\mu} \varphi_\mu. \tag{F.7}$$

This follows from Eq.(F.4) directly. For second order the corresponding formulae are derived similarly. Taking $\psi_2^{k_i}$ orthogonal to $\mathrm{Eig}(H_0, E_0)$, we obtain

$$\begin{aligned} E_2^{k_i} &= \sum_\mu \frac{|\langle \varphi_\mu H' \psi_0^{k_i} \rangle|^2}{E_0 - \epsilon_\mu}, \\ \psi_2^{k_i} &= \sum_{\mu,\nu} \frac{\langle \varphi_\mu H' \varphi_\nu \rangle - E_1 \delta_{\mu\nu}}{(E_0 - \epsilon_\mu)(E_0 - \epsilon_\nu)} \langle \varphi_\nu H' \psi_0^{k_i} \rangle \, \varphi_\mu. \end{aligned} \tag{F.8}$$

F.2 Examples with Degeneracy

In this section I will focus on the behavior of a two-dimensional eigenspace $\mathrm{Eig}(H_0, E_0)$ in perturbation theory. The examples illustrate how the splitting of the eigenvalues is achieved and what it is that determines the new eigenvectors.

The first example is a real symmetric 2×2 matrix with small entries on the off-diagonal

$$H = \begin{pmatrix} A & a \\ a & A \end{pmatrix}. \tag{F.9}$$

To zeroth order this is trivially degenerate. Evaluation of Eq.(F.5) gives

$$\left(E_1^1 = a, \quad \psi_0^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) \quad , \quad \left(E_1^2 = -a, \quad \psi_0^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right). \tag{F.10}$$

Here, the eigenvectors are fixed without ambiguity, the degeneracy is broken down in first order and ψ_0^1 is automatically orthogonal to ψ_0^2. This is the well known case where the subsequent calculation of higher order corrections is done for each eigenvector separately. I will skip this part as it is a straightforward application of the given formulae.

The second example is somewhat more subtle since the degeneracy is preserved to first order. Consider the following real symmetric 3×3 matrix with small parameters b and c

$$H = \begin{pmatrix} A & b & c \\ b & D & \\ c & & D \end{pmatrix}. \tag{F.11}$$

To zeroth order the most general orthonormal basis in terms of eigenvectors is given by

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \cos\alpha \\ -\sin\alpha \end{pmatrix}, \begin{pmatrix} 0 \\ \sin\alpha \\ \cos\alpha \end{pmatrix}, \tag{F.12}$$

where the last two states belong to the eigenvalue D. This degeneracy is not destroyed in first order because $PH'P = 0$, hence $E_1 = 0$. It is therefore necessary to continue with the general expression for the basis of $\mathrm{Eig}(H_0, D)$, i.e. with the undetermined angle α. Evaluating the first and second order corrections to these two eigenvectors leads to

$$\begin{pmatrix} \frac{b\cos\alpha - c\sin\alpha}{D-A} \\ \cos\alpha \\ -\sin\alpha \end{pmatrix}, \begin{pmatrix} \frac{b\sin\alpha + c\cos\alpha}{D-A} \\ \sin\alpha \\ \cos\alpha \end{pmatrix}. \tag{F.13}$$

The angle α will be determined by the requirement that these two states have to be orthogonal to each other. Thus, one finds

$$\tan 2\alpha = \frac{2bc}{c^2 - b^2}. \tag{F.14}$$

In general, the necessity of orthogonality fixes the eigenstates completely, provided that the perturbation has been considered to an order in which the degeneracy is broken down.

Appendix G

Fields and Mass Matrices

In supersymmetry it is sometimes hard to keep track of all notations needed to formulate the theory. Especially one has to distinguish between interaction and mass eigenstates for all types of fields, i.e. neutral colorless, charged colorless and colored bosons and fermions. The following table gives an overview of the conventions used in this thesis[1]. For the sake of clearness, some indices which are discussed thoroughly in the chapters are omitted.

		Neutral Colorless	**Charged Colorless**	**Colored**
Fermion:	Flavor⟶Mass	$\psi^0 \longrightarrow \chi^0$	$\psi^+ \longrightarrow \chi^+$	$d_\mathrm{D} \longrightarrow d_\mathrm{D}$
	Transformation Matrix	N	U, V	1
	Mass Matrix	m_ν	m_e	m_d
Boson:	Flavor	ϕ^0	ϕ^+	ϕ
	Transformation Matrix	W	$\tilde{V}^e$	$\tilde{V}^d$
	Mass Matrix	M_ν^2	M_e^2	M_d^2

The neutral colorless fermions are composed of the neutralinos and the neutrinos. The charged analog is given by the charginos and the charged leptons. Due to the choice of basis made in Chapter 5, the flavor and mass eigenstates of the down quarks coincide; one does not have to consider up quarks as they do not enter the calculations of neutrino masses. With regard to the bosons, the most troublesome part is the mixing of the neutral Higgses with the sneutrinos. Concerning the charged fields, one can simplify the considerations by neglecting generation mixing. Thus, only the two selectrons or down squarks of one flavor mix.

[1]The references to the equations where the individual expressions are introduced can be taken from the list below:

		Neutral Colorless	**Charged Colorless**	**Colored**
Fermion:	Flavor⟶Mass	(6.2),(10.1)	(8.1),(9.1)	(8.19)
	Transformation Matrix	(10.1)	(9.1)	(8.19)
	Mass Matrix	(6.10)	(8.3)	(8.20)
Boson:	Flavor	(10.7)	(8.6)	(8.22)
	Transformation Matrix	(10.17)	(8.12)	(8.27)
	Mass Matrix	(10.9)	(8.8)	(8.24)

Appendix H

SPS-Scenarios

The Snowmass points provide a list of numerical values for the low energy parameters of R-parity conserving supersymmetry. These are given explicitly in this appendix together with some standard model parameters needed for the calculation of the neutrino mass matrix. The resulting neutrino mass spectra are shown in the second section.

H.1 Snowmass Points

The Snowmass points and slopes are specified by a few high energy input parameters. Here, different "SPS"-scenarios are considered. SPS 1 − SPS 6 are minimal supergravity scenarios, whereas SPS 7 and SPS 8 are gauge-mediated supersymmetry breaking theories and SPS 9 is an anomaly-mediated one. As a standard to obtain the corresponding low energy MSSM parameters via renormalization group running the program ISAJET 7.58 has been chosen [69]. The numerical values used in this diploma thesis are taken from [85] and summarized in Table H.1. As the A parameters of the first and second generation do not show considerable renormalization group running they can be taken from the high energy theory. In the case of gauge- and anomaly-mediated supersymmetry breaking there are no A parameters at the high energy scale. Therefore, I set them to zero for the first two generations.

The standard model parameters used in the calculation of the neutrino mass matrix are listed below [55]. The masses of the charged leptons are

$$m_e = 0.0005 \text{ GeV} \quad , \quad m_\mu = 0.106 \text{ GeV} \quad , \quad m_\tau = 1.777 \text{ GeV}. \tag{H.1}$$

The down type quark masses are taken to be

$$m_d = 0.006 \text{ GeV} \quad , \quad m_s = 0.12 \text{ GeV} \quad , \quad m_b = 4.2 \text{ GeV}. \tag{H.2}$$

The parameters of the spontaneously broken $SU(2)_\mathrm{L} \times U(1)_\mathrm{Y}$ gauge theory are

$$g' = 0.36 \quad , \quad g = 0.65 \quad , \quad v = 246 \text{ GeV}. \tag{H.3}$$

The more common standard model parameters θ_W and m_Z, the weak mixing angle and the mass of the Z boson can be calculated by Eq.(6.9).

Parameter	**SPS 1**	**SPS 2**	**SPS 3**	**SPS 4**	**SPS 5**	**SPS 6**	**SPS 7**	**SPS 8**	**SPS 9**
$\tan\beta$	10	10	10	50	5	10	15	15	10
μ_0	352	125	509	377	640	394	300	398	870
$(M_{\tilde{Q}})_{11,22}$	540	1534	818	732	644	641	861	1082	1219
$(M_{\tilde{Q}})_{33}$	496	1295	761	640	535	591	836	1043	1112
$(M_{\tilde{D}})_{11,22}$	520	1530	789	714	623	622	829	1029	1238
$(M_{\tilde{D}})_{33}$	517	1520	786	673	621	619	827	1026	1232
$(M_{\tilde{L}})_{11,22}$	197	1456	283	446	252	261	257	353	316
$(M_{\tilde{L}})_{33}$	196	1450	282	395	250	260	257	353	307
$(M_{\tilde{E}})_{11,22}$	136	1451	173	414	187	233	120	170	300
$(M_{\tilde{E}})_{33}$	134	1439	170	289	181	230	118	167	281
$(A_D)_{011,022}$	-100	0	0	0	-1000	0	-	-	-
$(A_D)_{033}$	-773	-797	-1042	-730	-1671	-811	-350	-330	216
$(A_E)_{011,022}$	-100	0	0	0	-1000	0	-	-	-
$(A_E)_{033}$	-254	-188	-246	-102	-1179	-213	-39	-37	1162
$(B)_0$	45	1654	65	13	145	55	31	44	95
M_1	99	120	163	121	121	196	169	140	-551
M_2	193	234	311	233	235	232	327	272	-176

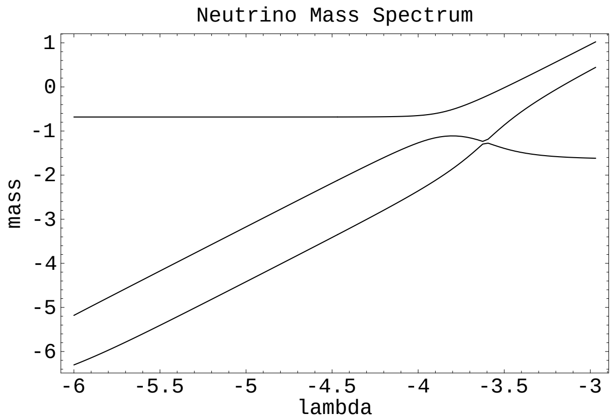
Table H.1: Low energy MSSM parameters of the Snowmass scenarios. All values are given in GeV, except for $\tan\beta$ which is dimensionless.

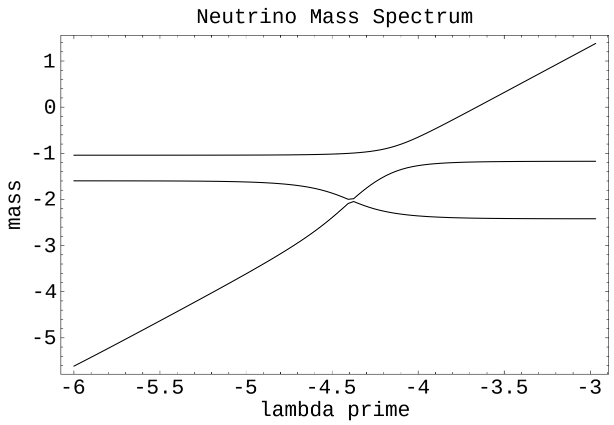
H.2 Neutrino Mass Spectra

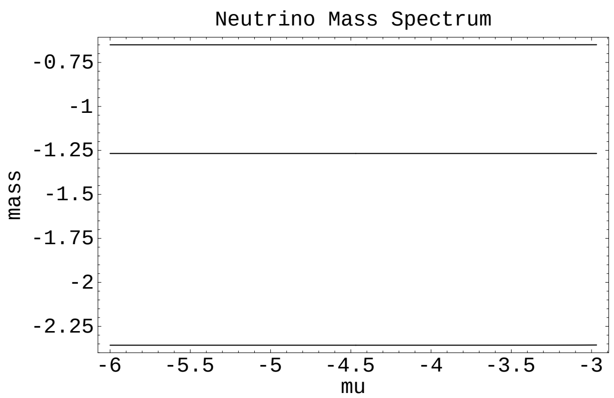
The resulting mass spectra are depicted for the various SPS-scenarios in the following. Here, one of the three R-parity violating parameters λ, λ' and μ is varied. The other two are fixed at the reference point defined in Eq.(11.5), i.e. 10^{-4} (GeV)[1]. As in Chapter 11 the units on the axes are the logarithm of the physical values. Thus, "-4" actually means 10^{-4}. The numerical values for the neutrino masses refer to eV whereas the μ-value is given in GeV. The similarity of the neutrino mass spectra for all SPS-models, except for SPS 2, is discussed in Chapter 11 and can be seen explicitly in the following diagrams.

[1] λ and λ' are dimensionless whereas μ is a mass parameter.

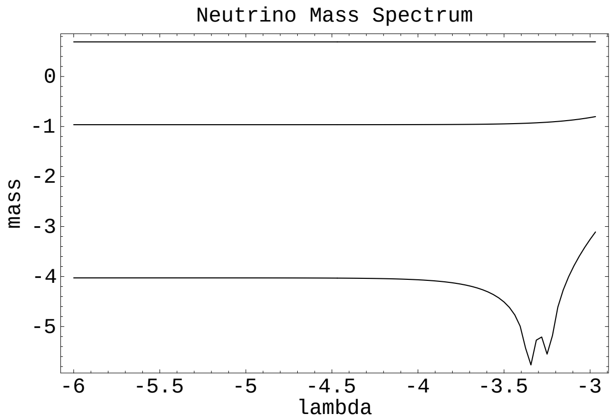
SPS 1

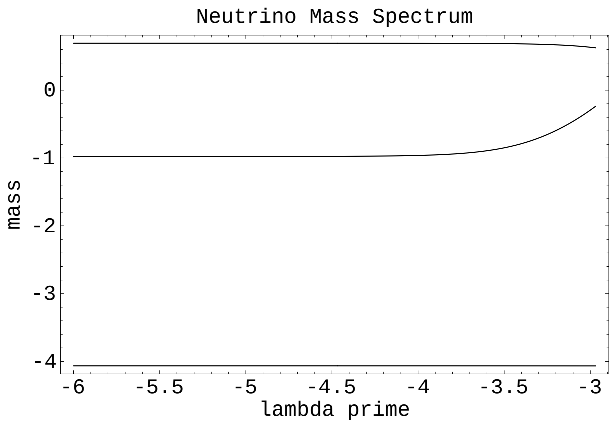

Neutrino Mass Spectrum
mass
lambda

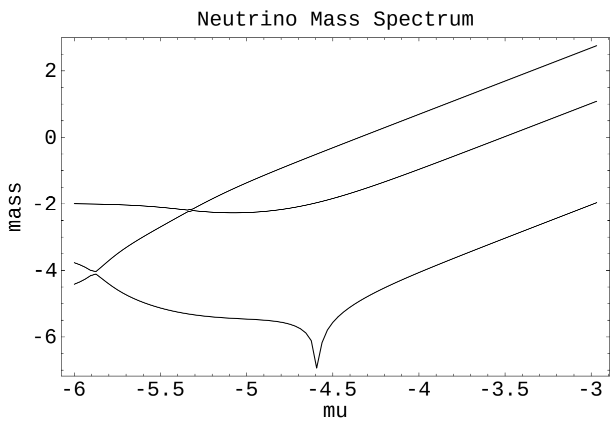

Neutrino Mass Spectrum
mass
lambda prime


Neutrino Mass Spectrum
mass
mu

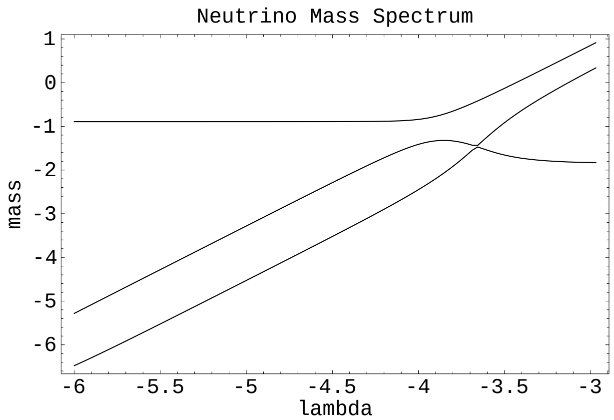
SPS 2

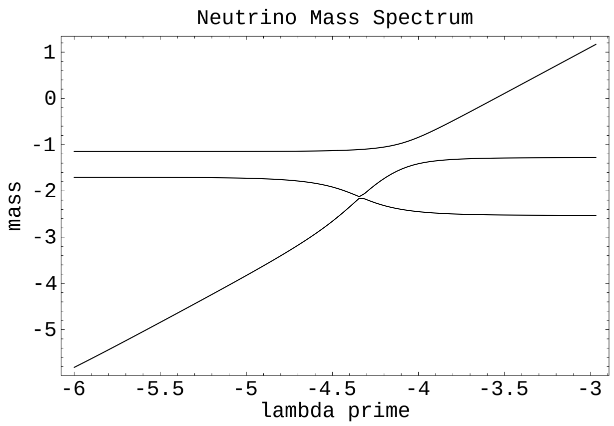

Neutrino Mass Spectrum
mass
lambda

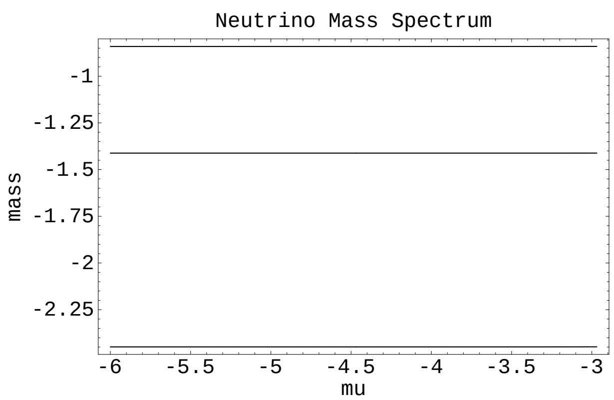

Neutrino Mass Spectrum
mass
lambda prime


Neutrino Mass Spectrum
mass
mu

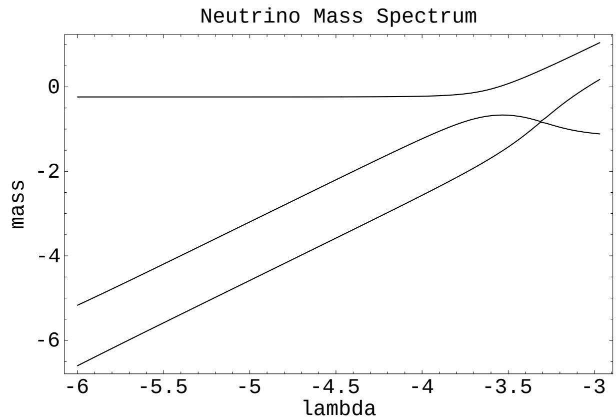
SPS 3

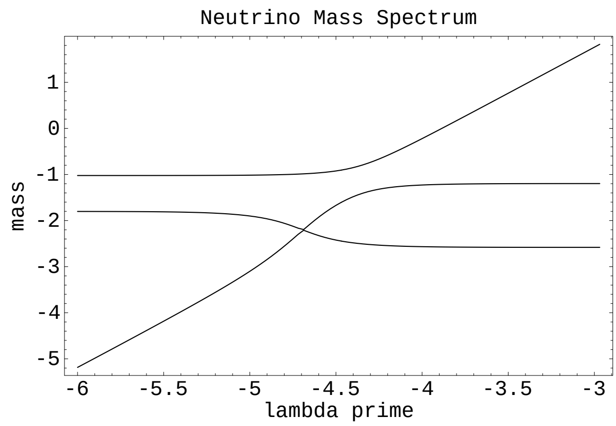

Neutrino Mass Spectrum
mass
lambda

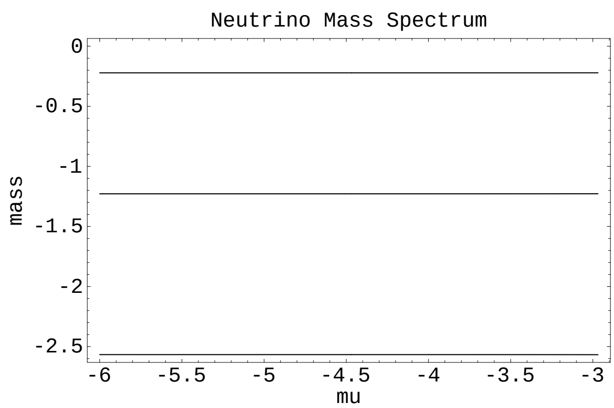

Neutrino Mass Spectrum
mass
lambda prime


Neutrino Mass Spectrum
mass
mu

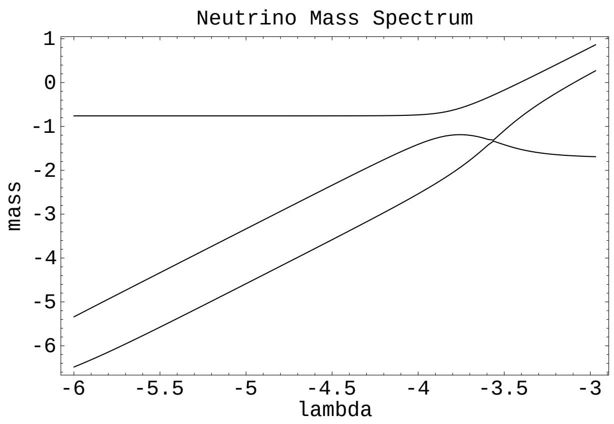
SPS 4

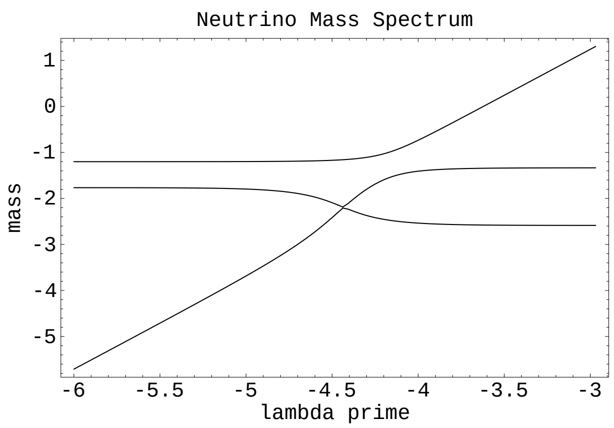

Neutrino Mass Spectrum
mass
lambda

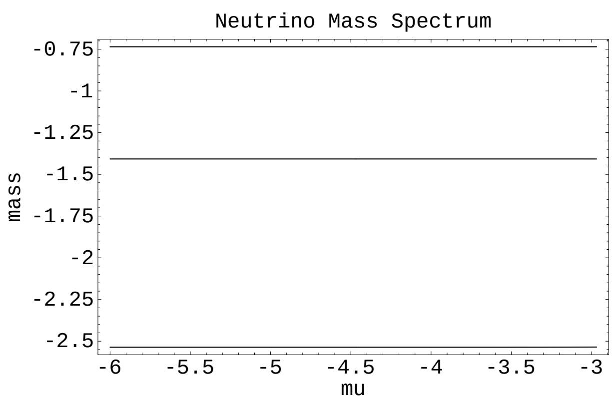

Neutrino Mass Spectrum
mass
lambda prime


Neutrino Mass Spectrum
mass
mu

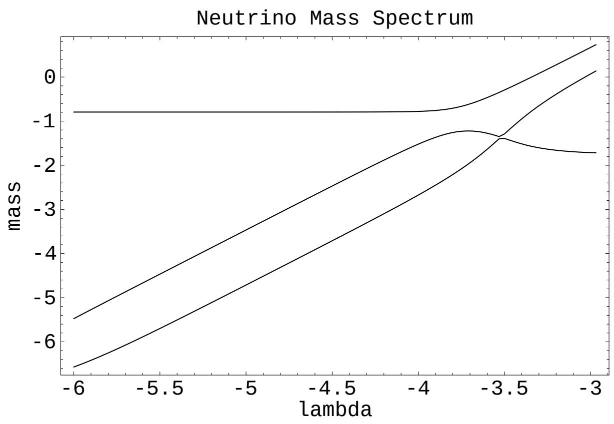
SPS 5

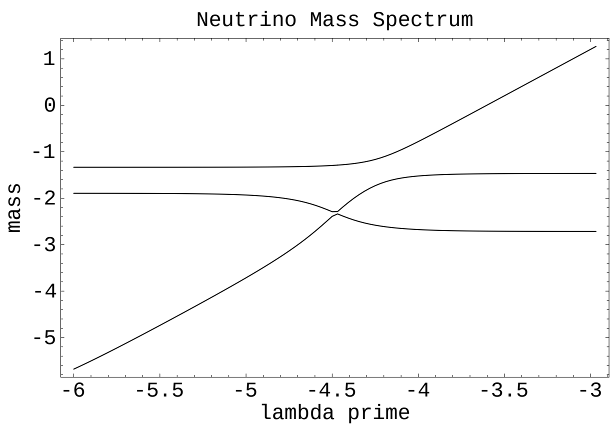

Neutrino Mass Spectrum
mass
lambda

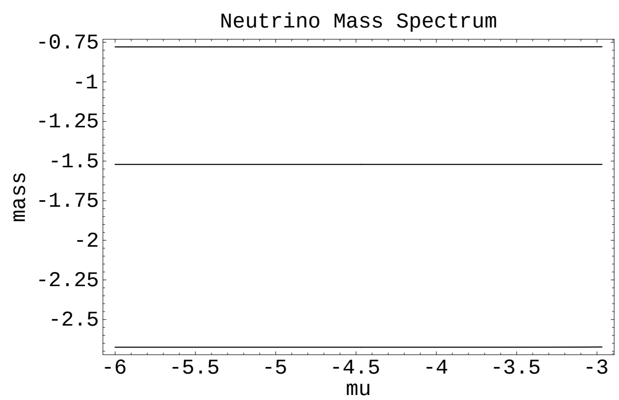

Neutrino Mass Spectrum
mass
lambda prime


Neutrino Mass Spectrum
mass
mu

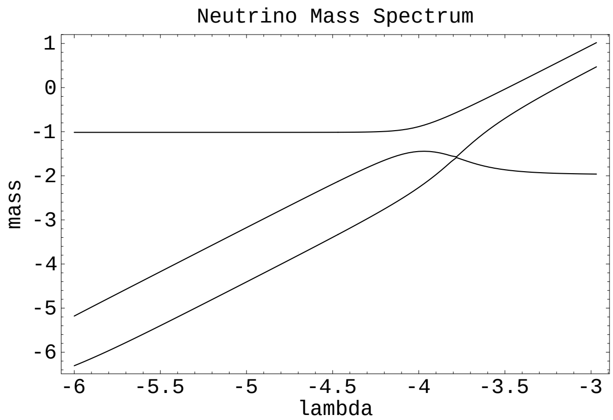
SPS 6

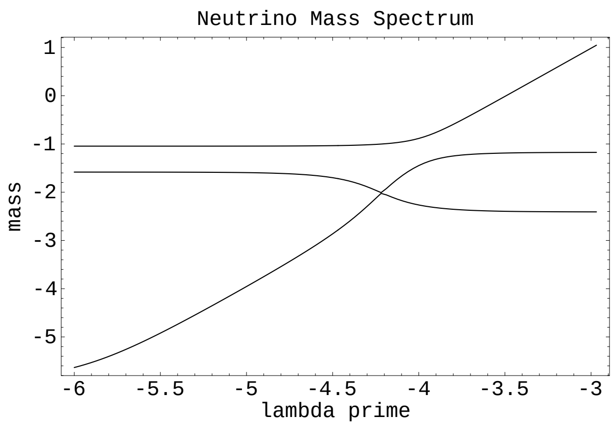

Neutrino Mass Spectrum
mass
lambda

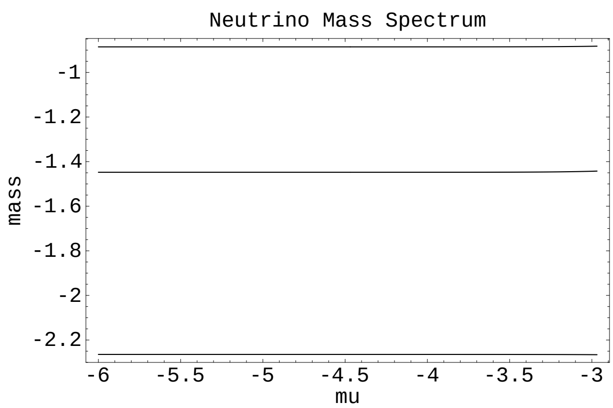

Neutrino Mass Spectrum
mass
lambda prime


Neutrino Mass Spectrum
mass
mu

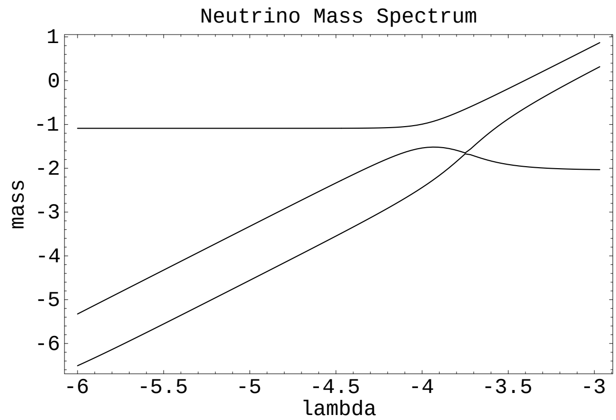
SPS 7

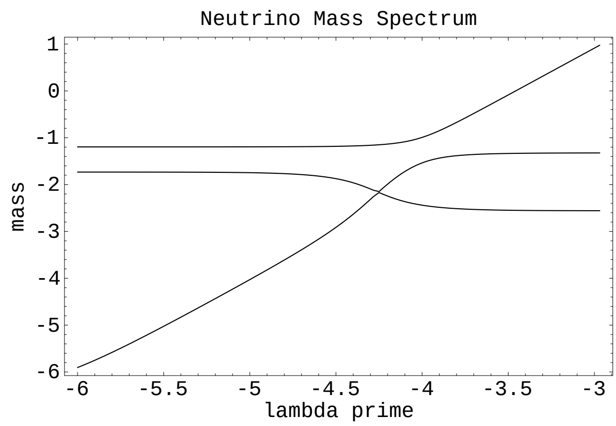

Neutrino Mass Spectrum
mass
1
0
-1
-2
-3
-4
-5
-6
-6 -5.5 -5 -4.5 -4 -3.5 -3
lambda

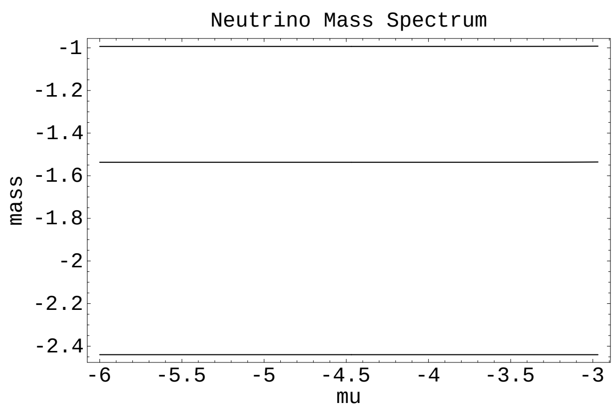

Neutrino Mass Spectrum
mass
1
0
-1
-2
-3
-4
-5
-6 -5.5 -5 -4.5 -4 -3.5 -3
lambda prime


Neutrino Mass Spectrum
mass
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-6 -5.5 -5 -4.5 -4 -3.5 -3
mu

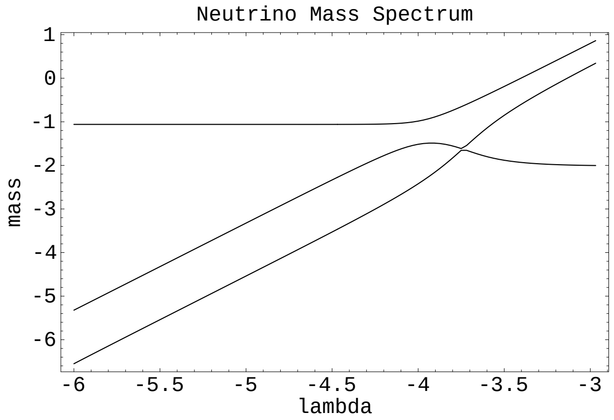
SPS 8


Neutrino Mass Spectrum
mass
1
0
-1
-2
-3
-4
-5
-6
-6 -5.5 -5 -4.5 -4 -3.5 -3
lambda

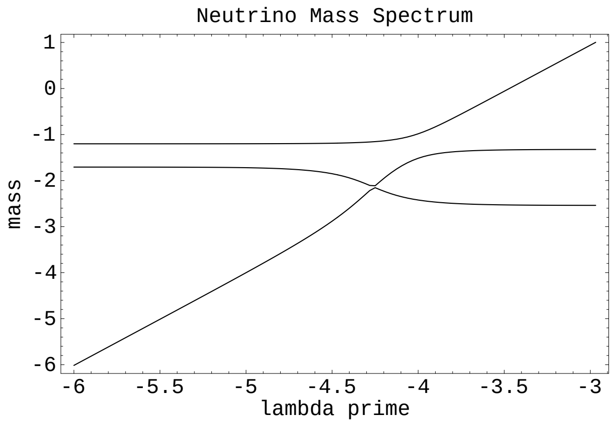

Neutrino Mass Spectrum
mass
1
0
-1
-2
-3
-4
-5
-6
-6 -5.5 -5 -4.5 -4 -3.5 -3
lambda prime


Neutrino Mass Spectrum
mass
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-6 -5.5 -5 -4.5 -4 -3.5 -3
mu

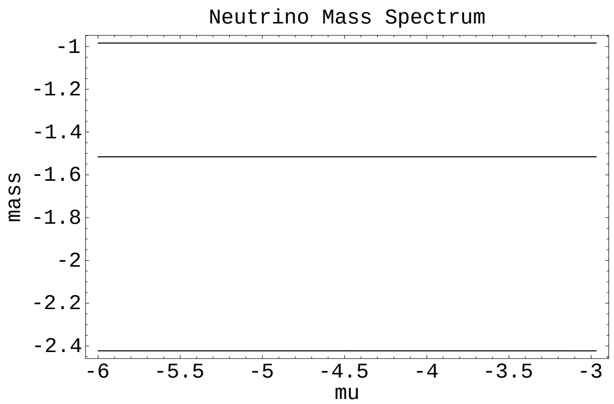
SPS 9



Neutrino Mass Spectrum
mass
lambda
1
0
-1
-2
-3
-4
-5
-6
-6
-5.5
-5
-4.5
-4
-3.5
-3



Neutrino Mass Spectrum
mass
lambda prime
1
0
-1
-2
-3
-4
-5
-6
-6
-5.5
-5
-4.5
-4
-3.5
-3



Neutrino Mass Spectrum
mass
mu
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-6
-5.5
-5
-4.5
-4
-3.5
-3

Bibliography

[1] R. Peccei. Neutrino Physics. hep-ph/9906509.

[2] R. Mohapatra and P. Pal. *Massive Neutrinos in Physics and Astrophysics.* Singapore: World Scientific (1998).

[3] A. Franklin. *Are there really Neutrinos? An Evidential History.* Cambridge, USA: Perseus Books (2001).

[4] W. Pauli. *Wissenschaftlicher Briefwechsel mit Bohr, Einstein, Heisenberg, u.a.* Berlin, Germany: Springer (1985).

[5] J. Chadwick. Intensitätsverteilung im magnetischen Spektrum der beta-Strahlen von Radium B + C. *Verhandlungen der deutschen physikalischen Gesellschaft 16*, 383 (1914).

[6] E. Fermi. An Attempt of a Theory of Beta Radiation. 1. *Z. Phys. 88*, 161 (1934).

[7] F. Reines and C. Cowan. Detection of the Free Neutrino. *Phys. Rev. 92*, 830 (1953).

[8] C. Cowan, F. Reines, F. Harrison, H. Kruse, and A. McGuire. Detection of the Free Neutrino: A Confirmation. *Science 124*, 103 (1956).

[9] S. Glashow. Partial Symmetries of Weak Interactions. *Nucl. Phys. 22*, 579 (1961).

[10] A. Salam and J. Ward. Electromagnetic and Weak Interactions. *Phys. Lett. 13*, 168 (1964).

[11] S. Weinberg. A Model of Leptons. *Phys. Rev. Lett. 19*, 1264 (1967).

[12] C. Weinheimer et al. High Precision Measurement of the Tritium Beta Spectrum Near its Endpoint and Upper Limit on the Neutrino Mass. *Phys. Lett. B460*, 219 (1999).

[13] A. Osipowicz et al. KATRIN: A Next Generation Tritium Beta Decay Experiment with Sub-eV Sensitivity for the Electron Neutrino Mass. hep-ex/0109033.

[14] R. Davis, D. Harmer, and K. Hoffman. Search for Neutrinos from the Sun. *Phys. Rev. Lett. 20*, 1205 (1968).

[15] R. Davis. A Review of the Homestake Solar Neutrino Experiment. *Prog. Part. Nucl. Phys. 32*, 13 (1994).

[16] Y. Suzuki. Kamiokande Solar Neutrino Results. *Nucl. Phys. Proc. Suppl. 38*, 54 (1995).

[17] D. Abdurashitov et al. Results from SAGE. *Phys. Lett. B328*, 234 (1994).

[18] P. Anselmann et al. GALLEX Results From the First 30 Solar Neutrino Runs. *Phys. Lett. B327*, 377 (1994).

[19] Q. Ahmad et al. Measurement of the Charged Current Interactions Produced by B-8 Solar Neutrinos at the Sudbury Neutrino Observatory. *Phys. Rev. Lett. 87*, 071301 (2001). nucl-ex/0106015.

[20] Y. Fukuda et al. Evidence for Oscillation of Atmospheric Neutrinos. *Phys. Rev. Lett. 81*, 1562 (1998). hep-ex/9807003.

[21] M. Sung. Final Neutrino Oscillation Results from LSND. *Int. J. Mod. Phys. A 16 1B*, 752 (2001).

[22] C. Giunti, C. Kim, and U. Lee. When Do Neutrinos Really Oscillate?: Quantum Mechanics of Neutrino Oscillations. *Phys. Rev. D44*, 3635 (1991).

[23] L. Wolfenstein. Neutrino Oscillations in Matter. *Phys. Rev. D17*, 2369 (1978).

[24] M. Gell-Mann, P. Ramond, and R. Slansky. Complex Spinors and Unified Theories. Print-80-0576 (CERN).

[25] H. Dreiner. An Introduction to Explicit R-Parity Violation. hep-ph/9707435.

[26] S. Weinberg. *The Quantum Theory of Fields. Vol. 1: Foundations.* Cambridge, UK: University Press (1995).

[27] S. Weinberg. *The Quantum Theory of Fields. Vol. 3: Supersymmetry.* Cambridge, UK: University Press (2000).

[28] H. P. Nilles. Supersymmetry, Supergravity and Particle Physics. *Phys. Rept. 110*, 1 (1984).

[29] L. Susskind. The Gauge Hierarchy Problem, Technicolor, Supersymmetry, and all that. (Talk). *Phys. Rept. 104*, 181 (1984).

[30] H. Murayama. Dynamical (Super)Symmetry Breaking. hep-ph/0010021.

[31] H. Haber. Low-Energy Supersymmetry: Prospects and Challenges. hep-ph/9510412.

[32] H. Murayama. Supersymmetry Phenomenology. hep-ph/0002232.

[33] L. Ibanez and G. Ross. SU(2)-L x U(1) Symmetry Breaking as a Radiative Effect of Supersymmetry Breaking in GUTs. *Phys. Lett. B110*, 215 (1982).

[34] E. Farhi and L. Susskind. Technicolor. *Phys. Rept. 74*, 277 (1981).

[35] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali. The Hierarchy Problem and New Dimensions at a Millimeter. *Phys. Lett. B429*, 263 (1998). hep-ph/9803315.

[36] S. Dimopoulos and H. Georgi. Softly Broken Supersymmetry and SU(5). *Nucl. Phys. B193*, 150 (1981).

[37] J. Ellis, S. Kelley, and D. Nanopoulos. Probing the Desert Using Gauge Coupling Unification. *Phys. Lett. B260*, 131 (1991).

[38] R. Haag, J. Lopuszanski, and M. Sohnius. All Possible Generators of Supersymmetries of the S Matrix. *Nucl. Phys. B88*, 257 (1975).

[39] S. Coleman and J. Mandula. All Possible Symmetries of the S Matrix. *Phys. Rev. 159*, 1251 (1967).

[40] D. Bailin and A. Love. *Supersymmetric Gauge Field Theory and String Theory.* Bristol, UK: Institute of Physics Publishing (1994).

[41] A. Salam and J. Strathdee. Supergauge Transformations. *Nucl. Phys. B76*, 477 (1974).

[42] H. Haber. *Introduction to Supersymmetry.* Copy of H. Haber's lecture notes (2000).

[43] H. Haber and G. Kane. The Search for Supersymmetry: Probing Physics Beyond the Standard Model. *Phys. Rept. 117*, 75 (1985).

[44] J. Gunion and H. Haber. Higgs Bosons in Supersymmetric Models. 1. *Nucl. Phys. B272*, 1 (1986).

[45] S. Ferrara and B. Zumino. Supergauge Invariant Yang-Mills Theories. *Nucl. Phys. B79*, 413 (1973).

[46] A. Salam and J. Strathdee. Supersymmetry and Nonabelian Gauges. *Phys. Lett. B51*, 353 (1974).

[47] J. Bagger and J. Wess. *Supersymmetry and Supergravity.* Princeton, UK: University Press (1992).

[48] H. Kalka and G. Soff. *Supersymmetrie. (In German).* Stuttgart, Germany: Teubner (1997).

[49] P. Vaudrevange. *Feynman Rules for Two-Dimensional Weyl Spinors in Supersymmetry.* Diploma thesis, Bonn University (2001).

[50] P. Ramond. *Journeys Beyond the Standard Model.* Cambridge, USA: Perseus Books (1999).

[51] D. Griffiths. *Introduction to Elementary Particles.* New York, USA: Wiley (1987).

[52] S. Weinberg. Supersymmetry at Ordinary Energies. 1. Masses and Conservation Laws. *Phys. Rev. D26*, 287 (1982).

[53] Y. Grossman and H. Haber. (S)neutrino Properties in R-Parity Violating Supersymmetry. I: CP-Conserving Phenomena. *Phys. Rev. D59*, 093008 (1999). hep-ph/9810536.

[54] J. Goity and M. Sher. Bounds on Delta B = 1 Couplings in the Supersymmetric Standard Model. *Phys. Lett. B346*, 69 (1995). hep-ph/9412208.

[55] D. Groom et al. Review of Particle Physics. *Eur. Phys. J. C15*, 1 (2000).

[56] B. Allanach, A. Dedes, and H. Dreiner. Bounds on R-Parity Violating Couplings at the Weak Scale and at the GUT Scale. *Phys. Rev. D60*, 075014 (1999). hep-ph/9906209.

[57] A. Abada and M. Losada. Constraints on Both Bilinear and Trilinear R-Parity Violating Couplings from Neutrino Laboratories and Astrophysics Data. *Phys. Lett. B492*, 310 (2000). hep-ph/0007041.

[58] L. Girardello and M. Grisaru. Soft Breaking of Supersymmetry. *Nucl. Phys. B194*, 65 (1982).

[59] E. Nardi. Renormalization Group Induced Neutrino Masses in Supersymmetry without R-Parity. *Phys. Rev. D55*, 5772 (1997). hep-ph/9610540.

[60] S. Davidson and M. Losada. Neutrino Masses in the R(P) Violating MSSM. *JHEP 05*, 021 (2000). hep-ph/0005080.

[61] Y. Kuno and Y. Okada. Muon Decay and Physics Beyond the Standard Model. *Rev. Mod. Phys. 73*, 151 (2001). hep-ph/9909265.

[62] S. Davidson and M. Losada. Basis Independent Neutrino Masses in the R(P) Violating MSSM. hep-ph/0010325.

[63] Y. Grossman and H. Haber. Neutrino Masses and Sneutrino Mixing in R-Parity Violating Supersymmetry. hep-ph/9906310.

[64] Y. Grossman and H. Haber. Sneutrino Mixing Phenomena. *Phys. Rev. Lett. 78*, 3438 (1997). hep-ph/9702421.

[65] S. Choi, J. Kalinowski, G. Moortgat-Pick, and P. Zerwas. Analysis of the Neutralino System in Supersymmetric Theories. *Eur. Phys. J. C22*, 563 (2001). hep-ph/0108117.

[66] Y. Grossman and H. Haber. Basis-Independent Analysis of the Sneutrino Sector in R-Parity Violating Supersymmetry. *Phys. Rev. D63*, 075011 (2001). hep-ph/0005276.

[67] S. Davidson, M. Losada, and N. Rius. Neutral Higgs Sector of the MSSM without R(P). *Nucl. Phys. B587*, 118 (2000). hep-ph/9911317.

[68] H. Haber. The Status of the Minimal Supersymmetric Standard Model and Beyond. *Nucl. Phys. Proc. Suppl. 62*, 469 (1998). hep-ph/9709450.

[69] B. Allanach et al. The Snowmass Points and Slopes: Benchmarks for Susy Searches. hep-ph/0202233.

[70] A. Abada and M. Losada. Constraints on a General 3-Generation Neutrino Mass Matrix from Neutrino Data: Application to the MSSM with R-Parity Violation. *Nucl. Phys. B585*, 45 (2000). hep-ph/9908352.

[71] K. Assamagan et al. Upper Limit of the Muon-Neutrino Mass and Charged Pion Mass from Momentum Analysis of a Surface Muon Beam. *Phys. Rev. D53*. 6065 (1996).

[72] R. Barate et al. An Upper Limit on the Tau Neutrino Mass from Three- and Five-Prong Tau Decays. *Eur. Phys. J. C2*, 395 (1998).

[73] G. Fogli, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo. Atmospheric, Solar, and CHOOZ Neutrinos: A Global Three Generation Analysis. hep-ph/0104221.

[74] M. Apollonio et al. Limits on Neutrino Oscillations from the CHOOZ Experiment. *Phys. Lett. B466*, 415 (1999). hep-ex/9907037.

[75] W. Furry. On Transition Probabilities in Double Beta-Disintegration. *Phys.Rev.56*. 1184 (1939).

[76] B. Kayser, F. Gibrat-Debu, and F. Perrier. *The Physics of Massive Neutrinos*. Singapore: World Scientific (1989).

[77] L. Baudis et al. The Heidelberg - Moscow Experiment: Improved Sensitivity for Ge-76 Neutrinoless Double Beta Decay. *Phys. Lett. B407*, 219 (1997).

[78] H. Klapdor-Kleingrothaus, A. Dietz, H. Harney, and I. Krivosheina. Evidence for Neutrinoless Double Beta Decay. *Mod. Phys. Lett. A16*, 2409 (2002). hep-ph/0201231.

[79] C. Aalseth et al. Comment on 'Evidence for Neutrinoless Double Beta Decay'. hep-ex/0202018.

[80] H. Pas and T. Weiler. Absolute Neutrino Mass Determination. *Phys. Rev. D63*, 113015 (2001).

[81] Z. Fodor, S. Katz, and A. Ringwald. Determination of Absolute Neutrino Masses from Z-Bursts. hep-ph/0105064.

[82] K. Greisen. End to the Cosmic Ray Spectrum? *Phys. Rev. Lett. 16*, 748 (1966).

[83] G. Passarino and M. Veltman. One Loop Corrections for e+ e- Annihilation into mu+ mu- in the Weinberg Model. *Nucl. Phys. B160*, 151 (1979).

[84] G. 't Hooft and M. Veltman. Scalar one Loop Integrals. *Nucl. Phys. B153*, 365 (1979).

[85] N. Ghodbane and H.-U. Martyn. Compilation of Susy Particle Spectra from Snowmass 2001 Benchmark Models. hep-ph/0201233.