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Feynman rules for Weyl spinors in supersymmetry

von
Pascal Marc Vaudrevange

We give a brief introduction to Weyl spinors and show how to derive Feynman rules. We construct the Lagrangian density of the MSSM **using the superfield formalism, derive the Feynman rules for this theory and perform some sample calculations.**

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Feynman Rules for Two-Dimensional Weyl Spinors in Supersymmetry

Pascal Vaudrevange

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Ich versichere, dass ich diese Arbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie die Zitate kenntlich gemacht habe.

Referent:	Prof. Dr. H.K. Dreiner
Koreferent:	Prof. Dr. H.P. Nilles

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Overview

In this thesis we derive the Feynman rules for Weyl spinors in the MSSM. First we will begin with a brief introduction to free Weyl spinors, showing their explicit form and how to express Dirac spinors in terms of them. Next we will review the process of deriving Feynman rules both by path integral methods and by making use of Wick's theorem. In the third chapter we will comment on the appropriate way to define SUSY generators as the (in)correct choice has been causing some confusion. In the fourth chapter we give a short introduction to supersymmetry and its invariants. The next chapter contains the derivation of the Lagrangian density of the MSSM which enables us to list the Feynman rules in chapter seven. Finally in the last chapter we present some sample calculations in the Weyl spinor formalism. For convenience we included most of the calculational details in the appendix which also contains a list of useful formulae in part B as well as the conventions used throughout this thesis.

Chapter 1

Introduction to Weyl Spinors

The proper Lorentz group $SO(1,3)$ is locally isomorphic to $SU(2)_L \otimes SU(2)_R$. Spinors which transform under $SU(2)_L$ are called left-chiral Weyl spinors and are represented by $\psi_\alpha, \alpha = 1, 2$. Those spinors transforming under $SU(2)_R$ are called right-chiral. They are written as $\bar{\psi}^{\dot{\alpha}}, \dot{\alpha} = \dot{1}, \dot{2}$. Their renormalizable Poincaré invariant combinations are given by

$$\psi^\alpha \phi_\alpha,\ \bar{\psi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}},\ \psi^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \bar{\phi}^{\dot{\alpha}} A_\mu,\ \bar{\psi}_{\dot{\alpha}} \bar{\sigma}^{\dot{\alpha}\alpha}_\mu \phi_\alpha A^\mu, \tag{1.1}$$

where A_μ is some four-vector. We raise and lower dotted and undotted indices with

$$\begin{aligned} \epsilon_{\alpha\beta} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \\ \epsilon_{\dot{\alpha}\dot{\beta}} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \end{aligned} \tag{1.2}$$

respectively. Complex conjugation acts on Weyl spinors as

$$(\psi_\alpha)^* = \bar{\psi}_{\dot{\alpha}}. \tag{1.3}$$

The Lagrangian density $\mathcal{L}$ for a free Weyl spinor is given by

$$\mathcal{L} = i\psi\sigma^\mu\partial_\mu\bar{\psi}. \tag{1.4}$$

Making the ansatz in analogy to the Dirac equation

$$\begin{aligned} \psi_\alpha &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(u_\alpha(p) a_{\vec{p}} e^{-ipx} + v_\alpha(p) b^\dagger_{\vec{p}} e^{ipx} \right), \\ \bar{\psi}_{\dot{\alpha}} &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(\bar{u}_{\dot{\alpha}}(p) a^\dagger_{\vec{p}} e^{ipx} + \bar{v}_{\dot{\alpha}}(p) b_{\vec{p}} e^{-ipx} \right), \end{aligned} \tag{1.5}$$

with $p = (E_{\vec{p}}, \vec{p})$ and $E_{\vec{p}} = \sqrt{m^2 + \vec{p}^2} \equiv p_0$ leads to the solution of the equations of motion (e.o.m.)

$$\begin{aligned} i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}^{\dot{\alpha}} &= 0, \\ \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix} \begin{pmatrix} \bar{u}^{\dot{1}} \\ \bar{u}^{\dot{2}} \end{pmatrix} &= 0, \\ \bar{u}^{\dot{1}} &= -\frac{p_0 - p_3}{p_1 + ip_2} \bar{u}^{\dot{2}}. \end{aligned} \tag{1.6}$$

The solution for the antiparticle is

$$\bar{v}^{\dot{1}} = -\frac{p_0 - p_3}{p_1 + ip_2}\bar{v}^{\dot{2}}. \tag{1.7}$$

We could have also taken the functional derivative with respect to $\bar{\psi}$ and we could have transformed $\mathcal{L}$ to $\mathcal{L} = i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi$. Those cases can be dealt with along similar lines (see appendix A.1) giving

$$\begin{aligned}
\bar{u}^{\dot{\alpha}} &= N^* \begin{pmatrix} -p_0 + p_3 \\ p_1 + ip_2 \end{pmatrix}, \quad \bar{v}^{\dot{\alpha}} = N'^* \begin{pmatrix} -p_0 + p_3 \\ p_1 + ip_2 \end{pmatrix}, \\
u^{\alpha} &= N \begin{pmatrix} -p_0 + p_3 \\ p_1 - ip_2 \end{pmatrix}, \quad v^{\alpha} = N' \begin{pmatrix} -p_0 + p_3 \\ p_1 - ip_2 \end{pmatrix}, \\
u_{\alpha} &= N \begin{pmatrix} p_1 - ip_2 \\ p_0 - p_3 \end{pmatrix}, \quad v_{\alpha} = N' \begin{pmatrix} p_1 - ip_2 \\ p_0 - p_3 \end{pmatrix}, \\
\bar{u}_{\dot{\alpha}} &= N^* \begin{pmatrix} p_1 + ip_2 \\ p_0 - p_3 \end{pmatrix}, \quad \bar{v}_{\dot{\alpha}} = N'^* \begin{pmatrix} p_1 + ip_2 \\ p_0 - p_3 \end{pmatrix}.
\end{aligned} \tag{1.8}$$

We remark that these spinors satisfy $\bar{u}^{\dot{\alpha}} = (u^\alpha)^* = (\epsilon^{\alpha\beta}u_\beta)^*$. To find the normalization N, N', we compare these results with the solutions of the Dirac equation. In the Dirac representation[1] the Dirac equation is solved by the following four component spinors[2] satisfying $u_D^\dagger u_D = 2E_{\vec{p}}$

$$\begin{aligned}
u_D^{(1)} &= \frac{1}{\sqrt{p_0}} \begin{pmatrix} p_0 \\ 0 \\ -p_3 \\ -p_1 - ip_2 \end{pmatrix}, \quad u_D^{(2)} = \frac{1}{\sqrt{p_0}} \begin{pmatrix} 0 \\ p_0 \\ -p_1 + ip_2 \\ p_3 \end{pmatrix}, \\
v_D^{(1)} &= \frac{1}{\sqrt{p_0}} \begin{pmatrix} p_1 - ip_2 \\ -p_3 \\ 0 \\ p_0 \end{pmatrix}, \quad v_D^{(2)} = \frac{1}{\sqrt{p_0}} \begin{pmatrix} -p_3 \\ -p_1 - ip_2 \\ -p_0 \\ 0 \end{pmatrix},
\end{aligned} \tag{1.9}$$

where $u_D^{(1)}$ is an electron with spin up, $u_D^{(2)}$ corresponds to an electron with spin down, $v_D^{(1)}$ represents a positron with spin down and $v_D^{(2)}$ denotes a positron with spin up. To convert spinors from the Dirac to the chiral representation one has[3]

$$\psi_{\text{chiral}} = U\psi_D = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \psi_D. \tag{1.10}$$

Therefore in the chiral representation the spinors are given by

$$\begin{aligned}
u_{\text{chiral}}^{(1)} &= \frac{1}{\sqrt{2p_0}} \begin{pmatrix} p_0 + p_3 \\ p_1 + ip_2 \\ p_0 - p_3 \\ -p_1 - ip_2 \end{pmatrix}, \quad u_{\text{chiral}}^{(2)} = \frac{1}{\sqrt{2p_0}} \begin{pmatrix} p_1 - ip_2 \\ p_0 - p_3 \\ -p_1 + ip_2 \\ p_0 + p_3 \end{pmatrix}, \\
v_{\text{chiral}}^{(1)} &= \frac{1}{\sqrt{2p_0}} \begin{pmatrix} -p_1 + ip_2 \\ -p_0 + p_3 \\ -p_1 + ip_2 \\ p_0 + p_3 \end{pmatrix}, \quad v_{\text{chiral}}^{(2)} = \frac{1}{\sqrt{2p_0}} \begin{pmatrix} p_0 + p_3 \\ p_1 + ip_2 \\ -p_0 + p_3 \\ p_1 + ip_2 \end{pmatrix}.
\end{aligned} \tag{1.11}$$

---

[1] In this representation the γ matrices are given by $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$.

[2] C.f. eq.(7.46) and eq.(7.50) in [Gri87], assuming massless particles and remembering $(\vec{p})_i = -p_i$.

[3] C.f.(A-26) in [IZ80].

u_α	$=$	$\frac{1}{\sqrt{2p_0}}\begin{pmatrix} p_0+p_3 \\ p_1+ip_2 \end{pmatrix}$	$\bar{u}_{\dot\alpha}$	$=$	$\frac{1}{\sqrt{2p_0}}\begin{pmatrix} p_0+p_3 \\ p_1-ip_2 \end{pmatrix}$
u^α	$=$	$\frac{1}{\sqrt{2p_0}}\begin{pmatrix} -p_1-ip_2 \\ p_0+p_3 \end{pmatrix}$	$\bar{u}^{\dot\alpha}$	$=$	$\frac{1}{\sqrt{2p_0}}\begin{pmatrix} -p_1+ip_2 \\ p_0+p_3 \end{pmatrix}$
v_α	$=$	$\frac{1}{\sqrt{2p_0}}\begin{pmatrix} -p_1+ip_2 \\ -p_0+p_3 \end{pmatrix}$	$\bar{v}_{\dot\alpha}$	$=$	$\frac{1}{\sqrt{2p_0}}\begin{pmatrix} -p_1-ip_2 \\ -p_0+p_3 \end{pmatrix}$
v^α	$=$	$\frac{1}{\sqrt{2p_0}}\begin{pmatrix} p_0-p_3 \\ -p_1+ip_2 \end{pmatrix}$	$\bar{v}^{\dot\alpha}$	$=$	$\frac{1}{\sqrt{2p_0}}\begin{pmatrix} p_0-p_3 \\ -p_1-ip_2 \end{pmatrix}$

Table 1.1: Weyl spinors

$u^{(1)}_{\text{chiral}}$	$=$	$\begin{pmatrix} u_\alpha \\ \bar{v}^{\dot\alpha} \end{pmatrix}$	$u^{(2)}_{\text{chiral}}$	$=$	$\begin{pmatrix} -v_\alpha \\ \bar{u}^{\dot\alpha} \end{pmatrix}$
$v^{(1)}_{\text{chiral}}$	$=$	$\begin{pmatrix} v_\alpha \\ \bar{u}^{\dot\alpha} \end{pmatrix}$	$v^{(2)}_{\text{chiral}}$	$=$	$\begin{pmatrix} u_\alpha \\ -\bar{v}^{\dot\alpha} \end{pmatrix}$

Table 1.2: Dirac spinors in terms of Weyl spinors

Now we can compare Dirac spinors and Weyl spinors. As an ansatz we choose

$$u^{(1)}_{\text{chiral}} = \begin{pmatrix} u_\alpha \\ \bar{v}^{\dot\alpha} \end{pmatrix} = \begin{pmatrix} N(p_1-ip_2) \\ N(p_0-p_3) \\ N'^*(-p_0+p_3) \\ N'^*(p_1+ip_2) \end{pmatrix}, \tag{1.12}$$

so that we have the following normalization from $u^{(1)\dagger}_{\text{chiral}} u^{(1)}_{\text{chiral}} = 2E_{\vec{p}}$

$$N = \frac{1}{\sqrt{2p_0}} \frac{p_1+ip_2}{p_0-p_3} \quad , \quad N'^* = -\frac{1}{\sqrt{2p_0}}. \tag{1.13}$$

This gives the final form of the Weyl spinors (table (1.1)). We can then express the other Dirac spinors in terms of Weyl spinors which is useful for knowing what diagrams to compute for a given particle setup (table (1.2))

$$\begin{aligned} u^{(1)}_{\text{chiral}} &= \begin{pmatrix} u_\alpha \\ \bar{v}^{\dot\alpha} \end{pmatrix}, \quad u^{(2)}_{\text{chiral}} \begin{pmatrix} -v_\alpha \\ \bar{u}^{\dot\alpha} \end{pmatrix}, \\ v^{(1)}_{\text{chiral}} &= \begin{pmatrix} v_\alpha \\ \bar{u}^{\dot\alpha} \end{pmatrix}, \quad v^{(2)}_{\text{chiral}} \begin{pmatrix} u_\alpha \\ -\bar{v}^{\dot\alpha} \end{pmatrix}. \end{aligned} \tag{1.14}$$

As a tool for calculations with Dirac spinors there are the completeness relations

$$\begin{aligned} \sum_s u^{(s)}\bar{u}^{(s)} &= \not{p} + m &= \begin{pmatrix} m & \sigma^\mu p_\mu \\ \bar\sigma^\mu p_\mu & m \end{pmatrix}, \\ \sum_s v^{(s)}\bar{v}^{(s)} &= \not{p} - m &= \begin{pmatrix} -m & \sigma^\mu p_\mu \\ \bar\sigma^\mu p_\mu & -m \end{pmatrix}, \end{aligned} \tag{1.15}$$

which can be expanded in terms of Weyl spinors

$$\begin{aligned}
\sum_s u^{(s)}_{\text{chiral}}\bar{u}^{(s)}_{\text{chiral}} &= u^{(1)}_{\text{chiral}}\bar{u}^{(1)}_{\text{chiral}} + u^{(2)}_{\text{chiral}}\bar{u}^{(2)}_{\text{chiral}} \\
&= \begin{pmatrix} u_\alpha v^\beta - v_\alpha u^\beta & u_\alpha \bar{u}_{\dot\beta} + v_\alpha \bar{v}_{\dot\beta} \\ \bar{v}^{\dot\alpha} v^\beta + \bar{u}^{\dot\alpha} u^\beta & \bar{v}^{\dot\alpha}\bar{u}_{\dot\beta} - \bar{u}^{\dot\alpha}\bar{v}_{\dot\beta} \end{pmatrix}, \\
\sum_s v^{(s)}_{\text{chiral}}\bar{v}^{(s)}_{\text{chiral}} &= \begin{pmatrix} v_\alpha u^\beta - u_\alpha v^\beta & v_\alpha \bar{v}_{\dot\beta} + u_\alpha \bar{u}_{\dot\beta} \\ \bar{u}^{\dot\alpha} u^\beta + \bar{v}^{\dot\alpha} v^\beta & \bar{u}^{\dot\alpha}\bar{v}_{\dot\beta} - \bar{v}^{\dot\alpha}\bar{u}_{\dot\beta} \end{pmatrix},
\end{aligned} \tag{1.16}$$

and we note that indeed the diagonal elements vanish as Weyl spinors are massless. The off-diagonal entries give

$$\begin{aligned}
u_\alpha \bar{u}_{\dot\beta} &= \frac{(p_0+p_3)(p_0-p_3)}{2p_0(p_0-p_3)} \begin{pmatrix} p_0+p_3 & p_1 - ip_2 \\ p_1+ip_2 & p_0-p_3 \end{pmatrix} = \frac{p_0+p_3}{2p_0}\sigma^\mu_{\alpha\dot\beta} p_\mu, \\
v_\alpha \bar{v}_{\dot\beta} &= \frac{p_0-p_3}{2p_0} \begin{pmatrix} p_0+p_3 & p_1 - ip_2 \\ p_1+ip_2 & p_0-p_3 \end{pmatrix} = \frac{p_0-p_3}{2p_0}\sigma^\mu_{\alpha\dot\beta} p_\mu, \\
\bar{u}^{\dot\alpha} u^\beta &= \frac{p_0+p_3}{2p_0} \begin{pmatrix} p_0-p_3 & -p_1 + ip_2 \\ -p_1-ip_2 & p_0+p_3 \end{pmatrix} = \frac{p_0+p_3}{2p_0}\bar{\sigma}^{\mu\dot\alpha\beta} p_\mu, \\
\bar{v}^{\dot\alpha} v^\beta &= \frac{p_0-p_3}{2p_0} \begin{pmatrix} p_0-p_3 & -p_1 + ip_2 \\ -p_1-ip_2 & p_0+p_3 \end{pmatrix} = \frac{p_0-p_3}{2p_0}\bar{\sigma}^{\mu\dot\alpha\beta} p_\mu.
\end{aligned} \tag{1.17}$$

We obtain the completeness relations for Weyl spinors

$$\begin{aligned}
u_\alpha \bar{u}_{\dot\alpha} + v_\alpha \bar{v}_{\dot\alpha} &= \sigma^\mu_{\alpha\dot\alpha} p_\mu, \\
\bar{u}^{\dot\alpha} u^\alpha + \bar{v}^{\dot\alpha} v^\alpha &= \bar{\sigma}^{\mu\dot\alpha\alpha} p_\mu.
\end{aligned} \tag{1.18}$$

These formulae are helpful when calculating Feynman diagrams.

Chapter 2

How to Derive Feynman Rules

Feynman diagrams are a way of visualising n-point Green's functions which are defined as

$$\Delta_{1,2,3,\ldots,n} \equiv \frac{1}{i^n}\frac{1}{Z[0]}\frac{\delta}{\delta J_1}\frac{\delta}{\delta J_2}\frac{\delta}{\delta J_3}\cdots\frac{\delta}{\delta J_n}Z[J]\Big|_{J=0}, \tag{2.1}$$

with the generating functional

$$Z[J] = N\int D\phi_1 D\phi_2 D\phi_3 \ldots D\phi_n\, e^{i\int d^4x(\mathcal{L}+J_x\phi_x)}. \tag{2.2}$$

$\mathcal{L}$ is the Lagrange density, ϕ is a real field, J is an arbitrary source term. The lower index symbolizes the variable on which the field depends, e.g. $\phi_x = \phi(x)$, $J_1 = J(z_1)$. A n-point Green's functions is represented by a blob diagram, e.g. figure (2.1). One can show that it corresponds to

$$\langle 0|T\phi_1\phi_2\phi_3\ldots\phi_n|0\rangle = \Delta_{1,2,3,\ldots,n}, \tag{2.3}$$

with T the time-ordering operator. This time ordered product can be expanded via the use of Wick's theorem[1]

$$\begin{aligned}\langle 0|T\phi_1\phi_2\phi_3\ldots\phi_n|0\rangle &= \langle 0|:\phi_1\phi_2\phi_3\ldots\phi_n:|0\rangle + \langle 0|:\overbrace{\phi_1\phi_2}\,\phi_3\ldots\phi_n:|0\rangle + \ldots\\ &\quad +\text{all possible contractions.}\end{aligned} \tag{2.4}$$

One can interprete the n-point Green's function as the amplitude for the creation and destruction of a total of n particles. A typical Green's function contains also some so called disconnected pieces (e.g. figure (2.2)) which do not contribute to the non-trivial part of the S-matrix. To get rid of them one introduces another generating functional

$$W[J] \equiv -i\ln Z[J] \qquad \Leftrightarrow \qquad Z[J] = e^{iW[J]}, \tag{2.5}$$

which replaces $Z[J]$ in eq.(2.1). In practice however we only compute the n-point function via the generator $Z[J]$ and drop the disconnected pieces.

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[1]: A : symbolizes a normal-ordered operator, i.e. all annihilation operators are standing right of the creation operators. The precise meaning of the contraction $\overbrace{\phi\phi}$ will be explained in section 2.2.

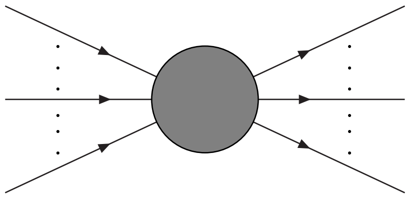
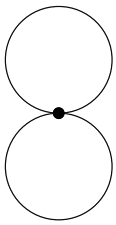
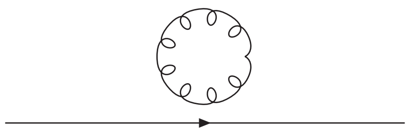


Figure 2.1: Graphical representation of a generic n-point Green's function



(a) Bubble diagram



(b) Disconnected 4-point function

Figure 2.2: Some disconnected Feynman diagrams

2.1 LSZ Reduction Formula

The LSZ reduction formula connects the S-matrix $S = \langle\phi_{\rm out}|\phi_{\rm in}\rangle$ and n-point Green's functions. The S matrix satisfies $\phi_{{\rm out},x} = S^\dagger \phi_{{\rm in},x} S$

$$\begin{aligned} \phi_{{\rm out},x} &= \lim_{t\to\infty}\phi(x) \\ \phi_{{\rm in},x} &= \lim_{t\to-\infty}\phi(x), \end{aligned}$$

with a real field ϕ. Its equation of motion is given by

$$(\Box_x + m^2)\phi_x = \frac{\delta\mathcal{L}_{\rm int}}{\delta\phi_x}, \tag{2.6}$$

with the Green's function $\Delta_{xy} \equiv \Delta(x-y)$ solving

$$(\Box_x + m^2)\Delta_{xy} = -i\delta_{xy},$$

and

$$\begin{aligned} \delta_{xy} &\equiv \delta^{(4)}(x-y), \\ \Box_x &\equiv \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x^\mu}. \end{aligned}$$

$\mathcal{L}_{\rm int}$ is the part of the Lagrange density containing the interactions. The solution ϕ_x is given by

$$\begin{aligned} \phi_x &= \phi_{{\rm in},x} + i\int d^4y\Delta^{\rm ret}_{xy}\frac{\delta\mathcal{L}_{\rm int}}{\delta\phi_y} \\ &= \phi_{{\rm in},x} + i\int d^4y\Delta^{\rm ret}_{xy}(\Box_y + m^2)\phi_y, \qquad (2.7) \\ \phi_x &= \phi_{{\rm out},x} + i\int d^4y\Delta^{\rm adv}_{xy}\frac{\delta\mathcal{L}_{\rm int}}{\delta\phi_y} \\ &= \phi_{{\rm out},x} + i\int d^4y\Delta^{\rm adv}_{xy}(\Box_y + m^2)\phi_y, \qquad (2.8) \end{aligned}$$

with $\left.\begin{array}{l}\Delta^{\rm ret}_{xy}=0\\ \Delta^{\rm adv}_{xy}=0\end{array}\right\}$ for $\left\{\begin{array}{l}x^0<y^0\\ x^0>y^0\end{array}\right.$ being the advanced and retarded propagator, respectively. We define

$$\begin{aligned} I[J] &\equiv Te^{i\int d^4x J_x\phi_x} \\ \Rightarrow \frac{1}{i}\frac{\delta I[J]}{\delta J_x} &= T\left(\phi_x I[J]\right). \end{aligned} \tag{2.9}$$

From eq.(2.1) and eq.(2.3) it follows that

$$\langle 0|I[J]|0\rangle = \frac{Z[J]}{Z[0]}. \tag{2.10}$$

Multiplying eq.(2.7) by $I[J]$ and T and using eq.(2.9)[2] we get

$$\frac{1}{i}\frac{\delta I[J]}{\delta J_x} = I[J]\phi_{{\rm in},x} + i\int d^4y\Delta^{\rm ret}_{xy}(\Box_y + m^2)\frac{1}{i}\frac{\delta I[J]}{\delta J_y}. \tag{2.11}$$

[2]We recall that $T(\phi_{{\rm in},x}I[J]) = I[J]\phi_{{\rm in},x}$ as $\phi_{\rm in}$ 'happens' before everything else.

The other solution, eq.(2.8), can be treated similarily

$$\frac{1}{i}\frac{\delta I[J]}{\delta J_x} = \phi_{\mathrm{out},x} I[J] + i\int d^4y \Delta^{\mathrm{adv}}_{xy}(\Box_y + m^2)\frac{1}{i}\frac{\delta I[J]}{\delta J_y}. \tag{2.12}$$

Substracting both equations and noting that $\Delta_{xy} = \Delta^{\mathrm{adv}}_{xy} - \Delta^{\mathrm{ret}}_{xy}$ gives

$$\phi_{\mathrm{out},x} I[J] - I[J]\phi_{\mathrm{in},x} = -\int d^4y \Delta_{xy}(\Box_y + m^2)\frac{\delta I[J]}{\delta J_y}. \tag{2.13}$$

Substituting $\phi_{\mathrm{out},x} = S^\dagger \phi_{\mathrm{in},x} S$ and multiplying by S from the left we get

$$[\phi_{\mathrm{in},x}, SI[J]] = -\int d^4y \Delta_{xy}(\Box_y + m^2)\frac{\delta SI[J]}{\delta J_y}. \tag{2.14}$$

A suitable ansatz for this equation is

$$SI[J] = e^{-\int d^4z \phi_{\mathrm{in},z}(\Box_z + m^2)\frac{\delta}{\delta J_z}} F[J], \tag{2.15}$$

with a yet-to-be-determined function $F[J]$. We insert this into the left-hand side[3]

$$[\phi_{\mathrm{in},x}, SI[J]] = \int d^4z \Delta_{xz}(\Box_z + m^2) e^{-\int d^4z \phi_{\mathrm{in},z}(\Box_z + m^2)\frac{\delta}{\delta J_z}} \frac{\delta F[J]}{\delta J_z}, \tag{2.16}$$

and the right-hand side gives

$$-\int d^4y \Delta_{xy}(\Box_y + m^2)\frac{\delta SI[J]}{\delta J_y} = -\int d^4y \Delta_{xy}(\Box_y + m^2) e^{\int d^4z \phi_{\mathrm{in},z}(\Box_z + m^2)\frac{\delta}{\delta J_z}} \frac{\delta F[J]}{\delta J_y}. \tag{2.17}$$

When we normal-order the expression eq.(2.15) for $SI[J]$ we can use the fact that $\langle 0| : e^A : |0\rangle = 1$ for any operator A. So we get

$$\langle 0|SI[J]|0\rangle = F[J]. \tag{2.18}$$

From eq.(2.10) we know that $\langle 0|SI[J]|0\rangle = \langle 0|I[J]|0\rangle = \frac{Z[J]}{Z[0]}$ and using $I[J] \stackrel{J\to 0}{\longrightarrow} 1$ we finally find the expression for S

$$S = \; : e^{-\int d^4z \phi_{\mathrm{in},z}(\Box_z + m^2)\frac{\delta}{\delta J_z}} : \left.\frac{Z[J]}{Z[0]}\right|_{J=0}. \tag{2.19}$$

In other words: To compute a certain S-matrix element simply calculate the corresponding Green's function and replace the external propagators by solutions of the free wave equation.

2.2 Wick Contractions

Despite its formal beauty the path integral formalism is not really convenient for deriving more than 2-point functions because the expressions of which one has to take the derivative become quite long. As already mentioned in eq.(2.4) there is also the possibility to make use of Wick's theorem.

[3] We use $[A, e^B] = [A, B]e^B$ for operators A, B with $[A, B]$ a c-number (this follows from the Baker-Campbell-Hausdorff formula) and $[\phi_{\mathrm{in},x}, \phi_{\mathrm{in},y}] = \Delta_{xy}$.

2.2.1 Real Bosonic Fields

For a real bosonic field ϕ the contraction is defined as

$$\begin{aligned}\overbrace{\phi_x \phi_y} &\equiv \begin{cases} [\phi_x^+, \phi_y^-] & \text{for } x^0 > y^0 \\ [\phi_y^+, \phi_x^-] & \text{for } y^0 > x^0 \end{cases} \\ &= \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-ip(x-y)}}{p^2 - m^2 + i\epsilon} \\ &= \Delta_{xy}^{\phi}, \end{aligned} \tag{2.20}$$

with

$$\begin{aligned}\phi_x &\equiv \phi_x^+ + \phi_x^- \\ &\equiv \int \frac{d^3p}{(2\pi)^3} \frac{a_{\vec{p}} e^{-ipx}}{\sqrt{2E_{\vec{p}}}} + \int \frac{d^3p}{(2\pi)^3} \frac{a_{\vec{p}}^\dagger e^{ipx}}{\sqrt{2E_{\vec{p}}}}, \end{aligned} \tag{2.21}$$

and $[a_{\vec{p}}, a_{\vec{q}}^\dagger] = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q})$, $[a_{\vec{p}}, a_{\vec{k}}] = [a_{\vec{p}}^\dagger, a_{\vec{k}}^\dagger] = 0$ and the ϕ on the Δ denoting the field the propagator belongs to.

2.2.2 Complex Bosonic Fields

For a complex bosonic field ϕ the contraction is defined as

$$\begin{aligned}\overbrace{\phi_x \phi_y^\dagger} &\equiv \begin{cases} [\phi_x^+, \phi_y^{-\dagger}] & \text{for } x^0 > y^0 \\ [\phi_y^{+\dagger}, \phi_x^-] & \text{for } y^0 > x^0 \end{cases} \\ &= \int \frac{d^4p}{(2\pi)^4} \frac{ie^{-ip(x-y)}}{p^2 - m^2 + i\epsilon} \\ &= \Delta_{xy}^{\phi}, \end{aligned} \tag{2.22}$$

with

$$\begin{aligned}\phi_x &\equiv \phi_x^+ + \phi_x^- \\ &\equiv \int \frac{d^3p}{(2\pi)^3} \frac{a_{\vec{p}} e^{-ipx}}{\sqrt{2E_{\vec{p}}}} + \int \frac{d^3p}{(2\pi)^3} \frac{b_{\vec{p}}^\dagger e^{ipx}}{\sqrt{2E_{\vec{p}}}}, \\ \phi_x^\dagger &\equiv \phi_x^{-\dagger} + \phi_x^{+\dagger} \\ &\equiv \int \frac{d^3p}{(2\pi)^3} \frac{a_{\vec{p}}^\dagger e^{ipx}}{\sqrt{2E_{\vec{p}}}} + \int \frac{d^3p}{(2\pi)^3} \frac{b_{\vec{p}} e^{-ipx}}{\sqrt{2E_{\vec{p}}}}, \end{aligned} \tag{2.23}$$

and $[a_{\vec{p}}, a_{\vec{k}}^\dagger] = [b_{\vec{p}}, b_{\vec{k}}^\dagger] = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{k})$, $[a_{\vec{p}}, a_{\vec{k}}] = [b_{\vec{p}}, b_{\vec{k}}] = [a_{\vec{p}}, b_{\vec{k}}^\dagger] = [a_{\vec{p}}, b_{\vec{k}}] = 0$.

2.2.3 Real Vector Fields

For a real vector field A_μ the contraction is defined as

$$\begin{aligned}\overbrace{A_{x\mu} A_{y\nu}^\dagger} &\equiv \begin{cases} [A_{x\mu}^+, A_{y\nu}^-] & \text{for } x^0 > y^0 \\ [A_{y\nu}^+, A_{x\mu}^-] & \text{for } y^0 > x^0 \end{cases} \\ &= \Delta_{\mu\nu xy}^{A}, \end{aligned} \tag{2.24}$$

with

$$
\begin{aligned}
A_{x\mu} &\equiv A^{+}_{x\mu} + A^{-}_{x\mu} \\
&\equiv \int \frac{d^3p}{(2\pi)^3}\epsilon_\mu(p)\frac{c_{\vec{p}}e^{-ipx}}{\sqrt{2E_{\vec{p}}}} + \int \frac{d^3p}{(2pi)^3}\epsilon^*_\mu(p)\frac{c^\dagger_{\vec{p}}e^{ipx}}{\sqrt{2E_{\vec{p}}}},
\end{aligned} \tag{2.25}
$$

and $[c_{\vec{p}}, c^\dagger_{\vec{q}}] = (2\pi)^3\delta^{(3)}(\vec{p}-\vec{q})$, $[c_{\vec{p}}, c_{\vec{k}}] = [c^\dagger_{\vec{p}}, c^\dagger_{\vec{k}}] = 0$. The explicit form of the propagator is derived in appendix A.3.2

2.2.4 Fermionic Fields

For a Weyl spinor field $\psi_\alpha(\alpha = 1,2)$ the contraction is defined as

$$
\begin{aligned}
\overbrace{\psi_{\alpha x}\bar{\psi}_{\dot{\alpha}y}} &\equiv \begin{cases} \{\psi^+_x, \bar{\psi}^-_y\} & \text{for } x^0 > y^0 \\ -\{\bar{\psi}^+_y, \psi^-_x\} & \text{for } y^0 > x^0 \end{cases} \\
&= \int \frac{d^4p}{(2\pi)^4}\frac{i\sigma^\mu_{\alpha\dot{\alpha}}p_\mu}{p^2+i\epsilon}e^{-ip(x-y)} \\
&= \Delta^\psi_{\alpha\dot{\alpha}xy},
\end{aligned} \tag{2.26}
$$

with (in analogy to the Dirac case)

$$
\begin{aligned}
\psi_\alpha &\equiv \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}(u_\alpha a^{(1)}_{\vec{p}} - v_\alpha a^{(2)}_{\vec{p}})e^{-ipx} + \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}(v_\alpha b^{(1)\dagger}_{\vec{p}} + u_\alpha b^{(2)\dagger}_{\vec{p}})e^{ipx} \\
&= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\sum_{r=1}^{2}\mathfrak{u}^{(r)}_{L\alpha}a^{(r)}_{\vec{p}}e^{-ipx} + \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\sum_{s=1}^{2}\mathfrak{v}^{(s)}_{L\alpha}b^{(s)\dagger}_{\vec{p}}e^{ipx} \\
&\equiv \psi^+_\alpha + \psi^-_\alpha, \\
\bar{\psi}_{\dot{\alpha}} &\equiv \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}(\bar{u}_{\dot{\alpha}} a^{(1)\dagger}_{\vec{p}} - \bar{v}_{\dot{\alpha}} a^{(2)\dagger}_{\vec{p}})e^{ipx} + \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}(\bar{v}_{\dot{\alpha}} b^{(1)}_{\vec{p}} + \bar{u}_{\dot{\alpha}} b^{(2)}_{\vec{p}})e^{-ipx} \\
&= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\sum_{r=1}^{2}\bar{\mathfrak{u}}^{(r)}_{\dot{\alpha}}a^{(r)\dagger}_{\vec{p}}e^{ipx} + \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\sum_{s=1}^{2}\bar{\mathfrak{v}}^{(s)}_{\dot{\alpha}}b^{(s)}_{\vec{p}}e^{-ipx} \\
&\equiv \bar{\psi}^-_{\dot{\alpha}} + \bar{\psi}^+_{\dot{\alpha}},
\end{aligned} \tag{2.27}
$$

and

$$
\begin{aligned}
\{a^{(r)}_{\vec{p}}, a^{(s)\dagger}_{\vec{k}}\} &= \{b^{(r)}_{\vec{p}}, b^{(s)\dagger}_{\vec{k}}\} = (2\pi)^3\delta^{rs}\delta^{(3)}(\vec{p}-\vec{k}), \\
\{a^{(r)}_{\vec{p}}, a^{(s)}_{\vec{k}}\} &= \{b^{(r)}_{\vec{p}}, b^{(s)}_{\vec{k}}\} = \{a^{(r)}_{\vec{p}}, b^{(s)\dagger}_{\vec{k}}\} = \{a^{(r)}_{\vec{p}}, b^{(s)}_{\vec{k}}\} = 0,
\end{aligned} \tag{2.28}
$$

and $u_\alpha, v_\alpha, \bar{u}_{\dot{\alpha}}, \bar{v}_{\dot{\alpha}}$ as defined in table (1.1). Analogously we find for the spinors with upper indices

$$
\begin{aligned}
\overbrace{\bar{\psi}^{\dot{\alpha}}_x\psi^\alpha_u} &\equiv \begin{cases} \{\bar{\psi}^{\dot{\alpha}+}_x, \psi^{\alpha-}_y\} & \text{for } x^0 > y^0 \\ -\{\psi^{\alpha+}_y, \bar{\psi}^{\dot{\alpha}-}_x\} & \text{for } y^0 > x^0 \end{cases} \\
&= \int \frac{d^4p}{(2\pi)^4}\frac{i\bar{\sigma}^{\alpha\dot{\alpha}}_\mu p^\mu}{p^2+i\epsilon}e^{-ip(x-y)} \\
&= \Delta^{\psi\dot{\alpha}\alpha}_{xy}.
\end{aligned} \tag{2.29}
$$

2.3 From the Lagrangian Density to the Feynman Rules

2.3.1 Using Wick's Theorem

At the beginning of this chapter we showed how to calculate the n-point Green's function as defined in eq.(2.1) by means of the a time ordered product. But as we already observed, these Green's functions contain unwanted disconnected pieces. Connected Green's functions can be calculated by means of time ordered products via

$$\langle\Omega|T\phi_1\phi_2\ldots\phi_n|\Omega\rangle = \frac{\langle 0|T\phi_1\phi_2\ldots\phi_n e^{i\int d^4z\mathcal{L}}|0\rangle}{\langle 0|Te^{i\int d^4z\mathcal{L}}|0\rangle}, \tag{2.30}$$

where $|\Omega\rangle$ is the vacuum of the theory including interactions and $|0\rangle$ is the vacuum of the free theory. The effect of the denominator is to remove the disconnected diagrams, which we will gracefully neglect in our calculations, instead we simply drop the disconnected parts. In order to obtain a Feynman rule for a specific process, all we have to do is expanding the following expression to first order:

$$\langle 0|T\phi_{\text{ext},1},\ldots,\phi_{\text{ext},n}e^{i\int d^4z\mathcal{L}_{\text{int}}}|0\rangle \approx \langle 0|T\phi_{\text{ext},1},\ldots,\phi_{\text{ext},n}\left(1+i\int d^4z\mathcal{L}_{\text{int}}\right)|0\rangle, \tag{2.31}$$

and carry out the contractions. The result is then composed of the expressions for the external particles and the vertex factor. We note that we have two choices for treating the external (i.e. incoming and outgoing) particles ϕ_{ext}:

- Make use of Wick's theorem and replace the contractions by propagators as derived in the previous sections. These can then be replaced by solutions to the free e.o.m. according to the LSZ reduction formula, so the vertex factor is rather easy to identify.

- Replace the ϕ_{ext} by appropriate creation and annihilation operators, then write everything in the correct temporal order. Then (anti-)commute the operators until everything is normal ordered. Finally identify the expressions for the external particles and the vertex factor.

Let us demonstrate the two possibilities: Suppose we have an interaction Lagrangian density

$$\mathcal{L}_{\text{int}} = g\psi^\alpha\sigma^\mu_{\alpha,\dot\alpha}\bar\psi^{\dot\alpha}A_\mu, \tag{2.32}$$

and would like to know the vertex factor for the process $\psi\to\bar\psi A$.

Possibility 1 The first way to go would be to evaluate

$$\begin{aligned}\langle 0|T\psi_1^\beta A_{\nu 2}\bar\psi_3^{\dot\beta}e^{i\int d^4zg\psi_z^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\psi_z^{\dot\alpha}A_{z\mu}}|0\rangle &= \langle 0|Tig\int d^4z\psi_1^\beta A_{\nu 2}\bar\psi_3^{\dot\beta}\psi_z^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\psi_z^{\dot\alpha}A_{z\mu}|0\rangle \\ &\approx ig\int d^4z(\psi_1^\beta A_{\nu 2}\bar\psi_3^{\dot\beta}\psi_z^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\psi_z^{\dot\alpha}A_{z\mu}+\psi_1^\beta A_\nu 2\bar\psi_3^{\dot\beta}\psi_z^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\psi_z^{\dot\alpha}A_{z\mu}) \\ &= ig\int d^4z(\Delta_{21}^{(\psi)\dot\beta\beta}\sigma_{\mu\alpha\dot\alpha}\Delta_{zz}^{(\psi)\dot\alpha\alpha}\Delta_{z3}^{(A)\mu\nu} \\ &\quad -\Delta_{2z}^{(\psi)\dot\beta\alpha}\sigma_{\mu\alpha\dot\alpha}\Delta_{z1}^{(\psi)\dot\alpha\beta}\Delta_{z3}^{(A)\mu\nu}),\end{aligned}$$

where the first term corresponds to a disconnected graph so that we are left with

$$-ig\int d^4z\Delta^{(\psi)\dot{\beta}\alpha}_{2z}\sigma^{\mu}_{\alpha\dot{\alpha}}\Delta^{(\psi)\dot{\alpha}\beta}_{z1}\Delta^{(A)\mu\nu}_{z3}.$$

Now if we remove the propagators we find the vertex factor

$$-ig\int d^4z\sigma^{\mu}_{\alpha\dot{\alpha}}. \tag{2.33}$$

Possibility 2 The other possibility would be to start with[4]

$$\begin{aligned}
&\langle 0|T\sqrt{2E_{p_1}}a^{(r)\dagger}_{p_1}\sqrt{2E_{p_2}}a^{(s)}_{p_2}\sqrt{2E_{p_2}}c_{p_3}e^{ig\int d^4z\psi_z\sigma^\mu\bar{\psi}_zA_{z\mu}}|0\rangle\\
=\;&\langle 0|Tig\int d^4z\sqrt{2E_{p_1}}a^{(r)\dagger}_{p_1}\sqrt{2E_{p_2}}a^{(s)}_{p_2}\sqrt{2E_{p_2}}c_{p_3}\psi_z\sigma^\mu\bar{\psi}_zA_{z\mu}|0\rangle\\
=\;&\langle 0|ig\int d^4z\sqrt{2E_{p_1}}\sqrt{2E_{p_2}}\sqrt{2E_{p_3}}a^{(s)}_{p_3}c_{p_2}\psi_z\sigma^\mu\bar{\psi}_zA_{z\mu}a^{(r)\dagger}_{p_1}|0\rangle
\end{aligned} \tag{2.34}$$

where we already dropped the disconnected piece. We note that $a^\dagger_{p_1}$ creates the incoming state so that they should happen before everything else. c_{p_3} creates the outgoing photon, a_{p_2} creates the outgoing fermion, so they happen after everything else. For simplicity we take $r = s = 1$ and obtain with eq.(2.24), ff

$$\begin{aligned}
&-ig\int d^4z\sqrt{2E_{p_1}}\sqrt{2E_{p_2}}\sqrt{2E_{p_3}}\langle 0|c_{p_2}A_{\mu z}\psi_z\sigma^\mu a^{(1)}_{p_3}\bar{\psi}_za^{(1)^\dagger}_{p_1}|0\rangle\\
=\;&-ig\int d^4z\sqrt{2E_{p_1}}\sqrt{2E_{p_2}}\sqrt{2E_{p_3}}\underbrace{[c_{p_2},A_{\mu z}]}_{\frac{1}{\sqrt{2E_{p_3}}}\epsilon_\mu(p_2)e^{-ip_2z}}\underbrace{\{\psi_z,a^{(1)\dagger}_{p_1}\}}_{\frac{1}{\sqrt{2E_{p_1}}}u(p_1)e^{-ip_1z}}\sigma^\mu\underbrace{\{a^{(1)}_{p_3},\bar{\psi}_z\}}_{\frac{1}{\sqrt{2E_{p_3}}}\bar{u}(p_3)e^{ip_3z}}\\
=\;&-ig\int d^4ze^{-ip_1z}e^{-ip_2z}e^{ip_3z}\epsilon_\mu(p_2)u(p_1)\sigma^\mu\bar{u}(p_3).
\end{aligned} \tag{2.35}$$

So we end up with the vertex factor

$$-ig\int d^4z\sigma^{\mu}_{\alpha\dot{\alpha}}, \tag{2.36}$$

after removing the external fields. In appendix A.3 we derive the vertex factors by using the first possibility, albeit in momentum space.

2.3.2 Using Path Integral Methods

The other possibility to derive the Feynman rules is to make use of the defining equation for the n-point Green's function eq.(2.1). As already mentioned, this is a rather messy business for $n > 2$. For 2-point functions however it is quite convenient to use this method. We refer to appendix A.3.2 for an example to illustrate this.

[4] The various factors $\sqrt{2E}$ occur because we want the incoming one-particle state $< p_1| = \langle 0|\sqrt{2E_{p_1}}a_{p_1}$ to be normalized to $< p_1|p_1 >= 2E_p$.

Chapter 3

How to Define SUSY Generators

In the literature (e.g.[BL94], [KS97]) it often remains unclear which generators of the SUSY algebra were chosen. Although given explicitly they sometimes do not seem to match the presented calculational results. So it might be interesting to investigate this a little further.

The SUSY algebra is given by

$$\begin{aligned}
\{Q_\alpha, \bar{Q}_{\dot\alpha}\} &= 2\sigma^\mu P_\mu, \\
\{Q_\alpha, Q_\beta\} &= 0, \\
\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} &= 0.
\end{aligned} \tag{3.1}$$

It is important to note that P_μ and not $\pm i\partial_\mu$ is the defining operator on the right hand side[1] as we will see in a moment. It is not enough to find some operators satisfying the SUSY algebra with $\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$ and defining $P_\mu = i\partial_\mu$ by hand. Instead one has to look carefully at what the generators do. In superspace $(x^\mu, \theta, \bar\theta)$, a SUSY transformation $e^{i(\zeta Q + \bar\zeta\bar Q)}$ with parameters $\zeta, \bar\zeta$ should act as

$$\begin{aligned}
x^\mu &\longrightarrow x^\mu + \delta x^\mu(\delta\theta, \delta\bar\theta, \theta, \bar\theta), \\
\theta^\alpha &\longrightarrow \theta^\alpha + \delta\theta^\alpha, \\
\bar\theta^{\dot\alpha} &\longrightarrow \bar\theta^{\dot\alpha} + \delta\bar\theta^{\dot\alpha},
\end{aligned} \tag{3.2}$$

where we restricted ourselves to global SUSY. Therefore $\delta\theta, \delta\bar\theta$ do not depend on x^μ. For our coordinates to remain real, δx^μ must be either of the form[2] $\delta x^\mu = \pm i(\delta\theta\sigma^\mu\bar\theta - \theta\sigma^\mu\delta\bar\theta)$ or $\delta x^\mu = \pm(\delta\theta\sigma^\mu\bar\theta + \theta\sigma^\mu\delta\bar\theta)$. Another requirement to be fulfilled is that $Q_\alpha, \bar{Q}_{\dot\alpha}$ should be conjugate to each other. We should first note that a suitable scalar product for superfields ϕ_1, ϕ_2 may be defined as

$$\langle \phi_1, \phi_2 \rangle = \int d^4x\; d^2\theta\; d^2\bar\theta\; \phi_1^\dagger \phi_2, \tag{3.3}$$

where $\dagger$ means the complex conjugate. Let us prove its invariance under SUSY transformations:

$$\begin{aligned}
\langle \phi_1', \phi_2' \rangle &= < e^{i(\zeta Q + \bar\zeta\bar Q)}\phi_1, e^{i(\zeta Q + \bar\zeta\bar Q)}\phi_2 > \\
&= \int d^4x\; d^2\theta\; d^2\bar\theta (\phi_1^\dagger \underbrace{e^{-i(\zeta^\dagger Q^\dagger + \bar\zeta^\dagger \bar Q^\dagger)}}_{\text{this still acts on } \phi_1^\dagger!}) e^{i(\zeta Q + \bar\zeta\bar Q)}\phi_2.
\end{aligned} \tag{3.4}$$

[1] Thanks to Christoph Luhn and Marc Thormeier for pointing me towards this.

[2] $(\delta\theta\sigma^\mu\bar\theta)^\dagger = \theta\sigma^\mu\delta\bar\theta$

Integrating by parts and dropping the boundary terms which is also formally valid for Grassmann numbers gives

$$\begin{aligned}
\langle \phi_1', \phi_2' \rangle &= \int d^4x \; d^2\theta \; d^2\bar{\theta} \phi_1^\dagger e^{i\left(\zeta^\dagger Q^\dagger + \bar{\zeta}^\dagger \bar{Q}^\dagger\right)} e^{i\left(\zeta Q + \bar{\zeta}\bar{Q}\right)} \phi_2 \\
&= \int d^4x \; d^2\theta \; d^2\bar{\theta} \phi_1^\dagger e^{i\left(\bar{\zeta} Q^\dagger + \zeta \bar{Q}^\dagger\right)} e^{i\left(\zeta Q + \bar{\zeta}\bar{Q}\right)} \phi_2 \\
&\overset{!}{=} \langle \phi_1, \phi_2 \rangle. \qquad (3.5)
\end{aligned}$$

Now if the integrand contained $e^{-i\text{Operator}} e^{i\text{Operator}}$ the invariance of the scalar product would be proven. Therefore we have to demand $Q^\dagger = -\bar{Q}$. Now if we perform a SUSY transformation on a function $f(x^\mu, \theta, \bar{\theta})$ we get

$$\begin{aligned}
f(x^\mu, \theta, \bar{\theta}) &\longrightarrow f(x^\mu + \delta x^\mu, \theta + \delta\theta, \bar{\theta} + \delta\bar{\theta}) \\
&= f(x^\mu, \theta, \bar{\theta}) + \left(\delta\theta \frac{\partial}{\partial \theta} + \delta\bar{\theta} \frac{\partial}{\partial \bar{\theta}} + \delta x^\mu \frac{\partial}{\partial x^\mu} \right) f(x^\mu, \theta, \bar{\theta}). \qquad (3.6)
\end{aligned}$$

Let us consider all possible combinations for δx^μ,$\delta\theta$,$\delta\bar{\theta}$ to see which ones satisfy the requirements[3]:

- choosing $\delta x^\mu = i(\zeta \sigma^\mu \bar{\theta} - \theta \sigma^\mu \bar{\zeta})$, $\delta\theta = \zeta$, $\delta\bar{\theta} = \bar{\zeta}$ gives

$$\delta f = \zeta \frac{\partial}{\partial \theta} f + \bar{\zeta} \frac{\partial}{\partial \bar{\theta}} f + i \left(\zeta \sigma^\mu \bar{\theta} \frac{\partial}{\partial x^\mu} - \theta \sigma^\mu \bar{\zeta} \partial_\mu \right) f = i(\zeta Q + \bar{\zeta}\bar{Q}) f. \qquad (3.7)$$

  So when we write our transformation as

$$f \longrightarrow e^{i\left(\zeta Q + \bar{\zeta}\bar{Q}\right)} f = e^{\zeta^\alpha \frac{\partial}{\partial \theta^\alpha} + \bar{\zeta}_{\dot{\alpha}} \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} + i\zeta\sigma^\mu\bar{\theta}\frac{\partial}{\partial x^\mu} - i\theta\sigma^\mu\bar{\zeta}\frac{\partial}{\partial x^\mu}} f, \qquad (3.8)$$

  we get

$$\begin{aligned}
Q_\alpha &= -i \frac{\partial}{\partial \theta^\alpha} + \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu \\
\bar{Q}_{\dot{\alpha}} &= i \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - \theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \\
\Rightarrow \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2i\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \\
\text{and} \qquad & Q_\alpha^\dagger = -\bar{Q}_{\dot{\alpha}}. \qquad (3.9)
\end{aligned}$$

- choosing $\delta x^\mu = -i(\zeta \sigma^\mu \bar{\theta} - \theta \sigma^\mu \bar{\zeta})$, $\delta\theta = \zeta$, $\delta\bar{\theta} = \bar{\zeta}$ we get

$$\begin{aligned}
Q_\alpha &= -i \frac{\partial}{\partial \theta^\alpha} - \sigma^\mu_{\alpha\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} \partial_\mu \\
\bar{Q}_{\dot{\alpha}} &= i \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + \theta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \\
\Rightarrow \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= -2i\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \\
\text{and} \qquad & Q_\alpha^\dagger = -\bar{Q}_{\dot{\alpha}}. \qquad (3.10)
\end{aligned}$$

- choosing $\delta x^\mu = \pm(\zeta \sigma^\mu \bar{\theta} + \theta \sigma^\mu \bar{\zeta})$, $\delta\theta = \zeta$, $\delta\bar{\theta} = \bar{\zeta}$ leads to $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 0$.

[3]We use $\left(\frac{\partial}{\partial \theta^\alpha}\right)^* = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}}$ because of $\delta_\alpha^\beta = \left(\frac{\partial}{\partial \theta^\alpha} \theta^\beta\right)^\dagger = \bar{\theta}^{\dot{\beta}} \left(\frac{\partial}{\partial \theta^\alpha}\right)^\dagger = -\left(\frac{\partial}{\partial \theta^\alpha}\right)^\dagger \bar{\theta}^{\dot{\beta}} \overset{!}{=} \delta_{\dot{\alpha}}^{\dot{\beta}}$.

So we are left with only two possibilities eq.(3.9) and eq.(3.10). We recall that the Minkowski space part of the transformations looks like a translation $e^{a^\mu \partial_\mu}$ with parameter $a^\mu = +i(\zeta\sigma^\mu\bar{\theta} - \theta\sigma^\mu\bar{\zeta})$. Now there are two possibilities of defining the generator $U(a)$ of a translation:

- $U(a^\mu) = e^{-iaP} \Rightarrow P_\mu = i\partial_\mu$,
- $U(a^\mu) = e^{iaP} \Rightarrow P_\mu = -i\partial_\mu$.

So for eq.(3.9) we have to choose $P_\mu = i\partial_\mu$, for eq.(3.10) we have to choose $P_\mu = -i\partial_\mu$. This choice also fixes the generator of translations.

We think that [BL94] has started with the $Q, \bar{Q}$ from eq.(3.10) which leads to a wrong sign between the kinetic term of a fermion and its mass term. By virtue of $P_\mu = i\partial_\mu$ this would have made the energy bounded from above instead of from below. Nevertheless their result is valid because in their notation $P_\mu = -i\partial_\mu$ must be chosen.

Chapter 4

Supersymmetry and its Invariants

The supersymmetry algebra is defined by

$$\begin{aligned} \{Q_\alpha, Q_\beta\} &= \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0 \\ \{Q_\alpha, \bar{Q}_{\dot\alpha}\} &= 2\sigma^\mu_{\alpha\dot\alpha} P_\mu . \end{aligned} \tag{4.1}$$

A possible choice of $Q, \bar{Q}, P$ is[1]

$$\begin{aligned} Q_\alpha &= -i\frac{\partial}{\partial\theta^\alpha} + \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu , \\ \bar{Q}_{\dot\alpha} &= i\frac{\partial}{\partial\bar\theta^{\dot\alpha}} - \theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu , \\ P_\mu &= i\partial_\mu . \end{aligned} \tag{4.2}$$

To find SUSY invariants it is useful to introduce fermionic derivatives which anti-commute with the SUSY generators:

$$\begin{aligned} D_\alpha &= \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \\ \bar{D}_{\dot\alpha} &= -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu \\ \{D_\alpha, Q_\beta\} &= \{D_\alpha, \bar{Q}_{\dot\beta}\} = 0 \\ D^*_\alpha &= \bar{D}_{\dot\alpha} . \end{aligned} \tag{4.3}$$

For a more detailed introduction to supersymmetry we refer the reader to the literature, especially [KS97].

4.1 Superfields

Superfields are functions over superspace depending on the real variables x^μ and the Grassmann variables $\theta^\alpha, \bar\theta^{\dot\alpha}$. The most general form of a superfield Φ is

$$\begin{aligned} \Phi &= A(x) + \theta B(x) + \theta^2 C(x) + \bar\theta D(x) + \bar\theta^2 E(x) + i\theta\sigma^\mu\bar\theta G_\mu(x) + \theta^2\bar\theta H(x) \\ &\quad + \bar\theta^2\theta I(x) + \bar\theta^2\theta^2 J(x), \end{aligned} \tag{4.4}$$

as all $\theta^n, \bar\theta^n$ vanish for $n > 2$. These fields can be subjected to different constraints.

[1] See chapter 3 for a more detailed discussion.

4.1.1 Chiral Superfields

Superfields $\Phi(x^\mu, \theta, \bar{\theta})$ which obey

$$\bar{D}_{\dot{\alpha}} \Phi = 0, \tag{4.5}$$

are called chiral superfields. A nice way to handle them is by noting that $\bar{D}_{\dot{\alpha}} y^\mu = 0$ with $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$. Therefore chiral superfields can be written as $\Phi(y^\mu, \theta)$.

$$\begin{aligned} \Phi(y^\mu, \theta) &= \varphi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y), \\ \Rightarrow \Phi(x^\mu, \theta, \bar{\theta}) &= \varphi(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi(x) + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\theta} \\ & -\frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi(x). \end{aligned} \tag{4.6}$$

Under SUSY transformations $\delta\Phi = i(\zeta Q + \bar{\zeta}\bar{Q})\Phi$ the component fields transform as[2]

$$\begin{aligned} \delta\varphi &= \sqrt{2}\zeta\psi, \\ \delta\psi &= \sqrt{2}\zeta F - i\sqrt{2}\sigma^\mu\bar{\zeta}\partial_\mu\varphi, \\ \delta F &= i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\zeta}. \end{aligned} \tag{4.7}$$

We note that the F-term, the coefficient of θ^2, stays invariant up to a total derivative. The product of two chiral superfields is also a chiral superfield. Its D-term[3] is invariant under SUSY transformations up to a total divergence because $\theta^n = 0$ for $n > 2$. The D-term of $(\Phi^\dagger\Phi)$ gives

$$\left(\Phi^\dagger\Phi\right)_D = \partial_\mu\varphi^\dagger\partial^\mu\varphi + i\partial_\mu\psi\sigma^\mu\bar{\psi} + F^\dagger F, \tag{4.8}$$

which looks like the Lagrange density for a boson and a fermion. So this is a first hint as to how to write down a Lagrange density for a realistic SUSY model.

4.1.2 Vector Superfields

Up to now we did not see how to include gauge fields in SUSY models. When examining the constraint to make a superfield real we will see that this introduces fields that transform like the usual vector potential under $U(1)$.

$$\begin{aligned} V^\dagger &= V \\ &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + \frac{i}{2}\theta^2\left(M(x) + iN(x)\right) - \frac{i}{2}\bar{\theta}^2\left(M(x) - iN(x)\right) \\ &\quad +\theta\sigma^\mu\bar{\theta}V_\mu(x) + i\theta^2\bar{\theta}\left(\bar{\lambda} + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\right) - i\bar{\theta}^2\theta\left(\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\right) \\ &\quad +\frac{1}{2}\theta^2\bar{\theta}^2\left(D(x) - \frac{1}{2}\Box C(x)\right). \end{aligned} \tag{4.9}$$

This is the so-called vector superfield which should transform under local $U(1)$ transformations as[4]

$$V \longrightarrow V + i(\Lambda - \Lambda^\dagger), \tag{4.10}$$

[2] See section A.2.1.

[3] The D-term is the coefficient of $\theta^2\bar{\theta}^2$.

[4] This was proposed by Wess and Zumino in [WZ74].

with Λ being a chiral superfield and

$$\begin{aligned} i(\Lambda-\Lambda^\dagger) &= i(\varphi_\Lambda-\varphi_\Lambda^\dagger)+i\sqrt{2}(\theta\psi_\Lambda-\bar\theta\bar\psi_\Lambda)+i(\theta^2F_\Lambda-\bar\theta^2F_\Lambda^\dagger)+\theta\sigma^\mu\bar\theta\partial_\mu\left(\varphi_\Lambda+\varphi_\Lambda^\dagger\right) \\ &\quad -\frac{1}{\sqrt{2}}\theta^2\partial_\mu\psi_\Lambda\sigma^\mu\bar\theta-\frac{1}{\sqrt{2}}\bar\theta^2\theta\sigma^\mu\partial_\mu\bar\psi_\Lambda-\frac{1}{4}\theta^2\bar\theta^2\Box\left(\varphi_\Lambda-\varphi_\Lambda^\dagger\right). \end{aligned} \tag{4.11}$$

In the Wess-Zumino gauge we choose $\phi_\Lambda,\psi_\Lambda,F_\Lambda$ such that the vector superfield can be written as

$$V(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta V_\mu(x)+i\theta^2\bar\theta\bar\lambda-i\bar\theta^2\lambda(x)+\frac{1}{2}\theta^2\bar\theta^2D(x). \tag{4.12}$$

From now on we will work in this gauge. Comparing eq.(4.9) and eq.(4.11) we see that under $U(1)$ transformations the fields $\lambda,\bar\lambda,D$ are invariant whereas V_μ transforms as the familiar $U(1)$ gauge field. We note that $V^n=0$ for $n>2$.

4.2 Coupling Matter and Gauge Fields

Now that we found both matter and gauge fields we face the problem of how to produce interactions between them. This can be done by observing that a chiral field Φ should transform under $U(1)$ as

$$\Phi \longrightarrow e^{-2ig\Lambda}\Phi, \tag{4.13}$$

with the same Λ as in the last section so that

$$(\Phi^\dagger e^{2gV}\Phi)_D, \tag{4.14}$$

with $V=t^aV^a$ and t^a the generator of the corresponding symmetry group, is gauge invariant as well as SUSY invariant. When expanding this into component fields we get

$$\begin{aligned} (\phi^\dagger e^{2gt^aV^a}\phi)_D &= (D_\mu\varphi)^\dagger(D^\mu\varphi)+ig\sqrt{2}(\varphi^\dagger t^a\lambda^a\psi-t^a\varphi\bar\lambda^a\bar\psi)-iD_\mu\psi\sigma^\mu\bar\psi \\ &\quad +g\varphi^\dagger t^aD^a\varphi+F^\dagger F, \end{aligned} \tag{4.15}$$

with the familiar covariant derivative $D_\mu=\partial_\mu+igt^aV_\mu^a$.

4.3 Kinetic Energy of the Gauge Fields

4.3.1 Abelian Fields

A SUSY invariant expression containing the field strength of the gauge fields is given by

$$(W^\alpha W_\alpha)_F = 32i\lambda\sigma^\mu\partial_\mu\bar\lambda+16D^2-8V_{\mu\nu}V^{\mu\nu}+\text{total divergence}, \tag{4.16}$$

with the superfield strength

$$W_\alpha = \bar D^2D_\alpha V. \tag{4.17}$$

4.3.2 Non-Abelian Fields

Next we have to incorporate the kinetic energy for the non-Abelian gauge fields. This is done by defining a supersymmetric gauge covariant derivative and comparing the resulting terms for Abelian and non-Abelian gauge theory. For the sake of brevity we define $V = 2gV^a t^a$ with t^a the generators of the gauge group. Let Λ be a chiral superfield. For the derivative ∇_A to be gauge covariant it has to satisfy

$$\begin{aligned} (\nabla_A \phi)' &= e^{-i\Lambda}(\nabla_A \phi), \qquad A \in \{\dot{\alpha}, \alpha\}, \\ \Rightarrow \nabla'_A &= e^{-i\Lambda}\nabla_A e^{i\Lambda}. \end{aligned} \tag{4.18}$$

Because of $\bar{D}_{\dot{\alpha}}\Lambda = 0$ we choose

$$\nabla_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}}. \tag{4.19}$$

Eq.(4.18) suggests ∇_α to be

$$\begin{aligned} \nabla_\alpha &= e^{-V} D_\alpha e^V \\ \Rightarrow \nabla'_A &= e^{-V'} D_\alpha e^{V'} \\ &= e^{-i\Lambda} e^V e^{i\Lambda^\dagger} D_\alpha e^{-i\Lambda^\dagger} e^V e^{i\Lambda} \\ &\overset{D_\alpha \Lambda^\dagger = 0}{=} e^{-i\Lambda} e^{-V} D_\alpha e^V e^{i\Lambda} \\ &= e^{-i\Lambda}\nabla_\alpha e^{i\Lambda}. \end{aligned} \tag{4.20}$$

Let us inspect ∇_α a little further

$$\begin{aligned} \nabla_\alpha \Phi &= e^{-V} D_\alpha e^V \Phi \\ &= D_\alpha \Phi + \underbrace{e^{-V}(D_\alpha e^V)}_{=:\Gamma_\alpha} \Phi. \end{aligned} \tag{4.21}$$

In the Wess-Zumino gauge we have

$$\begin{aligned} \Gamma_\alpha &= (1 - V + \frac{1}{2}V^2)\left(D_\alpha(1 + V + \frac{1}{2}V^2)\right) \\ &= D_\alpha + \frac{1}{2}D_\alpha V^2 - V(D_\alpha V) \\ &= D_\alpha + \frac{1}{2}(D_\alpha V)V + \frac{1}{2}V(D_\alpha V) - V(D_\alpha V) \\ &= D_\alpha + \frac{1}{2}[D_\alpha V, V], \end{aligned} \tag{4.22}$$

which in the Abelian case reduces to

$$\Gamma_\alpha = 2gD_\alpha V. \tag{4.23}$$

So a suitable ansatz for W_α is

$$W_\alpha = \frac{1}{2g}\bar{D}^2\Gamma_\alpha, \tag{4.24}$$

which leads to

$$\begin{aligned}
W_\alpha &= \frac{1}{2g}\bar{D}^2\Gamma_\alpha \\
&= \frac{1}{2g}\bar{D}^2 D_\alpha V + \frac{1}{4g}\bar{D}^2(\underbrace{[t^a,t^b]}_{=if^{abc}t^c}(D_\alpha V^a)V^b)4g^2 \\
&= \frac{1}{2g}\bar{D}^2 D_\alpha V + igf^{abc}\bar{D}^2((D_\alpha V^b)V^c)t^a. \qquad (4.25)
\end{aligned}$$

Calculating $(W^{a\alpha}W^a_\alpha)_F$ gives the usual expression for the kinetic energy of gauge fields plus some extra terms. So for each gauge group the Lagrangian density will contain a term

$$\begin{aligned}
\mathcal{L} &= \frac{1}{32}(W^\alpha W_\alpha)_F \\
&= -\frac{1}{4}V^a_{\mu\nu}V^{a\mu\nu} + i\lambda^a\sigma^\mu\tilde{D}^{ac}_\mu\bar{\lambda}^c + \frac{1}{2}D^aD^a, \qquad (4.26)
\end{aligned}$$

with

$$\begin{aligned}
V^a_{\mu\nu} &= \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - gf^{abc}V^b_\mu V^c_\nu \\
\tilde{D}^{ac}_\mu &= \partial_\mu\delta^{ac} - gf^{abc}V^b_\mu. \qquad (4.27)
\end{aligned}$$

4.4 The Superpotential

As in a renormalizable Lagrangian density the mass dimension[5] of every product of fields should be ≤ 4, the most general expression for $\mathcal{L}$ is

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}W^{ij}(\phi,\phi^\dagger)\psi_i\psi_j + W^i(\phi,\phi^\dagger)F_i + \text{h.c.}, \qquad (4.28)$$

with $W^{ij}(\phi,\phi^\dagger)$ at most linear and $W^i(\phi,\phi^\dagger)$ at most quadratic in $\phi,\phi^\dagger$. Under SUSY transformations[6] this changes to

$$\begin{aligned}
\delta\mathcal{L}_{\text{int}} = &-\frac{1}{2}\frac{\delta W^{ij}}{\delta\phi_k}\delta\phi_k\psi_i\psi_j - \frac{1}{2}W^{ij}(-i)\sqrt{2}(\sigma^\mu\bar{\epsilon})^\alpha(\partial_\mu\phi_i)\psi_{j\alpha} - \frac{1}{2}\sqrt{2}F_iW^{ij}\epsilon\psi_j \\
&-\frac{1}{2}\frac{\delta W^{ij}}{\delta\phi^\dagger_k}\delta\phi^\dagger_k\psi_i\psi_j - \frac{1}{2}W^{ij}(-i)\sqrt{2}\psi_i\sigma^\mu\bar{\epsilon}\partial_\mu\phi_j - \frac{1}{2}\sqrt{2}W^{ij}\psi_i\epsilon F_j \\
&+\frac{\delta W^i}{\delta\phi_k}\delta\phi_kF_i + W^i i\sqrt{2}\partial_\mu\psi_i\sigma^\mu\bar{\epsilon} + \frac{\delta W^i}{\delta\phi^\dagger_k}\delta\phi^\dagger_kF_i. \qquad (4.29)
\end{aligned}$$

For $\mathcal{L}$ to be invariant these terms have to vanish.

$$-\frac{1}{2}\frac{\delta W^{ij}}{\phi_k}\delta\phi_k\psi_i\psi_j = -\frac{1}{\sqrt{2}}\frac{W^{ij}}{\delta\phi_k}(\epsilon\psi_k)(\psi_i\psi_j) \stackrel{!}{=} 0. \qquad (4.30)$$

As there is no other term of this structure it can only vanish because of the Fierz identity

$$(\epsilon\psi_k)\psi_i\psi_j + (\epsilon\psi_i)\psi_j\psi_k + (\epsilon\psi_j)\psi_k\psi_i = 0 \qquad (4.31)$$

[5] $[\psi] = \frac{3}{2}, [\phi] = 1, [F] = 2$

[6] $\delta\phi_i = \sqrt{2}\epsilon\psi_i,\ \delta\psi_{i\alpha} = \sqrt{2}\epsilon F_i - i\sqrt{2}(\sigma^\mu\bar{\epsilon})_\alpha\partial_\mu\phi_i,\ \delta F_i = i\sqrt{2}\partial_\mu\psi_i\sigma^\mu\bar{\epsilon}$

Therefore $\frac{\delta W^{ij}}{\delta\phi_k}$ must be symmetric in i, j. For the fourth term

$$-\frac{1}{2}\frac{\delta W^{ij}}{\delta\phi_k^\dagger}(\epsilon\bar{\psi}_k)\psi_i\psi_j \tag{4.32}$$

there is no suitable Fierz identity. For this to vanish for arbitrary fields, $\frac{\delta W^{ij}}{\delta\phi_k^\dagger} = 0$ has to hold, i.e. $W^{ij} = W^{ij}(\phi)$ does not depend on $\phi^\dagger$. Together with the symmetry in i, j it is clear that W^{ij} can be written as

$$W^{ij}(\phi) = \frac{\delta^2}{\delta\phi_i\delta\phi_j}W, \tag{4.33}$$

with

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k, \tag{4.34}$$

and M^{ij}, y^{ijk} being totally symmetric. Now we inspect the sum of the terms number 2, 5 and 8:

$$\begin{aligned}
0 &\stackrel{!}{=} iW^{ij}\psi_i\sigma^\mu\bar{\epsilon}\partial_\mu\phi_j + iW^i\partial_\mu\psi_i\sigma^\mu\bar{\epsilon} \\
&= i\partial_\mu\left(\frac{\delta W^i}{\delta\phi_j}\right)\psi_i\sigma^\mu\bar{\epsilon} + iW^i\partial_\mu\psi_i\sigma^\mu\bar{\epsilon} \\
&= i\left(\partial_\mu\frac{\delta W^i}{\delta\phi_j}\right)\psi_i\sigma^\mu\bar{\epsilon} - i\partial_\mu W^i\psi_i\sigma^\mu\bar{\epsilon} \\
&\Rightarrow W^i = \frac{\delta W}{\delta\phi_i}.
\end{aligned} \tag{4.35}$$

In the second line we used $\partial_\mu\left(\frac{\delta W}{\delta\phi_i}\right) = \frac{\delta^2 W}{\delta\phi_i\delta\phi_j}\partial_\mu\phi_j$. The sum of the $3^{\text{rd}}, 6^{\text{th}}, 7^{\text{th}}$ terms vanishes again due to the symmetry in $i \leftrightarrow j$. The last term is automatically zero as W does not depend on $\phi^\dagger$. All in all the interaction Lagrangian density does only depend on the derivatives of the superpotential W given by eq.(4.34).

4.5 SUSY-Breaking Terms

As there are no supersymmetric particles of the same mass as the particles of the standard model observed in nature, SUSY has to be broken. Yet it would be highly desirable if this breaking happened in a way which still led to the cancellation of the quadratic infinities in loop diagrams (soft SUSY-breaking). This has been investigated by Girardello and Grisaru [GG82] who showed that all soft breaking terms are of the form[7]

$$\tilde{M}_1\mathfrak{Re}(\phi^2) + \tilde{M}_2\mathfrak{Im}(\phi^2) + c(\phi^3 + \text{h.c.}) + \tilde{M}_3(\lambda^2 + \text{h.c.}) + d^{ijk}(\phi_i^*\phi_j\phi_k + \text{h.c.}) \tag{4.36}$$

where the ϕ are the sfermions and the λ are the superpartners of the gauge fields. We note that we will neglect the last term as it is very difficult to construct a mechanism for spontaneous SUSY breaking in which this term is not very small and as it may also lead to quadratic divergences in case the bosons ϕ are complete gauge-singlets. We do not add mass terms for fermions because they can be absorbed by performing some redefinitions (c.f.[Mar97]).

[7] $\mathfrak{Re}(x)$ is the real part of x, $\mathfrak{Im}(x)$ its imaginary part.

Chapter 5

The Lagrangian of the MSSM

5.1 Particle Content

After having reviewed the terms which are invariant under SUSY transformations we now give the particle content of the MSSM in table 5.1. We choose to work only with spinors which transform left-chirally under Lorentz transformations so every right-chiral particle gets complex conjugated (denoted by X_R^* for scalars and $\bar{X}_R$ for spinors) so that a Dirac spinor for e.g. an electron would be written as $\psi_e = \begin{pmatrix} e_L \\ e_R \end{pmatrix}$. Therefore the Weyl spinors for right-chiral fermions get a dotted index, e.g. $u_R^{\dot{\alpha}}$, and the barred spinors have undotted indices, e.g.$\bar{e}_R^{\alpha}$. The electric charge $Q_{\rm el}$ is given by $Q_{\rm el} = \frac{\sigma^3}{2} + Y$, with Y being the hypercharge. We note that unlike in the standard model, there are two Higgs doublets. Among others the reason for this is that it would not be possible to give masses to up and to down type quarks as the superpotential may not contain a superfield as well as its conjugate.

5.2 Lagrangian

5.2.1 Kinetic Terms

For every particle we need a term of the form (4.15), giving for the leptons

$$\begin{aligned}
\mathcal{L}_{\rm kin} \;=\;& -iD_\mu L\sigma^\mu\bar{L} + i(D_\mu\tilde{L})^\dagger(D^\mu L) \\
& +i\sqrt{2}[\tilde{L}^\dagger(g_Y\tilde{B}^0 Y + g_L\tilde{W}^a\tau^a)L - \tilde{L}(g_Y\bar{\tilde{B}}^0 Y + g_L\bar{\tilde{W}}^a\tau^a)\bar{L}] \\
& +g_Y\tilde{L}^\dagger D_Y Y\tilde{L} + g_L\tilde{L}^\dagger D_L^a\tau^a\tilde{L} + F_{e_L}^\dagger F_{e_L} + F_{\nu_e}^\dagger F_{\nu_e} \\
& -iD_\mu e_R\sigma^\mu\bar{e}_R + (D_\mu\tilde{e}_R)^*(D^\mu\tilde{e}_R) + i\sqrt{2}[\tilde{e}_R^* g_Y\tilde{B}^0 Y e_R - \tilde{e}_R g_Y\bar{\tilde{B}}^0 Y\bar{e}_R] \\
& +g_Y\tilde{e}_R^* D_Y Y\tilde{e}_R + F_{e_R}^\dagger F_{e_R},
\end{aligned}$$

for the quarks

$$\begin{aligned}
& -iD_\mu Q_L\sigma^\mu\bar{Q}_L + (D_\mu\tilde{Q}_L)^\dagger(D^\mu\tilde{Q}_L) \\
& +i\sqrt{2}[\tilde{Q}_L^\dagger(g_y\tilde{B}^0 Y + g_L\tilde{W}^a\tau^a + g_C\tilde{G}^a t^a)Q_L - \tilde{Q}(g_Y\bar{\tilde{B}}^0 Y + g_L\bar{\tilde{W}}^a\tau^a + g_C\bar{\tilde{G}}^a t^a)\bar{Q}_L] \\
& +g_Y\tilde{Q}_L^\dagger D_Y Y\tilde{Q}_L + g_L\tilde{Q}_L^\dagger D_L^a\tau^a\tilde{Q}_L + g_C\tilde{Q}_L^\dagger D_C^a t^a\tilde{Q}_L + F_{u_L}^\dagger F_{u_L} + F_{d_L}^\dagger F_{d_L}
\end{aligned}$$

symbol	spin 0	spin $\frac{1}{2}$	$SU(3)_C \times SU(2)_L \times U(1)_Y$	auxiliary field
Q_L	$\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	F_{u_L} F_{d_L}
$\bar{u}_R$	$\tilde{u}^*_R$	$\bar{u}_R$	$(\mathbf{3}, \mathbf{1}, -\frac{2}{3})$	F_{u_R}
$\bar{d}_R$	$\tilde{d}^*_R$	$\bar{d}_R$	$(\mathbf{3}, \mathbf{1}, \frac{1}{3})$	F_{d_R}
L	$\begin{pmatrix} \tilde{\nu}_e \\ \tilde{e}_L \end{pmatrix}$	$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	F_{ν_e} F_e
$\bar{e}_R$	$\tilde{e}^*_R$	$\bar{e}_R$	$(\mathbf{1}, \mathbf{1}, 1)$	F_{e_R}
H_u	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$	$F_{H_u^+}$ $F_{H_u^0}$
H_d	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$F_{H_d^0}$ $F_{H_d^-}$

Table 5.1: Particle content of the MSSM for one generation

gauge group	generator	spin $\frac{1}{2}$	spin 1	field strength	charge	aux.field
$U(1)_Y$	1	$\tilde{B}^0$	$B^{0\mu}$	$B^{0\mu\nu}$	g_Y	D_Y
$SU(2)_L$	$\tau^a = \frac{\sigma^a}{2}$	$\tilde{W}^a$	$W^{a\mu}$	$W^{\mu\nu}$	g_L	D_L
$SU(3)_C$	$t^a = \frac{\lambda^a}{2}$	$\tilde{G}^a$	$G^{a\mu}$	$G^{\mu\nu}$	g_C	D_C
$SU(2)_L \times U(1)_Y$ $\to U(1)_{\text{em}}$		$\tilde{B}^0$ $\tilde{W}^\pm, \tilde{W}^3$	A^μ $W^{\pm\mu}, Z^{0\mu}$	$A^{\mu\nu}$ $W^{\mu\nu}$		

Table 5.2: Gauge boson content of the MSSM

$$
\begin{aligned}
&-iD_\mu u_R \sigma^\mu \bar{u}_R + (D_\mu \tilde{u}_R)^*(D^\mu \tilde{u}_R) \\
&+i\sqrt{2}[\tilde{u}^*_R(g_Y \tilde{B}^0 Y + g_C \tilde{G}^a t^a)u_R - \tilde{u}_R(g_Y \bar{\tilde{B}}^0 Y + g_C \bar{\tilde{G}}^a t^a)\bar{u}_R] \\
&+g_Y \tilde{u}^*_R D_Y Y \tilde{u}_R + g_C \tilde{u}^*_R D^a_C t^a \tilde{u}_R + F^\dagger_{u_R} F_{u_R} \\
&-iD_\mu d_R \sigma^\mu \bar{d}_R + (D_\mu \tilde{d}_R)^*(D^\mu \tilde{d}_R) \\
&+i\sqrt{2}[\tilde{d}^*_R(g_Y \tilde{B}^0 Y + g_C \tilde{G}^a t^a)d_R - \tilde{d}_R(g_Y \bar{\tilde{B}}^0 Y + g_C \bar{\tilde{G}}^a t^a)\bar{d}_R] \\
&+g_Y \tilde{d}^*_R D_Y Y \tilde{d}_R + g_C \tilde{d}^*_R D^a_c t^a \tilde{d}_R + F^\dagger_{d_R} F_{d_R},
\end{aligned} \tag{5.1}
$$

and finally for the Higgses

$$
\begin{aligned}
&-iD_\mu \tilde{H}_u \sigma^\mu \bar{\tilde{H}}_u + (D_\mu H_u)^\dagger(D^\mu H_u) \\
&+i\sqrt{2}[H^\dagger_u(g_Y \tilde{B}^0 Y + g_L \tilde{W}^a \tau^a)\tilde{H}_u - H_u(g_Y \bar{\tilde{B}}^0 Y + g_L \bar{\tilde{W}}^a \tau^a)\bar{\tilde{H}}_u] \\
&+g_Y H^\dagger_u D_Y Y H_u + g_L H^\dagger_u D^a_L \tau^a H_u + F^\dagger_{H_u^+} F_{H_u^+} + F^\dagger_{H_u^0} F_{H_u^0} \\
&-iD_\mu \tilde{H}_d \sigma^\mu \bar{\tilde{H}}_d + (D_\mu H_d)^\dagger(D^\mu H_d) + i\sqrt{2}[H^\dagger_d(g_Y \tilde{B}^0 Y + g_L \tilde{W}^a \tau^a)\tilde{H}_d \\
&-H_d(g_Y \bar{\tilde{B}}^0 Y + g_L \bar{\tilde{W}}^a \tau^a)\bar{\tilde{H}}_d] + g_Y H^\dagger_d D_Y Y H_d + g_L H^\dagger_d D^a_L \tau^a H_d + F^\dagger_{H_d^-} F_{H_d^-} + F^\dagger_{H_d^0} F_{H_d^0}
\end{aligned} \tag{5.2}
$$

Further on we need the kinetic terms (4.26) of the gauge bosons of each gauge group:

$$\begin{aligned}\mathcal{L}_{\text{kin,gauge}} &= -\frac{1}{4}B^0_{\mu\nu}B^{0\mu\nu} + i\tilde{B}^0\sigma^\mu\partial_\mu\bar{\tilde{B}}^0 + \frac{1}{2}D_Y D_Y - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} + i\tilde{W}^a\sigma^\mu\tilde{D}^{ac}_\mu\bar{\tilde{W}}^c + \frac{1}{2}D^a_L D^a_L \\ &\quad -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + i\tilde{G}^a\sigma^\mu\tilde{D}^{ac}_\mu\bar{\tilde{G}}^c + \frac{1}{2}D^a_C D^a_C. \end{aligned} \tag{5.3}$$

5.2.2 Superpotential

Finally we add the term which comes from the superpotential (4.34)

$$\mathcal{L}_{\text{int}} = -\frac{1}{2}W^{ij}(\phi,\phi^\dagger)\psi_i\psi_j + W^i(\phi,\phi^\dagger)F_i + \text{h.c.}, \tag{5.4}$$

with $F_i^\dagger = -W^i$ from the Euler-Lagrange equations, $W^i = \frac{\partial W}{\partial\phi_i}$, W^{ij} correspondingly and the superpotential of the MSSM is given by

$$W = h^e_{ij}\tilde{e}^*_{Ri}\tilde{L}^r_j H^s_d\epsilon^{rs} + h^u_{ij}\tilde{u}^*_{Ri}\tilde{Q}^r_j H^s_u\epsilon^{rs} + h^d_{ij}\tilde{d}^*_{Ri}\tilde{Q}^r_j H^s_d\epsilon^{rs} + \mu H^r_u H^s_d\epsilon^{rs}, \tag{5.5}$$

with $\epsilon^{12} = 1$. We note that we have written W in terms of the bosonic fields.

5.2.3 Soft-Breaking Terms

We restrict ourselves to the following soft SUSY-breaking terms which conserve R-parity[1] and assume soft-breaking universality.[2]

$$\begin{aligned}\mathcal{L}_{\text{soft}} &= -\big((M^2_{\tilde{Q}_L})_{ii}\tilde{Q}^r_{Li}\tilde{Q}^{s*}_{Li}\epsilon^{rs} + (M^2_{\tilde{L}})_{ii}\tilde{L}^r_i\tilde{L}^{s*}_i\epsilon^{rs} + (M^2_{\tilde{u}_R})_{ii}\tilde{u}_{Ri}\tilde{u}^*_{Ri} + (M^2_{\tilde{d}_R})_{ii}\tilde{d}_{Ri}\tilde{d}^*_{Ri} \\ &\quad +(M^2_{\tilde{e}_R})_{ii}\tilde{e}_{Ri}\tilde{e}^*_{Ri}\big) - m^2_{H_u}H^*_u H_u - m^2_{H_d}H^*_d H_d - \big(B\mu H^r_u H^s_d\epsilon^{rs} + \text{h.c.}\big) \\ &\quad -\frac{1}{2}M_1\tilde{A}^0\tilde{A}^0 - \frac{1}{2}M_2\tilde{B}^a\tilde{B}^a - \frac{1}{2}M_3\tilde{G}^a\tilde{G}^a + \text{h.c} \\ &\quad -h^e_{ij}A^e\tilde{L}^r_i\tilde{e}^*_{Rj}H^s_d\epsilon^{rs} - h^u_{ij}A^u\tilde{Q}^r_{Li}\tilde{u}^*_{Rj}H^s_u\epsilon^{rs} - h^d_{ij}A^d\tilde{Q}^r_{Li}\tilde{d}^*_{Rj}H^s_d\epsilon^{rs} + \text{h.c.}\end{aligned}$$

5.2.4 Complete Lagrangian

$$\mathcal{L}_{\text{susy}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{soft}} \tag{5.6}$$

Now we add up all three contributions to the Lagrangian. We note that in L_{int} the term $-W_i^\dagger W_i$ occurs twice[3] so that the $\sum_i F_i^\dagger F_i = \sum_i W_i^\dagger W_i$ term from L_{kin} is canceled and one term $-W_i^\dagger W_i$ survives. Furthermore we have to eliminate the auxiliary fields D_i by applying their equations of motion. A similar argument holds for these giving the remaining term $-\frac{1}{2}\sum_{i\in\{Y,L,C\}}D_i^2$. We break $SU(2)_L\times U(1)_Y \to U(1)_{\text{em}}$ by giving vevs[4] to the Higgses (Table (5.3)). Defining $\tan\theta_W = \frac{g_Y}{g_L}$, $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, $W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$, $Z^0_\mu = c_W W^3_\mu - s_W B_\mu$, $A_\mu = c_W B_\mu + s_W W^3_\mu$, $\tilde{W}^\pm = \frac{1}{\sqrt{2}}(\tilde{W}^1 \mp i\tilde{W}^2)$ and writing out all the component fields, we finally

[1]This is the conservation of $R_P = (-1)^{3B+L+2S}$ with baryon number B, lepton number L and spin S.

[2]This means that the M_i^2 do not mix different generations and that the coupling for the ternary terms is proportional to the coupling appearing in the superpotential with proportionality factor $A^i, i\in\{e,u,d\}$.

[3]This is due to h.c.

[4]Vev is shorthand for vacuum expectation value.

Higgs	vev
H_u	$\begin{pmatrix} H_u^{(+)} \\ v_u + h_u^{(0)} \end{pmatrix}$
H_d	$\begin{pmatrix} v_d + h_d^{(0)} \\ H_d^{(-)} \end{pmatrix}$

Table 5.3: Vevs for the Higgses

arrive at the complete Lagrangian density of the MSSM (cf. appendix A.4 for calculational details). The kinetic terms for the leptons are given by

$$
\begin{aligned}
\mathcal{L}_{\text{MSSM}} \;=\;& -i\partial_\mu \nu_e \sigma^\mu \bar{\nu}_e + \frac{1}{2} Z^0_\mu (g_Y s_W + g_L c_W) \nu_e \sigma^\mu \bar{\nu}_e + \frac{g_L}{\sqrt{2}} W^+_\mu e_L \sigma^\mu \bar{\nu}_e + \frac{g_L}{\sqrt{2}} W^-_\mu \nu_e \sigma^\mu \bar{e}_L \\
& -i\partial_\mu e_L \sigma^\mu \bar{e}_L - A_\mu g_Y c_W e_L \sigma^\mu \bar{e}_L + \frac{1}{2} Z^0_\mu (g_Y s_W - g_L c_W) e_L \sigma^\mu \bar{e}_L \\
& +\partial_\mu \tilde{\nu}^*_e \partial^\mu \tilde{\nu}_e + \left(\frac{i}{2} Z^0_\mu (g_Y s_W + g_L c_W) \partial^\mu \tilde{\nu}^*_e \tilde{\nu}_e + \frac{i g_L}{\sqrt{2}} W^+_\mu \tilde{e}_L \partial^\mu \tilde{\nu}^*_e + \text{h.c.} \right) \\
& +\frac{1}{4} (g_Y s_W + g_L c_W)^2 Z^0_\mu Z^{0\mu} \tilde{\nu}^*_e \tilde{\nu}_e + \left(\frac{1}{2\sqrt{2}} (g_Y s_W + g_L c_W) g_L Z^{0\mu} W^+_\mu \tilde{\nu}^*_e \tilde{e}_L + \text{h.c.} \right) \\
& +\frac{g_L^2}{2} W^+_\mu W^{-\mu} \tilde{\nu}^*_e \tilde{\nu}_e + \frac{g_L^2}{2} W^+_\mu W^{-\mu} \tilde{e}^*_L \tilde{e}_L \\
& +(\partial_\mu \tilde{e}^*_L) \partial^\mu \tilde{e}_L + g_Y^2 c_W^2 A_\mu A^\mu \tilde{e}^*_L \tilde{e}_L + \frac{1}{4} (-g_Y s_W + g_L c_W)^2 Z^0_\mu Z^{0\mu} \tilde{e}^*_L \tilde{e}_L \\
& + \left(-\frac{i g_L}{\sqrt{2}} W^+_\mu \tilde{\nu}^*_e \partial^\mu \tilde{e}_L - \frac{g_L g_Y c_W}{\sqrt{2}} W^+_\mu A^\mu \tilde{\nu}^*_e \tilde{e}_L + \frac{g_L}{2\sqrt{2}} (g_Y s_W - g_L c_W) W^+_\mu Z^{0\mu} \tilde{\nu}^*_e \tilde{e}_L \right. \\
& \qquad - i A_\mu g_Y c_W (\partial^\mu \tilde{e}^*_L) \tilde{e}_L + \frac{i}{2} (g_Y s_W - g_L c_W) Z^0_\mu (\partial^\mu \tilde{e}^*_L) \tilde{e}_L \\
& \qquad \left. + \frac{1}{2} (-g_Y s_W + g_L c_W) g_Y c_W Z^0_\mu A^\mu \tilde{e}^*_L \tilde{e}_L + \text{h.c.} \right) \\
& + \left(\frac{i}{\sqrt{2}} \tilde{\nu}^*_e (g_L \tilde{W}^3 - g_Y \tilde{B}^0) \nu_e + i g_L \tilde{\nu}^*_e \tilde{W}^+ e_L + i g_L \tilde{e}^*_L \tilde{W}^- \nu_e - \frac{i}{\sqrt{2}} \tilde{e}^*_L (g_L \tilde{W}^3 + g_Y \tilde{B}^0) e_L \right. \\
& \qquad \left. + \text{h.c.} \right) - i\partial_\mu \bar{e}_R \sigma^\mu e_R + g_Y \bar{e}_R (c_W A_\mu - s_W Z^0_\mu) \sigma^\mu e_R + \partial_\mu \tilde{e}^*_R \partial^\mu \tilde{e}_R \\
& + g_Y^2 c_W^2 A_\mu A^\mu \tilde{e}^*_R \tilde{e}_R + g_Y^2 s_W^2 Z^0_\mu Z^{0\mu} \tilde{e}^*_R \tilde{e}_R \\
& + \left(i g_Y c_W (\partial^\mu \tilde{e}^*_R) A_\mu \tilde{e}_R - i g_Y s_W (\partial^\mu \tilde{e}^*_R) Z^0_\mu \tilde{e}_R - g_Y^2 c_W s_W A^\mu Z^0_\mu \tilde{e}^*_R \tilde{e}_R + \text{h.c.} \right) \\
& + i\sqrt{2} \left(\tilde{e}_R g_Y \tilde{B}^0 \bar{e}_R + \text{h.c.} \right) .
\end{aligned}
$$

The kinetic terms for the quarks are given by

$$
\begin{aligned}
& -i\partial_\mu u_L \sigma^\mu \bar{u}_L + \frac{2}{3} g_Y c_W A_\mu u_L \sigma^\mu \bar{u}_L + \frac{1}{2} (g_L c_W - \frac{1}{3} g_Y s_W) Z^0_\mu u_L \sigma^\mu \bar{u}_L \\
& + \frac{1}{2} g_C \lambda^a G^a_\mu u_L \sigma^\mu \bar{u}_L + \frac{1}{\sqrt{2}} g_L W^+_\mu d_L \sigma^\mu \bar{u}_L + \frac{1}{\sqrt{2}} g_L W^-_\mu u_L \sigma^\mu \bar{d}_L
\end{aligned}
$$

$$
\begin{aligned}
&+\partial_\mu d_L \sigma^\mu \bar{d}_L + \frac{1}{2}(-g_L s_W + \frac{1}{3} g_Y c_W) A_\mu d_L \sigma^\mu \bar{d}_L - \frac{1}{2}(g_L c_W + \frac{1}{3} g_Y s_W) Z^0_\mu d_L \sigma^\mu \bar{d}_L \\
&+\frac{1}{2} g_C \lambda^a G^a_\mu d_L \sigma^\mu \bar{d}_L + \partial_\mu \tilde{u}^*_L \partial^\mu \tilde{u}^*_L + \frac{4}{9} g_Y^2 c_W^2 A_\mu A^\mu \tilde{u}^*_L \tilde{u}_L \\
&+\frac{1}{4}(\frac{1}{3} g_Y s_W - g_L c_W)^2 Z^0_\mu Z^{0\mu} \tilde{u}^*_L \tilde{u}_L + \frac{1}{2} g_L^2 W^-_\mu W^{+\mu} \tilde{d}^*_L \tilde{d}_L \\
&+\frac{1}{2} g_L^2 W^-_\mu W^{+\mu} \tilde{u}^*_L \tilde{u}_L + \partial^\mu \tilde{d}^*_L \partial_\mu \tilde{d}_L + \frac{1}{4} g_C^2 \tilde{u}^*_L \lambda^{b\dagger} \lambda^a \tilde{u}_L G^b_\mu G^{a\mu} \\
&-\frac{1}{3} g_Y^2 c_W^2 A_\mu A^\mu \tilde{d}^*_L \tilde{d}_L + \frac{1}{4}\left(\frac{1}{3} g_Y s_W - g_L c_W\right) Z^0_\mu Z^{0\mu} \tilde{d}^*_L \tilde{d}_L + \frac{1}{4} g_C^2 \tilde{d}^*_L \lambda^{b\dagger} \lambda^a \tilde{d}_L G^b_\mu G^{a\mu} \\
&+ \Bigg((\partial_\mu \tilde{u}^*_L) \frac{2i}{3} g_Y c_W A^\mu \tilde{u}_L + \frac{i}{2}(g_L c_W - \frac{1}{3} g_Y s_W)(\partial^\mu \tilde{u}^*_L) Z^0_\mu \tilde{u}_L + \frac{i}{\sqrt{2}} g_L (\partial^\mu \tilde{u}^*_L) W^+_\mu \tilde{d}_L \\
&\qquad + \frac{i}{2} g_C G^a_\mu (\partial^\mu \tilde{u}^*_L) \lambda^a \tilde{u}_L + \frac{1}{3} g_C g_Y c_W A^\mu G^a_\mu \tilde{u}^*_L \lambda^a \tilde{u}_L \\
&\qquad + \frac{1}{4}\left(g_L c_W - \frac{1}{3} g_Y s_W\right) Z^0_\mu G^{a\mu} \tilde{u}^*_L \lambda^a \tilde{u}_L + \frac{1}{2\sqrt{2}} W^-_\mu G^{a\mu} \tilde{d}^*_L \lambda^a \tilde{u}_L \\
&\qquad + \frac{1}{3} g_Y c_W (g_L c_W - \frac{1}{3} g_Y s_W) A_\mu Z^{0\mu} \tilde{u}^*_L \tilde{u}_L + \frac{\sqrt{2}}{3} g_L g_Y c_W \tilde{u}^*_L A^\mu W^+_\mu \tilde{d}_L \\
&\qquad + \frac{1}{2\sqrt{2}} g_L (g_L c_W - \frac{1}{3} g_Y s_W) \tilde{u}^*_L Z^0_\mu W^{+\mu} \tilde{d}_L - \frac{i}{\sqrt{2}} g_L W^+_\mu \tilde{u}^*_L (\partial^\mu \tilde{d}_L) \\
&\qquad - \frac{1}{3\sqrt{2}} g_L g_Y c_W W^+_\mu A^\mu \tilde{u}^*_L \tilde{d}_L - \frac{1}{2\sqrt{2}} g_L \left(\frac{1}{3} g_Y s_W + g_L c_W\right) W^+_\mu Z^{0\mu} \tilde{u}^*_L \tilde{d}_L \\
&\qquad - \frac{i}{3} g_Y c_W A_\mu \partial^\mu \tilde{d}^*_L \tilde{d}_L - \frac{i}{2}\left(\frac{1}{3} g_Y s_W + g_L c_W\right) Z^0_\mu \partial^\mu \tilde{d}^*_L \tilde{d}_L \\
&\qquad + \frac{1}{2\sqrt{2}} g_L g_C W^+_\mu G^{a\mu} \tilde{u}^*_L \lambda \tilde{d}_L + \frac{1}{2} g_C G^a_\mu (\partial^\mu \tilde{d}^*_L) \lambda^a \tilde{d}_L \\
&\qquad - \frac{1}{6} g_L g_Y c_W A^\mu G^a_\mu \tilde{d}^*_L \lambda^a \tilde{d}_L - \frac{1}{4} g_C \left(\frac{1}{3} g_Y s_W + g_L c_W\right) Z^0_\mu G^{a\mu} \tilde{d}^*_L \lambda^a \tilde{d}_L \\
&\qquad + \frac{1}{2} g_Y c_W \left(\frac{1}{3} g_Y s_W + g_L c_W\right) Z^0_\mu A^\mu \tilde{d}^*_L \tilde{d}_L + \text{h.c.} \Bigg) \\
&+ \Bigg(\frac{i}{3\sqrt{2}} g_Y \tilde{u}^*_L \tilde{B}^0 u_L + \frac{i}{\sqrt{2}} g_L \tilde{u}^*_L \tilde{W}^3 u_L + \frac{i}{\sqrt{2}} g_C \tilde{u}^*_L \lambda^a \tilde{G}^a u_L + i g_L \tilde{u}^*_L \tilde{W}^+ d_L \\
&\qquad + i g_L \tilde{d}^*_L \tilde{W}^- u_L + \frac{i}{\sqrt{2}} \tilde{d}^*_L \left(\frac{1}{3} g_Y \tilde{B}^0 - g_L \tilde{W}^3 + g_C \lambda^a \tilde{G}^a\right) d_L + \text{h.c.} \Bigg) \\
&-i\partial_\mu \bar{u}_R \sigma^\mu u_R + \frac{2}{3} g_Y c_W A_\mu \bar{u}_R \sigma^\mu u_R - \frac{2}{3} g_Y s_W Z^0_\mu \bar{u}_R \sigma^\mu u_R + \frac{1}{2} g_C \bar{u}_R \lambda^a G^a_\mu \sigma^\mu u_R \\
&+\partial_\mu \tilde{u}^*_R \partial^\mu \tilde{u}_R + \frac{4}{9} g_Y^2 c_W^2 A_\mu A^\mu \tilde{u}^*_R \tilde{u}_R - \frac{8}{9} g_Y^2 c_W s_W A_\mu Z^{0\mu} \tilde{u}^*_R \tilde{u}_R \\
&+\frac{4}{9} g_Y^2 s_W^2 Z^0_\mu Z^{0\mu} \tilde{u}^*_R \tilde{u}_R + \frac{1}{4} g_C^2 \tilde{u}^*_R \lambda^{b\dagger} \lambda^a \tilde{u}_R G^a_\mu G^{b\mu} \\
&+ \Bigg(\frac{2i}{3} g_Y (\partial_\mu \tilde{u}^*_R) c_W A^\mu \tilde{u}_R - \frac{2i}{3} g_Y (\partial_\mu \tilde{u}^*_R) s_W Z^{0\mu} \tilde{u}_R + \frac{i}{2} g_C G^a_\mu (\partial^\mu \tilde{u}^*_R) \lambda^a \tilde{u}_R \\
&\qquad + \frac{1}{3} g_Y g_C c_W A^\mu \tilde{u}^*_R \lambda^a G^a_\mu \tilde{u}_R - \frac{1}{3} g_Y g_C s_W Z^{0\mu} \tilde{u}^*_R \lambda^a G^a_\mu \tilde{u}_R + \text{h.c.} \Bigg) \\
&+ \Bigg(\frac{i2\sqrt{2}}{3} g_Y \tilde{u}_R \tilde{B}^0 \bar{u}_R + \frac{i}{\sqrt{2}} g_C \tilde{G}^a \tilde{u}_R \lambda^a \bar{u}_R + \text{h.c.} \Bigg)
\end{aligned}
$$

$$
\begin{aligned}
&-\partial_\mu \bar{d}_R \sigma^\mu d_R - \frac{1}{3} g_Y c_W A_\mu \bar{d}_R \sigma^\mu d_R + \frac{1}{3} g_Y s_W Z^0_\mu \bar{d}_R \sigma^\mu d_R + \frac{1}{2} g_C \bar{d}_R \lambda^a G^a_\mu \sigma^\mu d_R \\
&+\partial_\mu \tilde{d}^*_R \partial^\mu \tilde{d}_R + \frac{1}{9} g_Y^2 c_W^2 A_\mu A^\mu \tilde{d}^*_R \tilde{d}_R - \frac{2}{9} g_Y^2 c_W s_W A_\mu Z^{0\mu} \tilde{d}^*_R \tilde{d}_R \\
&+\frac{1}{9} g_Y^2 s_W^2 Z^0_\mu Z^{0\mu} \tilde{d}^*_R \tilde{d}_R + \frac{1}{4} g_C^2 \tilde{d}^*_R \lambda^{b\dagger} \lambda^a \tilde{d}_R G^a_\mu G^{b\mu} \\
&+ \Bigg(-\frac{i}{3} g_Y (\partial_\mu \tilde{d}^*_R) c_W A^\mu \tilde{d}_R + \frac{i}{3} g_Y (\partial_\mu \tilde{d}^*_R) s_W Z^{0\mu} \tilde{d}_R + \frac{i}{2} g_C G^a_\mu (\partial^\mu \tilde{d}^*_R) \lambda^a \tilde{d}_R \\
&\qquad - \frac{1}{6} g_Y g_C c_W A^\mu \tilde{d}^*_R \lambda^a G^a_\mu \tilde{d}_R + \frac{1}{6} g_Y g_C s_W Z^{0\mu} \tilde{d}^*_R \lambda^a G^a_\mu \tilde{d}_R + \text{h.c.} \Bigg) \\
&+i\sqrt{2} \left(-\frac{1}{3} g_Y \tilde{d}_R \tilde{B}^0 \bar{d}_R + \frac{1}{2} g_C \tilde{d}_R \tilde{G}^a \lambda^a \bar{d}_R + \text{h.c.} \right).
\end{aligned}
$$

The kinetic terms for the Higgses are given by

$$
\begin{aligned}
&-i\partial_\mu \tilde{H}^{(+)}_u \sigma^\mu \bar{\tilde{H}}^{(+)}_u + g_Y c_W A_\mu \tilde{H}^{(+)}_u \sigma^\mu \bar{\tilde{H}}^{(+)}_u + \frac{1}{2}(g_L c_W - g_Y s_W) Z^0_\mu \tilde{H}^{(+)}_u \sigma^\mu \bar{\tilde{H}}^{(+)}_u \\
&+\frac{1}{\sqrt{2}} g_L W^+_\mu \tilde{H}^{(0)}_u \sigma^\mu \bar{\tilde{H}}^{(+)}_u + \frac{1}{\sqrt{2}} g_L W^-_\mu \tilde{H}^{(+)}_u \sigma^\mu \bar{\tilde{H}}^{(0)}_u - i\partial_\mu \tilde{H}^{(0)}_u \sigma^\mu \bar{\tilde{H}}^{(0)}_u \\
&-\frac{1}{2}(g_Y s_W + g_L c_W) Z^0_\mu \tilde{H}^{(0)}_u \sigma^\mu \bar{\tilde{H}}^{(0)}_u \\
&+\partial_\mu H^{(+)*}_u \partial^\mu H^{(+)}_u + g_Y^2 c_W^2 A_\mu A^\mu H^{(+)*}_u H^{(+)}_u + \frac{1}{4}(g_Y s_W - g_L c_W)^2 Z^{0\mu} Z^0_\mu H^{(+)*}_u H^{(+)}_u \\
&+\frac{1}{2} g_L^2 W^-_\mu W^{+\mu} \left(|v_u|^2 + |h^{(0)}_u|^2 + v_u^* h^{(0)}_u + v_u h^{(0)*}_u \right) + \frac{1}{2} g_L^2 W^-_\mu W^{+\mu} H^{(+)*}_u h^{(+)}_u \\
&+\partial_\mu h^{(0)*}_u \partial^\mu h^{(0)}_u + \frac{1}{4}(g_Y s_W + g_L c_W)^2 Z^0_\mu Z^{0\mu} h^{(0)*}_u h^{(0)}_u \\
&+ \Bigg(i g_Y c_W (\partial_\mu H^{(+)*}_u) A^\mu H^{(+)}_u + \frac{i}{2}(g_L c_W - g_Y s_W)(\partial_\mu H^{(+)*}_u) Z^{0\mu} H^{(+)}_u \\
&\quad + \frac{i g_L}{\sqrt{2}} (\partial_\mu H^{(+)*}_u) W^{+\mu} (v_u + h^{(0)}_u) + \frac{1}{2} g_L c_W (g_L c_W - g_Y s_W) A_\mu Z^{0\mu} H^{(+)*}_u H^{(+)}_u \\
&\quad + \frac{1}{\sqrt{2}} g_L g_Y c_W A_\mu W^{+\mu} H^{(+)*}_u (v_u + h^{(0)}_u) + \frac{1}{2\sqrt{2}} g_L (g_L c_W - g_Y s_W) Z^0_\mu W^{+\mu} H^{(+)*}_u (v_u + h^{(0)}_u) \\
&\quad - \frac{i}{\sqrt{2}} W^+_\mu H^{(+)*}_u \partial^\mu h^{(0)}_u - \frac{1}{2\sqrt{2}} (g_Y s_W + g_L c_W) W^+_\mu Z^{0\mu} H^{(+)*}_u (v_u + h^{(0)}_u) \\
&\quad - \frac{i}{2}(g_Y s_W + g_L c_W) Z^{0\mu} (\partial_\mu h^{(0)*}_u)(v_u + h^{(0)}_u) + \text{h.c.} \Bigg) \\
&+ \Bigg(\frac{i}{\sqrt{2}} H^{(+)*}_u \left(g_Y \tilde{B}^0 + g_L \tilde{W}^3 \right) \tilde{H}^{(+)}_u + i g_L (v_u^* + h^{(0)*}_u) \tilde{W}^- \tilde{H}^{(+)}_u + i g_L H^{(+)*}_u \tilde{W}^+ \tilde{H}^{(0)}_u \\
&\qquad + \frac{i}{\sqrt{2}} (v_u^* + h^{(0)*}_u)(g_Y \tilde{B}^0 - g_L \tilde{W}^3) \tilde{H}^{(0)}_u + \text{h.c.} \Bigg) \\
&-i\partial_\mu \tilde{H}^{(0)}_d \sigma^\mu \bar{\tilde{H}}^{(0)}_d + \frac{1}{2}(g_Y s_W + g_L c_W) Z^0_\mu \tilde{H}^{(0)}_d \sigma^\mu \bar{\tilde{H}}^{(0)}_d + \frac{1}{\sqrt{2}} g_L W^+_\mu \tilde{H}^{(-)}_d \sigma^\mu \bar{\tilde{H}}^{(0)}_d \\
&+\frac{1}{\sqrt{2}} g_L W^-_\mu \tilde{H}^{(0)}_d \sigma^\mu \bar{\tilde{H}}^{(-)}_d - i\partial_\mu \tilde{H}^{(-)}_d \sigma^\mu \bar{\tilde{H}}^{(-)}_d - g_Y c_W A_\mu \tilde{H}^{(-)}_d \sigma^\mu \bar{\tilde{H}}^{(-)}_d \\
&-\frac{1}{2}(g_L c_W - g_Y s_W) Z^0_\mu \tilde{H}^{(-)}_d \sigma^\mu \bar{\tilde{H}}^{(-)}_d
\end{aligned}
$$

$$
\begin{aligned}
&+\partial_\mu h_d^{(0)*}\partial^\mu h_d^{(0)} + \frac{1}{4}(g_Y s_W + g_L c_W)^2 Z^0_\mu Z^{0\mu}(|v_d|^2 + |h_d^{(0)}|^2 + v_d^* h_d^{(0)} + v_d h_d^{(0)*}) \\
&+\frac{1}{2}g_L^2 W^-_\mu W^{+\mu} H_d^{(-)*}H_d^{(-)} + \frac{1}{2}g_L^2 W^+_\mu W^{-\mu}(|v_d|^2 + |h_d^{(0)}|^2 + v_d^* h_d^{(0)} + v_d h_d^{(0)*}) \\
&+\partial_\mu H_d^{(-)*}\partial^\mu H_d^{(-)} + g_Y^2 c_W^2 A_\mu A^\mu H_d^{(-)*}H_d^{(-)} + \frac{1}{4}(g_L c_W - g_Y s_W)^2 Z^0_\mu Z^{0\mu} H_d^{(-)*}H_d^{(-)} \\
&+\big(\frac{i}{2}(g_Y s_W + g_L c_W) Z^0_\mu(\partial^\mu h_d^{(0)*})(v_d + h_d^{(0)}) + \frac{i}{\sqrt{2}} g_L(\partial_\mu h_d^{(0)*})W^{+\mu}H_d^{(-)} \\
&\quad + \frac{1}{2\sqrt{2}} g_L(g_Y s_W + g_L c_W) Z^0_\mu W^{+\mu}(v_d^* + h_d^{(0)*})H_d^{(-)} - \frac{i}{\sqrt{2}} g_L W^+_\mu (v_d^* + h_d^{(0)*})(\partial^\mu H_d^{(-)}) \\
&\quad - \frac{1}{\sqrt{2}} g_L g_Y c_W W^+_\mu A^\mu (v_d^* + h_d^{(0)*})H_d^{(-)} - \frac{1}{2\sqrt{2}} g_L(g_L c_W - g_Y s_W) W^+_\mu Z^{0\mu}(v_d^* + h_d^{(0)*})H_d^{(-)} \\
&\quad - i g_Y c_W(\partial_\mu H_d^{(-)*})A^\mu H_d^{(-)} - \frac{i}{2}(g_L c_W - g_Y s_W) Z^{0\mu}(\partial_\mu H_d^{(-)*})H_d^{(-)} \\
&\quad + \frac{1}{2} g_Y c_W(g_L c_W - g_Y s_W) A^\mu Z^0_\mu H_d^{(-)*}H_d^{(-)} + \text{h.c.}\big) \\
&+\big(\frac{i}{\sqrt{2}}(v_d^* + h_d^{(0)*})(g_L \tilde{W}^3 - g_Y \tilde{B}^0)\tilde{H}_d^{(0)} + i g_L H_d^{(-)*}\tilde{W}^-\tilde{H}_d^{(0)} \\
&\quad + i g_L(v_d^* + h_d^{(0)*})\tilde{W}^+\tilde{H}_d^{(-)} - \frac{i}{\sqrt{2}} H_d^{(-)*}(g_Y \tilde{B}^0 + g_L \tilde{W}^3)\tilde{H}_d^{(-)} + \text{h.c.}\big).
\end{aligned}
$$

The kinetic terms of the gauge fields are given by (with f^{abc} the structure constant of $SU(3)$)

$$
\begin{aligned}
&-\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right) - \frac{1}{4}\left(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu\right)\left(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}\right) \\
&-\frac{1}{4}\left(\partial_\mu Z^0_\nu - \partial_\nu Z^0_\mu\right)\left(\partial^\mu Z^{0\nu} - \partial^\nu Z^{0\mu}\right) \\
&+i\frac{g}{2}\big(s_W(\partial_\mu W^+_\nu)W^{\mu-}A^\nu + c_W(\partial_\mu W^+_\nu)W^{\mu-}Z^{0\nu} - s_W(\partial_\mu W^+_\nu)A^\mu W^{\nu-} \\
&\quad - c_W(\partial_\mu W^+_\nu)Z^{0\mu}W^{\nu-} - s_W(\partial_\mu W^-_\nu)W^{\mu+}A^\nu - c_W(\partial_\mu W^-_\nu)W^{\mu+}Z^{0\nu} \\
&\quad + s_W(\partial_\mu W^-_\nu)A^\mu W^{\nu+} + c_W(\partial_\mu W^-_\nu)Z^{0\mu}W^{\nu+} - s_W(\partial_\mu A_\nu)W^{\mu-}W^{\nu+} \\
&\quad + s_W(\partial_\mu A_\nu)W^{\nu-}W^{\mu+} - c_W(\partial_\mu Z^0_\nu)W^{\mu-}W^{\nu+} + c_W(\partial_\mu Z^0_\nu)W^{\nu-}W^{\mu+}\big) \\
&+\frac{g^2}{2}\left(W^+_\mu W^{\mu+}W^-_\nu W^{\nu-} - W^+_\mu W^{\mu-}W^+_\nu W^{\nu-}\right) + g^2 s_W^2\left(W^+_\mu A^\mu W^-_\nu A^\nu - W^+_\mu W^{\mu-}A_\nu A^\nu\right) \\
&+g^2 s_W c_W\left(-2W^+_\mu W^{\mu-}A_\nu Z^{0\nu} + W^+_\mu A^\mu W^-_\nu Z^{0\nu} + W^-_\mu A^\mu W^+_\nu Z^{0\nu}\right) \\
&+g^2 c_W^2\left(W^+_\mu Z^{0\mu}W^-_\nu Z^{0\nu} - W^+_\mu W^{\mu-}Z^0_\nu Z^{0\nu}\right) - \frac{1}{4}\left(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu\right)\left(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu}\right) \\
&-\frac{g}{2}f^{abc}(\partial_\mu G^a_\nu)G^{b\mu}G^{c\nu} - \frac{g^2}{4}f^{abc}f^{ade}G^b_\mu G^c_\nu G^{d\mu}G^{e\nu} + i\tilde{B}^0\sigma^\mu\partial_\mu\bar{\tilde{B}}^0 + i\tilde{W}^a\sigma^\mu\partial_\mu\bar{\tilde{W}}^a \\
&+i\tilde{G}^a\sigma^\mu\partial_\mu\bar{\tilde{G}}^a - i g_C f^{abc}\tilde{G}^a\sigma^\mu G^b_\mu\bar{\tilde{G}}^c - i g_L\big(\tilde{W}^-\sigma^\mu\bar{\tilde{W}}^3 W^+_\mu - \tilde{W}^3\sigma^\mu\bar{\tilde{W}}^+ W^+_\mu \\
&+s_W\tilde{W}^+\sigma^\mu\bar{\tilde{W}}^+ A_\mu + c_W\tilde{W}^+\sigma^\mu\bar{\tilde{W}}^+ Z^0_\mu - \text{h.c}\big)
\end{aligned}
$$

The D-term contribution is given by

$$
-\frac{1}{8}(g_Y^2 + g_L^2)\left((\tilde{\nu}_e^*\tilde{\nu}_e)^2 + (\tilde{e}_L^*\tilde{e}_L)^2 + 2\tilde{\nu}_e^*\tilde{\nu}_e\tilde{e}_L^*\tilde{e}_L\right)^2 - \frac{g_Y^2}{2}\left(\tilde{e}_R^*\tilde{e}_R\right)
$$

$$
\begin{aligned}
&-\left(\frac{g_Y^2}{72}+\frac{g_L^2}{8}+\frac{g_C^2}{6}\right)\Big((\tilde{u}_L^*\tilde{u}_L)^2+(\tilde{d}_L^*\tilde{d}_L)^2+2\tilde{u}_L^*\tilde{u}_L\tilde{d}_L^*\tilde{d}_L\Big)-\left(\frac{2g_Y^2}{9}+\frac{g_C^2}{6}\right)(\tilde{u}_R^*\tilde{u}_R)^2\\
&-\left(\frac{g_Y^2}{18}+\frac{g_C^2}{6}\right)\left(\tilde{d}_R^*\tilde{d}_R\right)^2-\left(\frac{g_Y^2}{8}+\frac{g_L^2}{8}\right)\Bigg(\;(H_u^{(+)*}H_u^{(+)})^2+(h_u^{(0)}h_u^{(0)})^2\\
&\quad+|v_u|^4+v_u^{*2}h_u(0)h_u^{(0)}+v_u^2h_u^{(0)*}h_u^{(0)*}+2H_u^{(+)*}H_u^{(+)}h_u^{(0)*}h_u^{(0)}+2|v_u|^2H_u^{(+)*}H_u^{(+)}\\
&\quad+2|v_u|^2h_u^{(0)*}h_u^{(0)}+2v_u*h_u^{(0)}H_u^{(+)*}H_u^{(+)}+2v_uh_u^{(0)*}H_u^{(+)*}H_u^{(+)}+4|v_u|^2h_u^{(0)*}h_u^{(0)}\\
&\quad+2v_u^*h_u^{(0)}h_u^{(0)*}h_u^{(0)}+2v_uh_u^{(0)*}h_u^{(0)*}h_u^{(0)}+2|v_u|^2v_u^*h_u^{(0)}+2|v_u|^2v_uh_u^{(0)*}\;\Bigg)\\
&-\left(\frac{g_Y^2}{8}+\frac{g_C^2}{8}\right)\Bigg(\;(H_d^{(-)*}H_d^{(-)})^2+(h_d^{(0)}h_d^{(0)})^2\\
&\quad+|v_d|^4+v_d^{*2}h_d^{(0)}h_d^{(0)}+v_d^2h_d^{(0)*}h_d^{(0)*}+2H_d^{(-)*}H_d^{(-)}h_d^{(0)*}h_d^{(0)}+2|v_d|^2H_d^{(-)*}H_d^{(-)}\\
&\quad+2|v_d|^2h_d^{(0)*}h_d^{(0)}+2v_d^*h_d^{(0)}H_d^{(-)*}H_d^{(-)}+2v_dh_d^{(0)*}H_d^{(-)*}H_d^{(-)}+4|v_d|^2h_d^{(0)*}h_d^{(0)}\\
&\quad+2v_d^*h_d^{(0)}h_d^{(0)*}h_d^{(0)}+2v_dh_d^{(0)*}h_d^{(0)*}h_d^{(0)}+2|v_d|^2v_d^*h_d^{(0)}+2|v_d|^2v_dh_d^{(0)*}\;\Bigg)\\
&+\frac{g_Y^2}{2}\tilde{e}_R^*\tilde{e}_R\left(\tilde{\nu}_e^*\tilde{\nu}_e+\tilde{e}_L^*\tilde{e}_L\right)+\left(\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)\tilde{\nu}_e^*\tilde{\nu}_e\tilde{u}_L^*\tilde{u}_L+\left(\frac{g_L^2}{4}+\frac{g_Y^2}{12}\right)\tilde{\nu}_e^*\tilde{\nu}_e\tilde{d}_L^*\tilde{d}_L\\
&+\left(\frac{g_L^2}{4}+\frac{g_Y^2}{12}\right)\tilde{e}_L^*\tilde{e}_L\tilde{u}_L^*\tilde{u}_L+\left(\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)\tilde{e}_L^*\tilde{e}_L\tilde{d}_L^*\tilde{d}_L-\frac{g_L^2}{2}\tilde{\nu}_e^*\tilde{e}_L\tilde{d}_L^*\tilde{u}_L-\frac{g_L^2}{2}\tilde{e}_L^*\tilde{\nu}_e\tilde{u}_L^*\tilde{d}_L\\
&-\frac{g_Y^2}{3}\tilde{u}_R^*\tilde{u}_R\left(\tilde{\nu}_e^*\tilde{\nu}_e+\tilde{e}_L^*\tilde{e}_L\right)+\frac{g_Y^2}{6}\tilde{d}_R^*\tilde{d}_R\left(\tilde{\nu}_e^*\tilde{\nu}_e+\tilde{e}_L^*\tilde{e}_L\right)\\
&+\left(\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)\tilde{\nu}_e^*\tilde{\nu}_eH_u^{(+)*}H_u^{(+)}+\left(\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)\tilde{\nu}_e^*\tilde{\nu}_eh_u^{(0)*}h_u^{(0)}+\left(\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)|v_u|^2\tilde{\nu}_e^*\tilde{\nu}_e\\
&+\left(\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)v_u^*\tilde{\nu}_e^*\tilde{\nu}_eh_u^{(0)}+\left(\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)v_u\tilde{\nu}_e^*\tilde{\nu}_eh_u^{(0)*}+\left(\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)\tilde{e}_L^*\tilde{e}_LH_u^{(+)*}H_u^{(+)}\\
&+\left(\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)\tilde{e}_L^*\tilde{e}_Lh_u^{(0)*}h_u^{(0)}+\left(\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)|v_u|^2\tilde{e}_L^*\tilde{e}_L+\left(\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)v_u^*\tilde{e}_L^*\tilde{e}_Lh_u^{(0)}\\
&+\left(\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)v_u\tilde{e}_L^*\tilde{e}_Lh_u^{(0)*}-\frac{g_L^2}{2}v_u^*\tilde{\nu}_e^*\tilde{e}_LH_u^{(+)}-\frac{g_L^2}{2}\tilde{\nu}_e^*\tilde{e}_Lh_u^{(0)*}H_u^{(+)}\\
&-\frac{g_L^2}{2}v_u\tilde{e}_L^*\tilde{\nu}_eH_u^{(+)*}-\frac{g_L^2}{2}\tilde{e}_L^*\tilde{\nu}_eH_u^{(+)*}h_u^{(0)}\\
&+\left(-\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)\tilde{e}_L^*\tilde{e}_LH_d^{(-)*}H_d^{(-)}+\left(-\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)\tilde{e}_L^*\tilde{e}_Lh_d^{(0)*}h_d^{(0)}+\left(-\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)|v_d|^2\tilde{e}_L^*\tilde{e}_L\\
&+\left(-\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)v_d^*\tilde{e}_L^*\tilde{e}_Lh_d^{(0)}+\left(-\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)v_d\tilde{e}_L^*\tilde{e}_Lh_d^{(0)*}+\left(-\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)\tilde{\nu}_e^*\tilde{\nu}_eH_d^{(-)*}H_d^{(-)}\\
&+\left(-\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)\tilde{\nu}_e^*\tilde{\nu}_eh_d^{(0)*}h_d^{(0)}+\left(-\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)|v_d|^2\tilde{\nu}_e^*\tilde{\nu}_e+\left(-\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)v_d^*\tilde{\nu}_e^*\tilde{\nu}_Lh_d^{(0)}\\
&+\left(-\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)v_d\tilde{\nu}_e^*\tilde{\nu}_eh_d^{(0)*}-\frac{g_L^2}{2}v_d^*\tilde{e}_L^*\tilde{\nu}_eH_d^{(-)}-\frac{g_L^2}{2}\tilde{e}_L^*\tilde{\nu}_eh_d^{(0)*}H_d^{(-)}\\
&-\frac{g_L^2}{2}v_d\tilde{\nu}_e^*\tilde{e}_LH_d^{(-)*}-\frac{g_L^2}{2}\tilde{\nu}_e^*\tilde{e}_LH_d^{(-)*}h_d^{(0)}-\frac{g_Y^2}{6}\tilde{e}_R^*\tilde{e}_R\left(\tilde{u}_L^*\tilde{u}_L+\tilde{d}_L^*\tilde{d}_L\right)+\frac{2g_Y^2}{3}\tilde{e}_R^*\tilde{e}_R\tilde{u}_R^*\tilde{u}_R
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{g_Y^2}{3}\tilde{e}_R^*\tilde{e}_R\tilde{d}_R^*\tilde{d}_R-\frac{g_Y^2}{2}\tilde{e}_R^*\tilde{e}_R\left(H_u^{(+)*}H_u^{(+)}+h_u^{(0)*}h_u^{(0)}+|v_u|^2+h_u^{(0)*}v_u+h_u^{(0)}v_u^*\right)\\
&+\frac{g_Y^2}{2}\tilde{e}_R^*\tilde{e}_R\left(H_d^{(-)*}H_d^{(-)}+h_d^{(0)*}h_d^{(0)}+|v_d|^2+h_d^{(0)*}v_d+h_d^{(0)}v_d^*\right)\\
&+\left(\frac{g_Y^2}{9}-\frac{g_C^2}{3}\right)\tilde{u}_R^*\tilde{u}_R\left(\tilde{u}_L^*\tilde{u}_L+\tilde{d}_L^*\tilde{d}_L\right)-\left(\frac{g_Y^2}{18}-\frac{g_C^2}{3}\right)\tilde{d}_R^*\tilde{d}_R\left(\tilde{u}_L^*\tilde{u}_L+\tilde{d}_L^*\tilde{d}_L\right)\\
&+\left(-\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)\tilde{u}_L^*\tilde{u}_LH_u^{(+)*}H_u^{(+)}+\left(-\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)\tilde{u}_L^*\tilde{u}_Lh_u^{(0)*}h_u^{(0)}+\left(-\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)|v_u|^2\tilde{u}_L^*\tilde{u}_L\\
&+\left(-\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)v_u^*\tilde{u}_L^*\tilde{u}_Lh_u^{(0)}+\left(-\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)v_u\tilde{u}_L^*\tilde{u}_Lh_u^{(0)*}+\left(-\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)\tilde{d}_L^*\tilde{d}_LH_u^{(+)*}H_u^{(+)}\\
&+\left(-\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)\tilde{d}_L^*\tilde{d}_Lh_u^{(0)*}h_u^{(0)}+\left(-\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)|v_u|^2\tilde{d}_L^*\tilde{d}_L+\left(-\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)v_u^*\tilde{d}_L^*\tilde{d}_Lh_u^{(0)}\\
&+\left(-\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)v_u\tilde{d}_L^*\tilde{d}_Lh_u^{(0)*}-\frac{g_L^2}{2}v_u^*\tilde{u}_L^*\tilde{d}_LH_u^{(+)}-\frac{g_L^2}{2}\tilde{u}_L^*\tilde{d}_Lh_u^{(0)*}H_u^{(+)}\\
&-\frac{g_L^2}{2}v_u\tilde{d}_L^*\tilde{u}_LH_u^{(+)*}-\frac{g_L^2}{2}\tilde{d}_L^*\tilde{u}_LH_u^{(+)*}h_u^{(0)}\\
&+\left(\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)\tilde{d}_L^*\tilde{d}_LH_d^{(-)*}H_d^{(-)}+\left(\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)\tilde{d}_L^*\tilde{d}_Lh_d^{(0)*}h_d^{(0)}+\left(\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)|v_d|^2\tilde{d}_L^*\tilde{d}_L\\
&+\left(\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)v_d^*\tilde{d}_L^*\tilde{d}_Lh_d^{(0)}+\left(\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)v_d\tilde{d}_L^*\tilde{d}_Lh_d^{(0)*}+\left(\frac{g_Y^2}{12}+\frac{g_L^2}{4}\right)\tilde{u}_L^*\tilde{u}_LH_d^{(-)*}H_d^{(-)}\\
&+\left(\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)\tilde{u}_L^*\tilde{u}_Lh_d^{(0)*}h_d^{(0)}+\left(\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)|v_d|^2\tilde{u}_L^*\tilde{u}_L+\left(\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)v_d^*\tilde{u}_L^*\tilde{u}_Lh_d^{(0)}\\
&+\left(\frac{g_Y^2}{12}-\frac{g_L^2}{4}\right)v_d\tilde{u}_L^*\tilde{u}_Lh_d^{(0)*}-\frac{g_L^2}{2}v_d^*\tilde{d}_L^*\tilde{u}_LH_d^{(-)}-\frac{g_L^2}{2}\tilde{d}_L^*\tilde{u}_Lh_d^{(0)*}H_d^{(-)}\\
&-\frac{g_L^2}{2}v_d\tilde{u}_L^*\tilde{d}_LH_d^{(-)*}-\frac{g_L^2}{2}\tilde{u}_L^*\tilde{d}_LH_d^{(-)*}h_d^{(0)}\\
&+\left(\frac{2g_Y^2}{9}-\frac{g_C^2}{3}\right)\tilde{u}_R^*\tilde{u}_R\tilde{d}_R^*\tilde{d}_R+\frac{g_Y^2}{3}\tilde{u}_R^*\tilde{u}_R\left(H_u^{(+)*}H_u^{(+)}+h_u^{(0)*}h_u^{(0)}+|v_u|^2+h_u^{(0)*}v_u+h_u^{(0)}v_u^*\right)\\
&-\frac{g_Y^2}{3}\tilde{u}_R^*\tilde{u}_R\left(H_d^{(-)*}H_d^{(-)}+h_d^{(0)*}h_d^{(0)}+|v_d|^2+h_d^{(0)*}v_d+h_d^{(0)}v_d^*\right)\\
&-\frac{g_Y^2}{6}\tilde{d}_R^*\tilde{d}_R\left(H_u^{(+)*}H_u^{(+)}+h_u^{(0)*}h_u^{(0)}+|v_u|^2+h_u^{(0)*}v_u+h_u^{(0)}v_u^*\right)\\
&+\frac{g_Y^2}{6}\tilde{d}_R^*\tilde{d}_R\left(H_d^{(-)*}H_d^{(-)}+h_d^{(0)*}h_d^{(0)}+|v_d|^2+h_d^{(0)*}v_d+h_d^{(0)}v_d^*\right)\\
&+\left(\frac{g_Y^2}{4}+\frac{g_L^2}{4}\right)\Bigg(H_u^{(+)*}H_u^{(+)}H_d^{(-)*}H_d^{(-)}+h_u^{(0)*}h_u^{(0)}h_d^{(0)*}h_d^{(0)}+|v_d|^2h_u^{(0)*}h_u^{(0)}+h_u^{(0)*}h_u^{(0)}h_d^{(0)*}v_d\\
&\quad+h_u^{(0)*}h_u^{(0)}h_d^{(0)}v_d^*+|v_u|^2h_d^{(0)*}h_d^{(0)}+|v_u|^2|v_d|^2+|v_u|^2v_dh_d^{(0)*}+|v_u|^2v_d^*h_d^{(0)}+h_u^{(0)*}v_uh_d^{(0)*}h_d^{(0)}\\
&\quad+|v_d|^2h_u^{(0)*}v_u+v_dh_u^{(0)*}v_uh_d^{(0)*}+v_d^*h_u^{(0)*}v_uh_d^{(0)}+v_u^*h_u^{(0)}h_d^{(0)*}h_d^{(0)}+|v_d|^2v_u^*h_u^{(0)}\\
&\quad+v_u^*v_dh_d^{(0)*}h_u^{(0)}+v_u^*v_d^*h_u^{(0)}h_d^{(0)}\Bigg)\\
&+\left(\frac{g_Y^2}{4}-\frac{g_L^2}{4}\right)\Bigg(H_u^{(+)*}H_u^{(+)}h_d^{(0)*}h_d^{(0)}+|v_d|^2H_u^{(+)*}H_u^{(+)}+v_dh_d^{(0)*}H_u^{(+)*}H_u^{(+)}+v_d^*h_d^{(0)}H_u^{(+)*}H_u^{(+)}
\end{aligned}
$$

$$+ h_u^{(0)*} h_u^{(0)} H_d^{(-)*} H_d^{(-)} + |v_u|^2 H_d^{(-)*} H_d^{(-)} + v_u h_u^{(0)*} H_d^{(-)*} H_d^{(-)} + v_u^* h_u^{(0)} H_d^{(-)*} H_d^{(-)} \Big)$$

$$-\frac{g_L^2}{2} \Big(v_u h_d^{(0)} H_u^{(+)*} H_d^{(-)*} + H_u^{(+)*} h_u^{(0)} H_d^{(-)*} h_d^{(0)} + v_u v_d H_u^{(+)*} H_d^{(-)*} + v_d h_u^{(0)} H_u^{(+)*} H_d^{(-)*} + \text{h.c.} \Big)$$

The ternary couplings coming from the superpotential are given by

$$\begin{aligned}
-\frac{1}{2} \Big(& h^e_{ij} \bar{e}_{Ri} \nu_j H_d^{(-)} - h^e_{ij} \bar{e}_{Ri} e_{Lj} v_d - h^e_{ij} \bar{e}_{Ri} e_{Lj} h_d^{(0)} + h^e_{ij} \tilde{e}^*_{Ri} \tilde{\nu}_j H_d^{(-)} - h^e_{ij} \bar{e}_{Ri} \tilde{e}_{Lj} \tilde{H}_d^{(0)} \\
& + h^e_{ij} \tilde{e}^*_{Ri} \nu_j \tilde{H}_d^{(-)} - h^e_{ij} \tilde{e}^*_{Ri} e_{Lj} \tilde{H}_d^{(0)} + h^u_{ij} \bar{u}_{Ri} u_{Lj} v_u + h^u_{ij} \bar{u}_{Ri} u_{Lj} h_u^{(0)} \\
& - h^u_{ij} \bar{u}_{Ri} d_{Lj} H_u^{(+)} + h^u_{ij} \bar{u}_{Ri} \tilde{u}_{Lj} \tilde{H}_u^{(0)} - h^u_{ij} \bar{u}_{Ri} \tilde{d}_{Lj} \tilde{H}_u^{(+)} + h^u_{ij} \tilde{u}^*_{Ri} u_{Lj} \tilde{H}_u^{(0)} - h^u_{ij} \tilde{u}^*_{Ri} d_{Lj} \tilde{H}_u^{(+)} \\
& + h^d_{ij} \bar{d}_{Ri} u_{Lj} H_d^{(-)} - h^d_{ij} \bar{d}_{Ri} d_{Lj} v_d - h^d_{ij} \bar{d}_{Ri} d_{Lj} h_d^{(0)} + h^d_{ij} \bar{d}_{Ri} \tilde{u}_{Lj} \tilde{H}_d^{(-)} - h^d_{ij} \bar{d}_{Ri} \tilde{d}_{Lj} \tilde{H}_d^{(0)} \\
& + h^d_{ij} \tilde{d}^*_{Ri} u_{Lj} \tilde{H}_d^{(-)} - h^d_{ij} \tilde{d}^*_{Ri} d_{Lj} \tilde{H}_d^{(0)} + \mu \tilde{H}_u^{(+)} \tilde{H}_d^{(-)} - \mu \tilde{H}_u^{(0)} \tilde{H}_d^{(0)} + \text{h.c} \Big) .
\end{aligned}$$

The quartic couplings originating in the superpotential are given by

$$\begin{aligned}
- \Big(& h^e_{ij} h^{e*}_{il} \tilde{\nu}_j \tilde{\nu}^*_l H_d^{(-)*} H_d^{(-)} - h^e_{ij} h^{e*}_{il} \tilde{\nu}_j H_d^{(-)} \tilde{e}^*_{Ll} v^*_d - h^e_{ij} h^{e*}_{il} \tilde{\nu}_j H_d^{(-)} \tilde{e}^*_{Ll} h_d^{(0)*} \\
& - h^e_{ij} h^{e*}_{il} \tilde{e}_{Lj} v_d \tilde{\nu}^*_l H_d^{(-)*} + h^e_{ij} h^{e*}_{il} \tilde{e}_{Lj} v_d \tilde{e}^*_{Ll} v^*_d + h^e_{ij} h^{e*}_{il} \tilde{e}_{Lj} v_d \tilde{e}^*_{Ll} h_d^{(0)*} \\
& - h^e_{ij} h^{e*}_{il} \tilde{e}_{Lj} h_d^{(0)} \tilde{\nu}^*_l H_d^{(-)*} + h^e_{ij} h^{e*}_{il} \tilde{e}_{Lj} h_d^{(0)} \tilde{e}^*_{Ll} v^*_d + h^e_{ij} h^{e*}_{il} \tilde{e}_{Lj} h_d^{(0)} \tilde{e}^*_{Ll} h_d^{(0)*} \\
& + h^e_{ij} h^{e*}_{jl} \tilde{e}^*_{Ri} \tilde{e}_{Rl} v^*_d v_d + h^e_{ij} h^{e*}_{jl} \tilde{e}^*_{Ri} \tilde{e}_{Rl} v_d h_d^{(0)*} + h^e_{ij} h^{e*}_{jl} \tilde{e}^*_{Ri} \tilde{e}_{Rl} v^*_d h_d^{(0)} \\
& + h^e_{ij} h^{e*}_{jl} \tilde{e}^*_{Ri} \tilde{e}_{Rl} h_d^{(0)*} h_d^{(0)} + h^e_{ij} h^{e*}_{jl} \tilde{e}^*_{Ri} \tilde{e}_{Rl} H_d^{(-)*} H_d^{(-)} + h^e_{ij} h^{e*}_{kl} \tilde{e}^*_{Ri} \tilde{e}_{Rk} \tilde{\nu}_j \tilde{\nu}^*_l \\
& + h^e_{ij} h^{e*}_{kl} \tilde{e}^*_{Ri} \tilde{e}_{Rk} \tilde{e}_{Lj} \tilde{e}^*_{Ll} \\
& + h^u_{ij} h^{u*}_{ik} \tilde{\nu}_j v_u \tilde{\nu}^*_k v^*_u + h^u_{ij} h^{u*}_{ik} \tilde{\nu}_j v_u \tilde{\nu}^*_k h_u^{(0)*} - h^u_{ij} h^{u*}_{ik} \tilde{\nu}_j v_u \tilde{e}^*_{Lk} H_u^{(+)*} \\
& + h^u_{ij} h^{u*}_{ik} \tilde{\nu}_j h_u^{(0)} \tilde{\nu}^*_k v^*_u + h^u_{ij} h^{u*}_{ik} \tilde{\nu}_j h_u^{(0)} \tilde{\nu}^*_k h_u^{(0)*} - h^u_{ij} h^{u*}_{ik} \tilde{\nu}_j h_u^{(0)} \tilde{e}^*_{Lk} H_u^{(+)*} \\
& - h^u_{ij} h^{u*}_{ik} \tilde{e}_{Lj} H_u^{(+)} \tilde{\nu}^*_k v^*_u - h^u_{ij} h^{u*}_{ik} \tilde{e}_{Lj} H_u^{(+)} \tilde{\nu}^*_k h_u^{(0)*} + h^u_{ij} h^{u*}_{ik} \tilde{e}_{Lj} H_u^{(+)} \tilde{e}^*_{Lk} H_u^{(+)*} \\
& + h^u_{ij} h^{u*}_{kj} \tilde{u}^*_{Ri} \tilde{u}_{Rk} v^*_u v_u + h^u_{ij} h^{u*}_{kj} \tilde{u}^*_{Ri} \tilde{u}_{Rk} v_u h_u^{(0)*} + h^u_{ij} h^{u*}_{kj} \tilde{u}^*_{Ri} \tilde{u}_{Rk} v^*_u h_u^{(0)} \\
& + h^u_{ij} h^{u*}_{kj} \tilde{u}^*_{Ri} \tilde{u}_{Rk} H_u^{(+)*} H_u^{(+)} + h^u_{ij} h^{u*}_{kl} \tilde{u}^*_{Ri} \tilde{u}_{Rk} \tilde{u}_{Lj} \tilde{u}^*_{Ll} + h^u_{ij} h^{u*}_{kl} \tilde{u}^*_{Ri} \tilde{u}_{Rk} \tilde{d}_{Lj} \tilde{d}^*_{Ll} \\
& + h^d_{ij} h^{d*}_{ik} \tilde{u}_{Lj} H_d^{(-)} \tilde{u}^*_{Lk} H_d^{(-)*} - h^d_{ij} h^{d*}_{ik} \tilde{u}_{Lj} H_d^{(-)} \tilde{d}^*_{Lk} v^*_d - h^d_{ij} h^{d*}_{ik} \tilde{u}_{Lj} H_d^{(-)} \tilde{d}^*_{Lk} h_d^{(0)*}
\end{aligned}$$

$$
\begin{aligned}
&- h^d_{ij} h^{d*}_{ik} \tilde{d}_{Lj} v_d \tilde{u}^*_{Lk} H^{(-)*}_d + h^d_{ij} h^{d*}_{ik} \tilde{d}_{Lj} v_d \tilde{d}^*_{Lk} v^*_d + h^d_{ij} h^{d*}_{ik} \tilde{d}_{Lj} v_d \tilde{d}^*_{Lk} h^{(0)*}_d \\
&- h^d_{ij} h^{d*}_{ik} \tilde{d}_{Lj} h^{(0)}_d \tilde{u}^*_{Lk} H^{(-)*}_d + h^d_{ij} h^{d*}_{ik} \tilde{d}_{Lj} h^{(0)}_d \tilde{d}^*_{Lk} v^*_d + h^d_{ij} h^{d*}_{ik} \tilde{d}_{Lj} h^{(0)}_d \tilde{d}^*_{Lk} h^{(0)*}_d \\
&+ h^d_{ij} h^{d*}_{kj} \tilde{d}^*_{Ri} \tilde{d}_{Rk} v^*_d v_d + h^d_{ij} h^{d*}_{kj} \tilde{d}^*_{Ri} \tilde{d}_{Rk} v_d h^{(0)*}_d + h^d_{ij} h^{d*}_{kj} \tilde{d}^*_{Ri} \tilde{d}_{Rk} v^*_d h^{(0)}_d \\
&+ h^d_{ij} h^{d*}_{kj} \tilde{d}^*_{Ri} \tilde{d}_{Rk} h^{(0)*}_d h^{(0)}_d + h^d_{ij} h^{d*}_{kj} \tilde{d}^*_{Ri} \tilde{d}_{Rk} H^{(-)*}_d H^{(-)}_d + h^d_{ij} h^{d*}_{kl} \tilde{d}^*_{Ri} \tilde{d}_{Rk} \tilde{u}_{Lj} \tilde{u}^*_{Lk} \\
&+ h^d_{ij} h^{d*}_{kl} \tilde{d}^*_{Ri} \tilde{d}_{Rk} \tilde{d}_{Lj} \tilde{d}^*_{Ll} \\
&+ \mu^2 v^*_u v_u + \mu^2 v^*_u h^{(0)}_u + \mu^2 v_u h^{(0)*}_u + \mu^2 h^{(0)}_u h^{(0)*}_u + \mu^2 H^{(+)*}_u H^{(+)}_u \\
&+ \mu^2 v^*_d v_d + \mu^2 v^*_d h^{(0)}_d + \mu^2 v_d h^{(0)*}_d + \mu^2 h^{(0)}_d h^{(0)*}_d + \mu^2 H^{(-)*}_d H^{(-)}_d \\
&+ h^e_{ij} h^{d*}_{kl} \tilde{e}^*_{Ri} \tilde{d}_{Rk} \tilde{\nu}_j \tilde{u}_{Ll} + h^e_{ij} h^{d*}_{kl} \tilde{e}^*_{Ri} \tilde{d}_{Rk} \tilde{e}_{Lj} \tilde{d}_{Ll} \\
&+ \Big(h^e_{ij} \mu \tilde{e}^*_{Ri} \tilde{\nu}_j H^{(+)*}_u + h^e_{ij} \mu v_u \tilde{e}^*_{Ri} \tilde{e}_{Lj} + h^e_{ij} \mu \tilde{e}^*_{Ri} \tilde{e}_{Lj} h^{(0)*}_u \\
&\quad + h^d_{ij} \mu \tilde{d}^*_{Ri} \tilde{u}_{Lj} H^{(+)*}_u + h^e_{ij} \mu v_u \tilde{d}^*_{Ri} \tilde{d}_{Lj} + h^e_{ij} \mu \tilde{d}^*_{Ri} \tilde{d}_{Lj} h^{(0)*}_u \\
&\quad + h^u_{ij} h^{d*}_{kl} v^*_d \tilde{u}^*_{Ri} \tilde{d}_{Rk} H^{(+)}_u + h^u_{ij} h^{d*}_{kl} \tilde{u}^*_{Ri} \tilde{d}_{Rk} H^{(+)}_u h^{(0)*}_d + h^u_{ij} h^{d*}_{kl} v_u \tilde{u}^*_{Ri} \tilde{d}_{Rk} H^{(-)*}_d \\
&\quad + h^u_{ij} h^{d*}_{kl} \tilde{u}^*_{Ri} \tilde{d}_{Rk} h^{(0)}_u H^{(-)*}_d + h^u_{ij} \mu v_d \tilde{u}^*_{Ri} \tilde{u}_{Lj} + h^u_{ij} \mu \tilde{u}^*_{Ri} \tilde{u}_{Lj} h^{(0)*}_d \\
&\quad + h^u_{ij} \mu \tilde{u}^*_{Ri} \tilde{u}_{Lj} H^{(-)*}_d + \text{h.c.} \Big) \Big) \,.
\end{aligned}
$$

The soft-breaking terms are given by

$$
\begin{aligned}
&-\big((M^2_{\tilde{Q}_L})_{ij} \tilde{Q}^r_{Li} \tilde{Q}^{s*}_{Lj} \epsilon^{rs} + (M^2_{\tilde{L}})_{ij} \tilde{L}^r_i \tilde{L}^{s*}_j \epsilon^{rs} + (M^2_{\tilde{u}_R})_{ij} \tilde{u}_{Ri} \tilde{u}^*_{Rj} + (M^2_{\tilde{d}_R})_{ij} \tilde{d}_{Ri} \tilde{d}^*_{Rj} \\
&+(M^2_{\tilde{e}_R})_{ij} \tilde{e}_{Ri} \tilde{e}^*_{Rj} + \text{h.c.}\big) - m^2_{H_u} H^*_u H_u - m^2_{H_d} H^*_d H_d - \big(B\mu H^r_u H^s_d \epsilon^{rs} + \text{h.c.}\big) \\
&-\frac{1}{2} M_1 \tilde{B}^0 \tilde{B}^0 - \frac{1}{2} M_2 \tilde{W}^a \tilde{W}^a - \frac{1}{2} M_3 \tilde{G}^a \tilde{G}^a + \text{h.c} \\
&-\big(h^e_{ij} A^e \tilde{\nu}_i \tilde{e}^*_{Rj} H^{(-)}_d - h^e_{ij} A^e \tilde{e}_{Li} \tilde{e}^*_{Rj} v_d - h^e_{ij} A^e \tilde{e}_{Li} \tilde{e}^*_{Rj} h^{(0)}_d \\
&\quad + h^u_{ij} A^u \tilde{u}_{Li} \tilde{u}^*_{Rj} v_u + h^u_{ij} A^u \tilde{u}_{Li} \tilde{u}^*_{Rj} h^{(0)}_u - h^u_{ij} A^u \tilde{d}_{Li} \tilde{u}^*_{Rj} H^{(+)}_u \\
&\quad + h^d_{ij} A^d \tilde{u}_{Li} \tilde{d}^*_{Rj} H^{(-)}_d - h^d_{ij} A^d \tilde{d}_{Li} \tilde{d}^*_{Rj} v_d - h^d_{ij} A^d \tilde{d}_{Li} \tilde{d}^*_{Rj} h^{(0)}_d + \text{h.c.}\big).
\end{aligned}
$$

Chapter 6

Feynman Rules

From the Lagrangian density in the last chapter we can extract all possible interactions together with their corresponding Feynman rule. We refer the reader to appendix A.3 for the calculational details and just summarize the final results in momentum space here.

6.1 External Particles

From the standard Feynman rules for Dirac spinors we know which Dirac spinor to identify with which external particle. Combining this with table (1.2) we define

$$\begin{array}{rclrclrclrcl}
\mathfrak{u}_L^{(1)} &=& u, & \mathfrak{u}_L^{(2)} &=& -v, & \mathfrak{u}_R^{(1)} &=& \bar{v}, & \mathfrak{u}_R^{(2)} &=& \bar{u}, \\
\mathfrak{v}_L^{(1)} &=& v, & \mathfrak{v}_L^{(2)} &=& u, & \mathfrak{v}_R^{(1)} &=& \bar{u}, & \mathfrak{v}_R^{(2)} &=& -\bar{v}, \\
\bar{\mathfrak{u}}_L^{(1)} &=& \bar{u}, & \bar{\mathfrak{u}}_L^{(2)} &=& -\bar{v}, & \bar{\mathfrak{u}}_R^{(1)} &=& v, & \bar{\mathfrak{u}}_R^{(2)} &=& u, \\
\bar{\mathfrak{v}}_L^{(1)} &=& \bar{v}, & \bar{\mathfrak{v}}_L^{(2)} &=& \bar{u}, & \bar{\mathfrak{v}}_R^{(1)} &=& u, & \bar{\mathfrak{v}}_R^{(2)} &=& -v,
\end{array} \tag{6.1}$$

and find the list of incoming and outgoing external particles:

particle	incoming	outgoing
left-chiral fermion	$\mathfrak{u}_L^{(s)}$	$\bar{\mathfrak{u}}_L^{(s)}$
right-chiral fermion	$\mathfrak{u}_R^{(s)}$	$\bar{\mathfrak{u}}_R^{(s)}$
left-chiral anti-fermion	$\mathfrak{v}_L^{(s)}$	$\bar{\mathfrak{v}}_L^{(s)}$
right-chiral anti-fermion	$\mathfrak{v}_R^{(s)}$	$\bar{\mathfrak{v}}_R^{(s)}$
scalar boson	1	1
vector boson	ϵ_μ	ϵ_μ^*

We recall the completeness relations:

$$\begin{array}{rcl}
\sum_{s=1}^{2} \bar{\mathfrak{u}}_L^{(s)\dot{\alpha}}(p)\mathfrak{u}_L^{(s)\alpha}(p) &=& \bar{\sigma}^{\mu\dot{\alpha}\alpha}p_\mu, \\
\sum_{s=1}^{2} \bar{\mathfrak{v}}_L^{(s)\dot{\alpha}}(p)\mathfrak{v}_L^{(s)\alpha}(p) &=& \bar{\sigma}^{\mu\dot{\alpha}\alpha}p_\mu,
\end{array} \tag{6.2}$$

$$\begin{aligned}
\sum_{s=1}^{2} \mathfrak{u}_R^{(s)\dot\alpha}\bar{\mathfrak{u}}_R^{(s)\alpha} &= \bar\sigma^{\mu\dot\alpha\alpha}p_\mu, \\
\sum_{s=1}^{2} \mathfrak{v}_R^{(s)\dot\alpha}\bar{\mathfrak{v}}_R^{(s)\alpha} &= \bar\sigma^{\mu\dot\alpha\alpha}p_\mu, \\
\sum_{\mathrm{Pol}} \epsilon_\mu(p)\epsilon_\nu^*(p) &= -g_{\mu\nu}, \quad \text{for massless vector bosons,} \\
\sum_{\mathrm{Pol}} \epsilon_\mu(p)\epsilon_\nu^*(p) &= \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}\right), \quad \text{for massive vector bosons.}
\end{aligned} \tag{6.3}$$

6.2 Mass Eigenstates

The physical mass eigenstates of the Higgses can be obtained by diagonalizing their mass matrices. We just quote the results from [Mar97]:

$$\begin{aligned}
H^+ &= -H_d^{(-)*}\sin\beta + H_u^{(+)}\cos\beta, \\
H^- &= -H_d^{(-)}\sin\beta + H_u^{(+)*}\cos\beta, \\
A^0 &= \sqrt{2}\left(-\mathfrak{Im}(h_d^{(0)} + v_d)\sin\beta + \mathfrak{Im}(h_u^{(0)} + v_u)\cos\beta\right), \\
H^0 &= \sqrt{2}\left(\mathfrak{Re}h_d^{(0)}\cos\alpha + \mathfrak{Re}h_u^{(0)}\sin\alpha\right), \\
h^0 &= \sqrt{2}\left(-\mathfrak{Re}h_d^{(0)}\sin\alpha + \mathfrak{Re}h_u^{(0)}\cos\alpha\right),
\end{aligned} \tag{6.4}$$

with

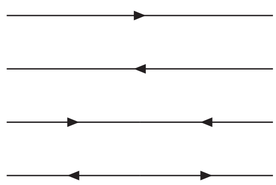
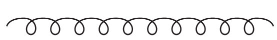
$$\begin{aligned}
\tan\beta &= \frac{v_u}{v_d}, \\
\frac{\cos 2\alpha}{\cos 2\beta} &= -\frac{m_{A^0}^2 - m_Z^2}{m_{H^0}^2 - m_{h^0}^2}, \\
m_{A^0}^2 &= \frac{2B\mu}{\sin 2\beta}, \\
m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2, \\
m_{h^0,H^0}^2 &= \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2\cos^2 2\beta}\right).
\end{aligned} \tag{6.5}$$

The mass eigenstates of the other particles are also mixtures of the interaction eigenstates (to be found e.g. in [HK85]) :

- the neutral gauginos and the neutral higgsinos mix to the neutralinos,
- the charged gauginos and the charged higgsinos mix to the charginos,
- the left- and right-chiral squarks mix,
- the left- and right-chiral sleptons mix.

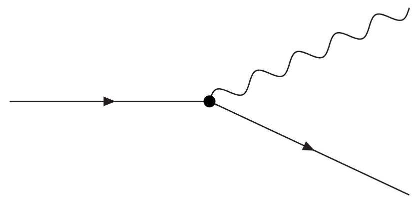
6.3 Propagators

f_L is a left-chiral fermion, f_R a right-chiral one, ϕ is a scalar boson and $\{A_\mu, W_\mu, G_\mu\}$ are the gauge bosons of the (broken) group $\{U(1), SU(2), SU(3)\}$ respectively. $\check{A}$ stands for the Fourier transform of A. The parameters ξ should be chosen to 0.

	$\begin{array}{c\|c\|l} \text{in} & \text{out} & \text{propagator} \\ \hline f_{L\alpha} & \bar{f}_{L\dot{\alpha}} & \check{\Delta}^{l}_{\alpha\dot{\alpha}}(p) = i\frac{p_\mu \sigma^\mu_{\alpha\dot{\alpha}}}{p^2-m^2} \\ f_R^{\dot{\alpha}} & \bar{f}_R^{\alpha} & \check{\Delta}^{r\dot{\alpha}\alpha}(p) = i\frac{p^\mu \bar{\sigma}^{\dot{\alpha}\alpha}_\mu}{p^2-m^2} \\ f_{L\alpha} & \bar{f}_R^{\beta} & \check{\Delta}^{lr}{}_{\alpha}{}^{\beta}(p) = \frac{-im}{p^2-m^2}\delta^\beta_\alpha \\ f_R^{\dot{\alpha}} & \bar{f}_{L\dot{\beta}} & \check{\Delta}^{rl}{}_{\dot{\beta}}{}^{\dot{\alpha}} = \frac{-im}{p^2-m^2}\delta^{\dot{\beta}}_{\dot{\alpha}} \end{array}$
	$\begin{array}{c\|c\|l} \text{in} & \text{out} & \text{propagator} \\ \hline \phi & \phi^* & \check{\Delta}(p) = \frac{i}{p^2-m^2} \end{array}$
	$\begin{array}{c\|c\|l} \text{in} & \text{out} & \text{propagator } \check{\Delta}_{\mu\nu}(p) \\ \hline A_\mu & A_\nu & \frac{-i}{p^2}\left(g_{\mu\nu} - (1-\xi_Y)\frac{p_\mu p_\nu}{p^2}\right) \end{array}$
	$\begin{array}{c\|c\|l} \text{in} & \text{out} & \text{propagator } \check{\Delta}_{\mu\nu}(p) \\ \hline W_\mu & W^*_\nu & \frac{-i}{p^2-m^2}\left(g_{\mu\nu} - (1-\xi_L)\frac{p_\mu p_\nu}{p^2}\right) \end{array}$
	$\begin{array}{c\|c\|l} \text{in} & \text{out} & \text{propagator } \check{\Delta}_{\mu\nu}(p) \\ \hline G^a_\mu & G^b_\nu & \frac{-i}{p^2}\left(g_{\mu\nu} - (1-\xi_C)\frac{p_\mu p_\nu}{p^2}\right)\delta^{ab} \end{array}$

6.4 Vertex Factors

As we see in appendix A.3.1 the vertex factors enter the matrix element multiplied by i except in the case of a pure gauge boson vertex. $\{g_Y, g_L, g_C\}$ are the coupling constants of $\{U(1)_Y, SU(2)_L, SU(3)_C\}$ respectively. The weak mixing angle θ_W is defined by $\tan\theta_W = \frac{g_Y}{g_L}$ and $c_W = \cos\theta_W, s_W = \sin\theta_W$ correspondingly. The electric unit of charge is given by $e = \frac{g_Y}{c_W} = \frac{g_L}{s_W}$. We note that the vertex factors are written down for the first generation only in case of no inter-generational mixing.



in	out	vertex factor
ν_e^α	$Z_\mu^0, \bar{\nu}_e^{\dot\alpha}$	$-\frac{1}{2}\left(g_Y s_W + g_L c_W\right)\sigma^\mu_{\alpha\dot\alpha}$
ν_e^α	$W_\mu^-, \bar{e}_L^{\dot\alpha}$	$-\frac{g_L}{\sqrt{2}}\sigma^\mu_{\alpha\dot\alpha}$
e_L^α	$W_\mu^+, \bar{\nu}_e^{\dot\alpha}$	$-\frac{g_L}{\sqrt{2}}\sigma^\mu_{\alpha\dot\alpha}$
e_L^α	$A_\mu, \bar{e}_L^{\dot\alpha}$	$g_Y c_W \sigma^\mu_{\alpha\dot\alpha}$
e_L^α	$Z_\mu^0, \bar{e}_L^{\dot\alpha}$	$-\frac{1}{2}\left(g_Y s_W - g_L c_W\right)\sigma^\mu_{\alpha\dot\alpha}$
$\bar{e}_R^\alpha$	$A_\mu, e_R^{\dot\alpha}$	$-g_Y c_W \sigma^\mu_{\alpha\dot\alpha}$
$\bar{e}_R^\alpha$	$Z_\mu^0, e_R^{\dot\alpha}$	$g_Y s_W \sigma^\mu_{\alpha\dot\alpha}$
u_L^α	$A_\mu, \bar{u}_L^{\dot\alpha}$	$-\frac{2}{3} g_Y c_W \sigma^\mu_{\alpha\dot\alpha}$
u_L^α	$Z_\mu^0, \bar{u}_L^{\dot\alpha}$	$-\frac{1}{2}\left(-g_L c_W + \frac{1}{3} g_Y s_W\right)\sigma^\mu_{\alpha\dot\alpha}$
u_L^α	$G_\mu^a, \bar{u}_L^{\dot\alpha}$	$-\frac{1}{2} g_C \lambda^a \sigma^\mu_{\alpha\dot\alpha}$
u_L^α	$W_\mu^-, \bar{d}_L^{\dot\alpha}$	$-\frac{1}{\sqrt{2}} g_L \sigma^\mu_{\alpha\dot\alpha}$
d_L^α	$W_\mu^+, \bar{u}_L^{\dot\alpha}$	$-\frac{1}{\sqrt{2}} g_L \sigma^\mu_{\alpha\dot\alpha}$
d_L^α	$A_\mu, \bar{d}_L^{\dot\alpha}$	$\frac{1}{3} g_Y c_W \sigma^\mu_{\alpha\dot\alpha}$
d_L^α	$Z_\mu^0, \bar{d}_L^{\dot\alpha}$	$\frac{1}{2}\left(g_L c_W + g\frac{1}{3} g_Y c_W\right)\sigma^\mu_{\alpha\dot\alpha}$
d_L^α	$G_\mu^a, \bar{d}_L^{\dot\alpha}$	$-\frac{1}{2} g_C \lambda^a \sigma^\mu_{\alpha\dot\alpha}$
$\bar{u}_R^\alpha$	$A_\mu, u_R^{\dot\alpha}$	$-\frac{2}{3} g_Y c_W \sigma^\mu_{\alpha\dot\alpha}$
$\bar{u}_R^\alpha$	$Z_\mu^0, u_R^{\dot\alpha}$	$\frac{2}{3} g_Y s_W \sigma^\mu_{\alpha\dot\alpha}$
$\bar{u}_R^\alpha$	$G_\mu^a, u_R^{\dot\alpha}$	$-\frac{1}{2} g_C \lambda^a \sigma^\mu_{\alpha\dot\alpha}$
$\bar{d}_R^\alpha$	$A_\mu, d_R^{\dot\alpha}$	$\frac{1}{3} g_Y c_W \sigma^\mu_{\alpha\dot\alpha}$
$\bar{d}_R^\alpha$	$Z_\mu^0, d_R^{\dot\alpha}$	$-\frac{1}{3} g_Y s_W \sigma^\mu_{\alpha\dot\alpha}$
$\bar{d}_R^\alpha$	$G_\mu^a, d_R^{\dot\alpha}$	$-\frac{1}{2} g_C \lambda^a G_\mu^a \sigma^\mu_{\alpha\dot\alpha}$
$\tilde{H}_u^{(+)\alpha}$	$A_\mu, \bar{\tilde{H}}_u^{(+)\dot\alpha}$	$-g_Y c_W \sigma^\mu_{\alpha\dot\alpha}$
$\tilde{H}_u^{(+)\alpha}$	$Z_\mu^0, \bar{\tilde{H}}_u^{(+)\dot\alpha}$	$-\frac{1}{2}\left(g_L c_W - g_Y s_W\right)\sigma^\mu_{\alpha\dot\alpha}$
$\tilde{H}_u^{(+)\alpha}$	$W_\mu^-, \bar{\tilde{H}}_u^{(0)\dot\alpha}$	$-\frac{1}{\sqrt{2}} g_L \sigma^\mu_{\alpha\dot\alpha}$
$\tilde{H}_u^{(0)\alpha}$	$W_\mu^+, \bar{\tilde{H}}_u^{(+)\dot\alpha}$	$-\frac{1}{\sqrt{2}} g_L \sigma^\mu_{\alpha\dot\alpha}$
$\tilde{H}_u^{(0)\alpha}$	$Z_\mu^0, \bar{\tilde{H}}_u^{(0)\dot\alpha}$	$\frac{1}{2}\left(g_Y s_W + g_L c_W\right)\sigma^\mu_{\alpha\dot\alpha}$

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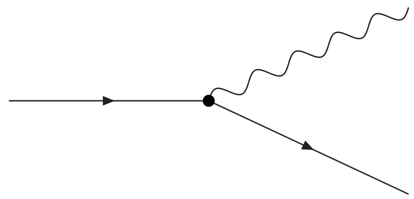
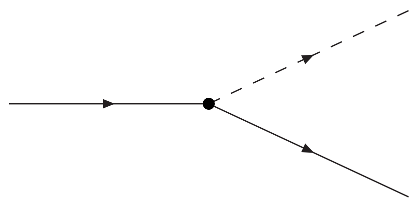
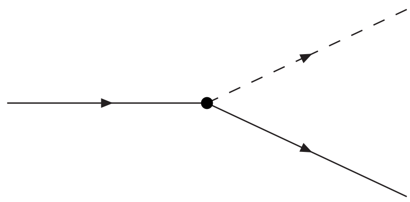


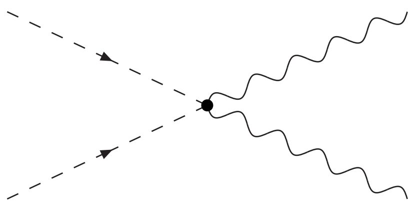
... continuation		
in	out	vertex factor
$\tilde{H}_d^{(0)\alpha}$	$Z^0_\mu, \bar{\tilde{H}}_d^{(0)\dot{\alpha}}$	$-\frac{1}{2}\left(g_Y s_W + g_L c_W\right)\sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{H}_d^{(0)\alpha}$	$W^-_\mu, \bar{\tilde{H}}_d^{(-)\dot{\alpha}}$	$-\frac{1}{\sqrt{2}} g_L \sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{H}_d^{(-)\alpha}$	$W^+_\mu, \bar{\tilde{H}}_d^{(0)\dot{\alpha}}$	$-\frac{1}{\sqrt{2}} g_L \sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{H}_d^{(-)\alpha}$	$A_\mu, \bar{\tilde{H}}_d^{(-)\dot{\alpha}}$	$g_Y c_W \sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{H}_d^{(-)\alpha}$	$Z^0_\mu, \bar{\tilde{H}}_d^{(-)\dot{\alpha}}$	$\frac{1}{2}\left(g_L c_W - g_Y s_W\right)\sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{W}^{-\alpha}$	$W^+_\mu, \bar{\tilde{W}}^{3\dot{\alpha}}$	$g_L \sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{W}^{3\alpha}$	$W^+_\mu, \bar{\tilde{W}}^{+\dot{\alpha}}$	$-g_L \sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{W}^{+\alpha}$	$A_\mu, \bar{\tilde{W}}^{+\dot{\alpha}}$	$i g_L s_W \sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{W}^{+\alpha}$	$Z^0_\mu, \bar{\tilde{W}}^{+\dot{\alpha}}$	$i g_L c_W \sigma^\mu_{\alpha\dot{\alpha}}$
$\tilde{G}^{a\alpha}$	$G^b_\mu, \bar{\tilde{G}}^{c\dot{\alpha}}$	$i g_C f^{abc} \sigma^\mu_{\alpha\dot{\alpha}}$



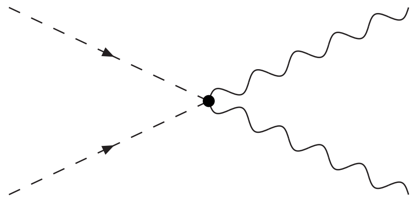
in	out	vertex factor
ν_e^α	$\tilde{\nu}_e^*, \tilde{W}^3_\alpha$	$-\frac{i}{\sqrt{2}} g_L$
ν_e^α	$\tilde{\nu}_e^*, \tilde{B}^0_\alpha$	$\frac{i}{\sqrt{2}} g_L$
e_L^α	$\tilde{\nu}_e^*, \tilde{W}^+_\alpha$	$-i g_L$
ν_e^α	$\tilde{e}_L^*, \tilde{W}^-_\alpha$	$-i g_L$
e_R^α	$\tilde{e}_R^*, \tilde{B}^0_\alpha$	$-i\sqrt{2} g_Y$
u_L^α	$\tilde{u}_L^*, \tilde{B}^0_\alpha$	$-\frac{i}{3\sqrt{2}} g_Y$
u_L^α	$\tilde{u}_L^*, \tilde{W}^3_\alpha$	$-\frac{i}{\sqrt{2}} g_L$
u_L^α	$\tilde{u}_L^*, \tilde{G}^a_\alpha$	$-\frac{i}{\sqrt{2}} g_C \lambda^a$
d_L^α	$\tilde{u}_L^*, \tilde{W}^+_\alpha$	$-i g_L$
u_L^α	$\tilde{d}_L^*, \tilde{W}^-_\alpha$	$-i g_L$
d_L^α	$\tilde{d}_L^*, tildeB^0_\alpha$	$-\frac{i}{3\sqrt{2}} g_Y$
		continued below ...



... continuation		
in	out	vertex factor
d_L^α	$\tilde{d}_L^*, \tilde{W}_\alpha^3$	$\frac{i}{\sqrt{2}} g_L$
d_L^α	$\tilde{d}_L^*, \tilde{G}_\alpha^a$	$-\frac{i}{\sqrt{2}} g_C \lambda^a$
$\tilde{u}_R$	$\bar{u}_R^\alpha, \tilde{B}_\alpha^0$	$-\frac{i2\sqrt{2}}{3} g_Y$
$\tilde{u}_R$	$\bar{u}_R^\alpha, \tilde{G}_\alpha^a$	$-\frac{i}{\sqrt{2}} g_C \lambda^a$
$\tilde{d}_R$	$\bar{d}_R^\alpha, \tilde{B}_\alpha^0$	$\frac{1}{3} g_Y$
$\tilde{d}_R$	$\bar{d}_R^\alpha, \tilde{G}_\alpha^a$	$-\frac{1}{2} g_C \lambda^a$
$\tilde{H}_u^{(+)\alpha}$	$H_u^{(+)*}, \tilde{B}_\alpha^0$	$-\frac{i}{\sqrt{2}} g_Y$
$\tilde{H}_u^{(+)\alpha}$	$H_u^{(+)*}, \tilde{W}_\alpha^3$	$-\frac{i}{\sqrt{2}} g_L$
$\tilde{H_u}^{(+)\alpha}$	$h^{(0)*}, \tilde{W}_\alpha^-$	$-i g_L$
$\tilde{H}^{(0)\alpha}$	$H_u^{(+)*}, \tilde{W}_\alpha^+$	$-i g_L$
$\tilde{H}^{(0)\alpha}$	$h_u^{(0)*}, \tilde{B}_\alpha^0$	$-\frac{i}{\sqrt{2}} g_Y$
$\tilde{H}^{(0)\alpha}$	$h_u^{(0)*}, \tilde{W}_\alpha^3$	$\frac{i}{\sqrt{2}} g_L$
$\tilde{H}_d^{(0)\alpha}$	$h_d^{(0)*}, \tilde{B}_\alpha^0$	$\frac{i}{\sqrt{2}} g_Y$
$\tilde{H}_d^{(0)\alpha}$	$h_d^{(0)*}, \tilde{W}_\alpha^3$	$-\frac{i}{\sqrt{2}} g_L$
$\tilde{H}_d^{(0)\alpha}$	$H_d^{(-)*}, \tilde{W}_\alpha^-$	$-i g_L$
$\tilde{H}_d^{(-)\alpha}$	$h_d^{(0)*}, \tilde{W}_\alpha^+$	$-i g_L$
$\tilde{H}_d^{(-)\alpha}$	$H_d^{(-)*}, \tilde{B}_\alpha^0$	$\frac{i}{\sqrt{2}} g_Y$
$\tilde{H}_d^{(-)\alpha}$	$H_d^{(-)*}, \tilde{W}_\alpha^3$	$\frac{i}{\sqrt{2}} g_L$



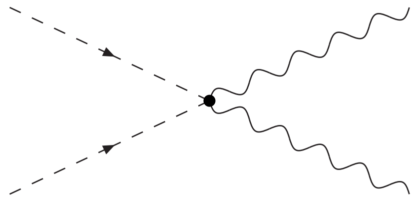
in	out	vertex factor
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$Z_\mu^0, Z^{0\mu}$	$\frac{1}{4}\left(g_Y s_W + g_L c_W\right)^2$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$W_\mu^+, W^{-\mu}$	$\frac{1}{2} g_L^2$
$\tilde{e}_L^*, \tilde{e}_L$	$W_\mu^+, W^{-\mu}$	$\frac{1}{2} g_L^2$
$\tilde{e}_L^*, \tilde{e}_L$	A_μ, A^μ	$g_Y^2 c_W^2$
		continued below ...



… continuation

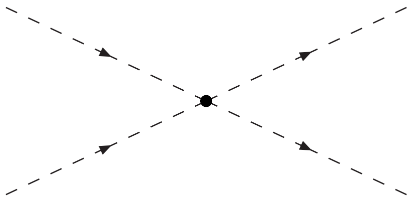
in	out	vertex factor
$\tilde{e}_L^*, \tilde{e}_L$	$Z^0_\mu, Z^{0\mu}$	$\frac{1}{4}\left(-g_Y s_W + g_L c_W\right)^2$
$\tilde{e}_L^*, \tilde{\nu}_e$	W^-_μ, A^μ	$-\frac{1}{\sqrt{2}} g_L g_Y c_W$
$\tilde{\nu}_e^*, \tilde{e}_L$	W^+_μ, A^μ	$-\frac{1}{\sqrt{2}} g_L g_Y c_W$
$\tilde{e}_L^*, \tilde{\nu}_e$	$W^-_\mu, Z^{0\mu}$	$\frac{1}{\sqrt{2}} g_L \left(g_Y s_W - g_L c_W\right)$
$\tilde{\nu}_e^*, \tilde{e}_L$	$W^+_\mu, Z^{0\mu}$	$\frac{1}{\sqrt{2}} g_L \left(g_Y s_W - g_L c_W\right)$
$\tilde{e}_L^*, \tilde{e}_L$	Z^0_μ, A^μ	$g_Y c_W (g_L c_W - g_Y s_W)$
$\tilde{e}_R^*, \tilde{e}_R$	A_μ, A^μ	$g_Y^2 c_W^2$
$\tilde{e}_R^*, \tilde{e}_R$	$Z^0_\mu, Z^{0\mu}$	$g_Y^2 s_W^2$
$\tilde{e}_R^*, \tilde{e}_R$	$Z^0_\mu A^\mu$	$-2 g_Y^2 c_W s_W$
$\tilde{u}_L^*, \tilde{u}_L$	A_μ, A^μ	$\frac{4}{9} g_Y^2 c_W^2$
$\tilde{u}_L^*, \tilde{u}_L$	$Z^0_\mu, Z^{0\mu}$	$\frac{1}{4}\left(\frac{1}{3} g_Y s_W - g_L c_W\right)^2$
$\tilde{d}_L^*, \tilde{d}_L$	$W^-_\mu, W^{+\mu}$	$\frac{1}{2} g_L^2$
$\tilde{u}_L^*, \tilde{u}_L$	$W^-_\mu, W^{+\mu}$	$\frac{1}{2} g_L^2$
$\tilde{u}_L^*, \tilde{u}_L$	$G^a_\mu, G^{b\mu}$	$\frac{1}{4} g_C^2 \lambda^{b\dagger} \lambda^a$
$\tilde{d}_L^*, \tilde{d}_L$	A_μ, A^μ	$-\frac{1}{3} g_Y^2 c_W^2$
$\tilde{d}_L^*, \tilde{d}_L$	$Z^0_\mu, Z^{0\mu}$	$\frac{1}{4}\left(\frac{1}{3} g_Y s_W + g_L c_W\right)$
$\tilde{d}_L^*, \tilde{d}_L$	$G^a_\mu, G^{b\mu}$	$\frac{1}{4} g_C^2 \lambda^{b\dagger} \lambda^a$
$\tilde{u}_L^*, \tilde{u}_L$	Z^0_μ, A^μ	$\frac{2}{3} g_Y c_W \left(g_L c_W - \frac{1}{3} g_Y s_W\right)$
$\tilde{u}_L^*, \tilde{d}_L$	W^+_μ, A^μ	$\frac{1}{3\sqrt{2}} g_L g_Y c_W$
$\tilde{d}_L^*, \tilde{u}_L$	W^-_μ, A^μ	$\frac{1}{3\sqrt{2}} g_L g_Y c_W$
$\tilde{u}_L^*, \tilde{d}_L$	$W^+_\mu, Z^{0\mu}$	$\frac{1}{\sqrt{2}} g_L \left(g_L c_W - \frac{1}{3} g_Y s_W\right)$
$\tilde{d}_L^*, \tilde{u}_L$	$W^-_\mu, Z^{0\mu}$	$\frac{1}{\sqrt{2}} g_L \left(g_L c_W - \frac{1}{3} g_Y s_W\right)$
$\tilde{u}_L^*, \tilde{d}_L$	W^+_μ, A^μ	$-\frac{1}{3\sqrt{2}} g_L g_Y c_W$
$\tilde{d}_L^*, \tilde{u}_L$	W^-_μ, A^μ	$-\frac{1}{3\sqrt{2}} g_L g_Y c_W$
$\tilde{d}_L^*, \tilde{d}_L$	Z^0_μ, A^μ	$g_Y c_W \left(\frac{1}{3} g_Y s_W + g_L c_W\right)$
$\tilde{u}_R^*, \tilde{u}_R$	A_μ, A^μ	$\frac{4}{9} g_Y^2 c_W^2$
$\tilde{u}_R^*, \tilde{u}_R$	Z^0_μ, A^μ	$-\frac{8}{9} g_Y^2 c_W s_W$
$\tilde{u}_R^*, \tilde{u}_R$	$Z^0_\mu, Z^{0\mu}$	$\frac{4}{9} g_Y^2 s_W^2$
$\tilde{u}_R^*, \tilde{u}_R$	$G^a_\mu, G^{b\mu}$	$\frac{1}{4} g_C^2 \lambda^{b\dagger} \lambda^a$
$\tilde{u}_R^*, \tilde{u}_R$	G^a_μ, A^μ	$\frac{2}{3} g_C g_Y c_W \lambda^a$
$\tilde{u}_R^*, \tilde{u}_R$	$G^a_\mu, Z^{0\mu}$	$-\frac{2}{3} g_C g_Y s_W \lambda^a$
$\tilde{d}_R^*, \tilde{d}_R$	A_μ, A^μ	$\frac{1}{9} g_Y^2 c_W^2$
$\tilde{d}_R^*, \tilde{d}_R$	Z^0_μ, A^μ	$-\frac{2}{9} g_Y^2 c_W s_W$
$\tilde{d}_R^*, \tilde{d}_R$	$Z^0_\mu, Z^{0\mu}$	$\frac{1}{9} g_Y^2 s_W^2$

continued below …



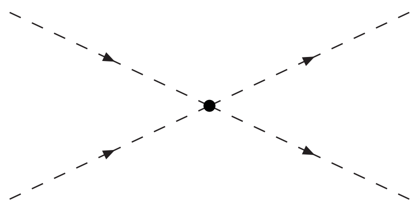
… continuation

in	out	vertex factor
$\tilde{d}_R^*, \tilde{d}_R$	$G^a_\mu, G^{b\mu}$	$\frac{1}{4} g_C^2 \lambda^{b\dagger} \lambda^a$
$\tilde{d}_R^*, \tilde{d}_R$	G^a_μ, A^μ	$-\frac{1}{3} g_C g_Y c_W \lambda^a$
$\tilde{d}_R^*, \tilde{d}_R$	$G^a_\mu, Z^{0\mu}$	$\frac{1}{3} g_C g_Y s_W \lambda^a$
$H_u^{(+)*}, H_u^{(+)}$	A_μ, A^μ	$g_Y^2 c_W^2$
$H_u^{(+)*}, H_u^{(+)}$	$Z^0_\mu, Z^{0\mu}$	$\frac{1}{4} \left(g_Y s_W - g_L c_W\right)^2$
$h_u^{(0)*}, h_u^{(0)}$	$W^-_\mu, W^{+\mu}$	$\frac{1}{2} g_L^2$
$H_u^{(+)*}, H_u^{(+)}$	$W^-_\mu, W^{+\mu}$	$\frac{1}{2} g_L^2$
$h_u^{(0)*}, h_u^{(0)}$	$Z^0_\mu, Z^{0\mu}$	$\frac{1}{4} \left(g_Y s_W + g_L c_W\right)^2$
$H_u^{(+)*}, H_u^{(+)}$	Z^0_μ, A^μ	$g_L c_W \left(g_L c_W - g_Y s_W\right)$
$H_u^{(+)*}, h_u^{(0)}$	W^+_μ, A^μ	$\frac{1}{\sqrt{2}} g_L g_Y c_W$
$h_u^{(0)*}, H_u^{(+)}$	W^-_μ, A^μ	$\frac{1}{\sqrt{2}} g_L g_Y c_W$
$H_u^{(+)*}, h_u^{(0)}$	$Z^0_\mu, W^{+\mu}$	$-\frac{1}{\sqrt{2}} g_L g_Y s_W$
$h_u^{(0)*}, H_u^{(+)}$	$Z^0_\mu, W^{-\mu}$	$-\frac{1}{\sqrt{2}} g_L g_Y s_W$
$h_d^{(0)*}, h_d^{(0)}$	$Z^0_\mu, Z^{0\mu}$	$\frac{1}{2} g_L^2$
$H_d^{(-)*}, H_d^{(-)}$	$W^-_\mu, W^{+\mu}$	$\frac{1}{2} g_L^2$
$h_d^{(0)*}, h_d^{(0)}$	$W^-_\mu, W^{+\mu}$	$\frac{1}{2} g_L^2$
$H_d^{(-)*}, H_d^{(-)}$	A_μ, A^μ	$g_Y^2 c_W^2$
$H_d^{(-)*}, H_d^{(-)}$	$Z^0_\mu, Z^{0\mu}$	$\frac{1}{4} \left(g_L c_w - g_Y s_W\right)^2$
$h_d^{(0)*}, H_d^{(-)}$	W^+_μ, A^μ	$-\frac{1}{\sqrt{2}} g_L g_Y c_W$
$H_d^{(-)*}, h_d^{(0)}$	W^-_μ, A^μ	$-\frac{1}{\sqrt{2}} g_L g_Y c_W$
$h_d^{(0)*}, H_d^{(-)}$	$Z^0_\mu, W^{+\mu}$	$\frac{1}{2} g_L g_Y s_W$
$H_d^{(-)*}, h_d^{(0)}$	$Z^0_\mu, W^{-\mu}$	$\frac{1}{2} g_L g_Y s_W$
$H_d^{(-)*}, H_d^{(-)}$	Z^0_μ, A^μ	$g_Y c_W \left(g_L c_W - g_Y s_W\right)$

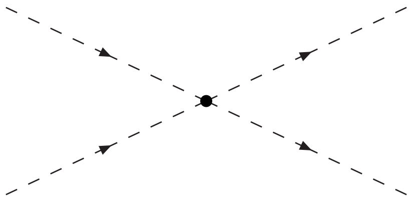


in	out	vertex factor
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$\tilde{\nu}_e^*, \tilde{\nu}_e$	$-\frac{1}{8}\left(g_Y^2 + g_L^2\right)$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$\tilde{e}_L^*, \tilde{e}_L$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$\tilde{e}_L^*, \tilde{e}_L$	$\tilde{e}_L^*, \tilde{e}_L$	$-\frac{1}{8}\left(g_Y^2 + g_L^2\right)$
$\tilde{e}_R^*, \tilde{e}_R$	$\tilde{e}_R^*, \tilde{e}_R$	$-\frac{1}{2}g_Y^2$
$\tilde{u}_L^*, \tilde{u}_L$	$\tilde{u}_L^*, \tilde{u}_L$	$-\frac{1}{2}\left(\frac{1}{36}g_Y^2 + \frac{1}{4}g_L^2 + \frac{1}{3}g_C^2\right)$
$\tilde{u}_L^*, \tilde{u}_L$	$\tilde{d}_L^*, \tilde{d}_L$	$-\frac{1}{2}\left(\frac{1}{18}g_Y^2 + \frac{1}{2}g_L^2 + \frac{1}{3}g_C^2\right)$
$\tilde{d}_L^*, \tilde{d}_L$	$\tilde{d}_L^*, \tilde{d}_L$	$-\frac{1}{2}\left(\frac{1}{36}g_Y^2 + \frac{1}{4}g_L^2 + \frac{1}{3}g_C^2\right)$
$\tilde{u}_R^*, \tilde{u}_R$	$\tilde{u}_R^*, \tilde{u}_R$	$-\frac{1}{3}\left(\frac{2}{3}g_Y^2 + \frac{1}{2}g_C^2\right)$
$\tilde{d}_R^*, \tilde{d}_R$	$\tilde{d}_R^*, \tilde{d}_R$	$-\frac{1}{6}\left(\frac{1}{3}g_Y^2 + g_C^2\right)$
$H_u^{(+)*}, H_u^{(+)}$	$H_u^{(+)*}, H_u^{(+)}$	$-\frac{1}{8}\left(g_Y^2 + g_L^2\right)$
$H_u^{(+)*}, H_u^{(+)}$	$h_u^{(0)*}, h_u^{(0)}$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$h_u^{(0)*}, h_u^{(0)}$	$h_u^{(0)*}, h_u^{(0)}$	$-\frac{1}{8}\left(g_Y^2 + g_L^2\right)$
$h_d^{(0)*}, h_d^{(0)}$	$h_d^{(0)*}, h_d^{(0)}$	$-\frac{1}{8}\left(g_Y^2 + g_L^2\right)$
$h_d^{(0)*}, h_d^{(0)}$	$H_d^{(-)*}, H_d^{(-)}$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$H_d^{(-)*}, H_d^{(-)}$	$H_d^{(-)*}, H_u^{(-)}$	$-\frac{1}{8}\left(g_Y^2 + g_L^2\right)$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$\tilde{e}_R^*, \tilde{e}_R$	$\frac{1}{2}g_Y^2$
$\tilde{e}_L^*, \tilde{e}_L$	$\tilde{e}_R^*, \tilde{e}_R$	$\frac{1}{2}g_Y^2$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$\tilde{u}_L^*, \tilde{u}_L$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 - g_L^2\right)$
$\tilde{e}_L^*, \tilde{e}_L$	$\tilde{d}_L^*, \tilde{d}_L$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 - g_L^2\right)$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$\tilde{d}_L^*, \tilde{d}_L$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 + g_L^2\right)$
$\tilde{e}_L^*, \tilde{e}_L$	$\tilde{u}_L^*, \tilde{u}_L$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 + g_L^2\right)$
$\tilde{\nu}_e^*, \tilde{e}_L$	$\tilde{d}_L^*, \tilde{u}_L$	$-\frac{1}{2}g_L^2$
$\tilde{e}_L^*, \tilde{\nu}_e$	$\tilde{u}_L^*, \tilde{d}_L$	$-\frac{1}{2}g_L^2$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$\tilde{u}_R^*, \tilde{u}_R$	$-\frac{1}{3}g_Y^2$
$\tilde{e}_L^*, \tilde{e}_L$	$\tilde{u}_R^*, \tilde{u}_R$	$-\frac{1}{3}g_Y^2$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$\tilde{d}_R^*, \tilde{d}_R$	$\frac{1}{6}g_Y^2$
$\tilde{e}_L^*, \tilde{e}_L$	$\tilde{d}_R^*, \tilde{d}_R$	$\frac{1}{6}g_Y^2$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$H_u^{(+)*}, H_u^{(+)}$	$\frac{1}{4}\left(g_Y^2 - g_L^2\right)$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$h_u^{(0)*}, h_u^{(0)}$	$\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$\tilde{e}_L^*, \tilde{e}_L$	$H_u^{(+)*}, H_u^{(+)}$	$\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$\tilde{e}_L^*, \tilde{e}_L$	$h_u^{(0)*}, h_u^{(0)}$	$\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$\tilde{\nu}_e^*, \tilde{e}_L$	$h_u^{(0)*}, H_u^{(+)}$	$-\frac{1}{2}g_L^2$
$\tilde{e}_L^*, \tilde{\nu}_e$	$H_u^{(+)*}, h_u^{(0)}$	$-\frac{1}{2}g_L^2$
$\tilde{e}_L^*, \tilde{e}_L$	$H_d^{(-)*}, H_d^{(-)}$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$\tilde{e}_L^*, \tilde{e}_L$	$h_d^{(0)*}, h_d^{(0)}$	$-\frac{1}{4}\left(g_Y^2 - g_L^2\right)$

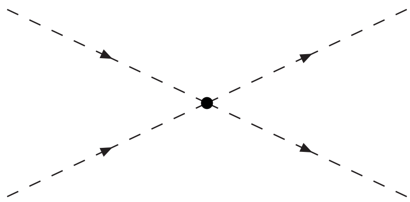
continued below ...



… continuation		
in	out	vertex factor
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$H_d^{(-)*}, H_d^{(-)}$	$-\frac{1}{4}\left(g_Y^2 - g_L^2\right)$
$\tilde{\nu}_e^*, \tilde{\nu}_e$	$h_d^{(0)*}, h_d^{(0)}$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$\tilde{e}_L^*, \tilde{\nu}_e$	$h_d^{(0)*}, H_d^{(-)}$	$-\frac{1}{2}g_L^2$
$\tilde{\nu}_e^*, \tilde{e}_L$	$H_d^{(-)*}, h_d^{(0)}$	$-\frac{1}{2}g_L^2$
$\tilde{u}_L^*, \tilde{u}_L$	$\tilde{e}_R^*, \tilde{e}_R$	$-\frac{1}{6}g_Y^2$
$\tilde{d}_L^*, \tilde{d}_L$	$\tilde{e}_R^*, \tilde{e}_R$	$-\frac{1}{6}g_Y^2$
$\tilde{u}_R^*, \tilde{u}_R$	$\tilde{e}_R^*, \tilde{e}_R$	$\frac{2}{3}g_Y^2$
$\tilde{d}_R^*, \tilde{d}_R$	$\tilde{e}_R^*, \tilde{e}_R$	$\frac{1}{3}g_Y^2$
$\tilde{e}_R^*, \tilde{e}_R$	$H_u^{(+)*}, H_u^{(+)}$	$-\frac{1}{2}g_L^2$
$\tilde{e}_R^*, \tilde{e}_R$	$h_u^{(0)*}, h_u^{(0)}$	$-\frac{1}{2}g_L^2$
$\tilde{e}_R^*, \tilde{e}_R$	$H_d^{(-)*}, H_d^{(-)}$	$\frac{1}{2}g_L^2$
$\tilde{e}_R^*, \tilde{e}_R$	$h_d^{(0)*}, h_d^{(0)}$	$\frac{1}{2}g_L^2$
$\tilde{u}_L^*, \tilde{u}_L$	$\tilde{u}_R^*, \tilde{u}_R$	$\frac{1}{3}\left(\frac{1}{3}g_Y^2 - g_C^2\right)$
$\tilde{d}_L^*, \tilde{d}_L$	$\tilde{u}_R^*, \tilde{u}_R$	$\frac{1}{3}\left(\frac{1}{3}g_Y^2 - g_C^2\right)$
$\tilde{u}_L^*, \tilde{u}_L$	$\tilde{d}_R^*, \tilde{d}_R$	$-\frac{1}{3}\left(\frac{1}{6}g_Y^2 + g_C^2\right)$
$\tilde{d}_L^*, \tilde{d}_L$	$\tilde{d}_R^*, \tilde{d}_R$	$-\frac{1}{3}\left(\frac{1}{6}g_Y^2 + g_C^2\right)$
$\tilde{u}_L^*, \tilde{u}_L$	$H_u^{(+)*}, H_u^{(+)}$	$-\frac{1}{4}\left(\frac{1}{3}g_Y^2 + g_L^2\right)$
$\tilde{u}_L^*, \tilde{u}_L$	$h_u^{(0)*}, h_u^{(0)}$	$-\frac{1}{4}\left(\frac{1}{3}g_Y^2 - g_L^2\right)$
$\tilde{d}_L^*, \tilde{d}_L$	$H_u^{(+)*}, H_u^{(+)}$	$-\frac{1}{4}\left(\frac{1}{3}g_Y^2 - g_L^2\right)$
$\tilde{d}_L^*, \tilde{d}_L$	$h_u^{(0)*}, h_u^{(0)}$	$-\frac{1}{4}\left(\frac{1}{3}g_Y^2 + g_L^2\right)$
$\tilde{u}_L^*, \tilde{d}_L$	$h_u^{(0)*}, H_u^{(+)}$	$-\frac{1}{2}g_L^2$
$\tilde{d}_L^*, \tilde{u}_L$	$H_u^{(+)*}, h_u^{(0)}$	$-\frac{1}{2}g_L^2$
$\tilde{d}_L^*, \tilde{d}_L$	$H_d^{(-)*}, H_d^{(-)}$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 - g_L^2\right)$
$\tilde{d}_L^*, \tilde{d}_L$	$h_d^{(0)*}, h_d^{(0)}$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 + g_L^2\right)$
$\tilde{u}_L^*, \tilde{u}_L$	$H_d^{(+)*}, H_d^{(+)}$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 + g_L^2\right)$
$\tilde{u}_L^*, \tilde{u}_L$	$h_d^{(0)*}, h_d^{(0)}$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 - g_L^2\right)$
$\tilde{d}_L^*, \tilde{u}_L$	$h_d^{(0)*}, H_d^{(-)}$	$-\frac{1}{2}g_L^2$
$\tilde{u}_L^*, \tilde{d}_L$	$H_d^{(-)*}, h_d^{(0)}$	$-\frac{1}{2}g_L^2$
$\tilde{u}_R^*, \tilde{u}_R$	$\tilde{d}_R^*, \tilde{d}_R$	$\frac{1}{3}\left(\frac{2}{3}g_Y^2 - g_C^2\right)$
$\tilde{u}_R^*, \tilde{u}_R$	$H_u^{(+)*}, H_u^{(+)}$	$\frac{1}{3}g_Y^2$
$\tilde{u}_R^*, \tilde{u}_R$	$h_u^{(0)*}, h_u^{(0)}$	$\frac{1}{3}g_Y^2$
$\tilde{u}_R^*, \tilde{u}_R$	$H_d^{(-)*}, H_d^{(-)}$	$-\frac{1}{3}g_Y^2$
$\tilde{u}_R^*, \tilde{u}_R$	$h_d^{(0)*}, h_d^{(0)}$	$-\frac{1}{3}g_Y^2$
		continued below …

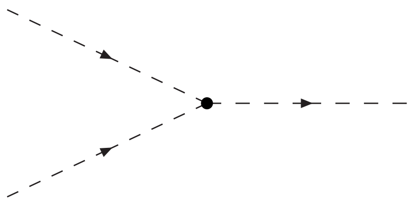


… continuation		
in	out	vertex factor
$\tilde{d}^*_R, \tilde{d}_R$	$H_u^{(+)*}, H_u^{(+)}$	$-\frac{1}{6}g_Y^2$
$\tilde{d}^*_R, \tilde{d}_R$	$h_u^{(0)*}, h_u^{(0)}$	$-\frac{1}{6}g_Y^2$
$\tilde{d}^*_R, \tilde{d}_R$	$H_d^{(-)*}, H_d^{(-)}$	$\frac{1}{6}g_Y^2$
$\tilde{d}^*_R, \tilde{d}_R$	$h_d^{(0)*}, h_d^{(0)}$	$\frac{1}{6}g_Y^2$
$H_u^{(+)*}, H_u^{(+)}$	$H_d^{(-)*}, H_d^{(-)}$	$\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$h_u^{(0)*}, h_u^{(0)}$	$h_d^{(0)*}, h_d^{(0)}$	$\frac{1}{4}\left(g_Y^2 + g_L^2\right)$
$H_u^{(+)*}, H_u^{(+)}$	$h_d^{(0)*}, h_d^{(0)}$	$\frac{1}{4}\left(g_Y^2 - g_L^2\right)$
$h_u^{(0)*}, h_u^{(0)}$	$H_d^{(-)*}, H_d^{(-)}$	$\frac{1}{4}\left(g_Y^2 - g_L^2\right)$
$H_u^{(+)*}, h_u^{(0)}$	$H_d^{(-)*}, h_d^{(0)}$	$-\frac{1}{2}g_L^2$
in addition to the couplings above there are those coming from the superpotential which mix among the different generations $i, j, k, l \in \{e, \mu\, \tau\}$ or $\{u, c, t\}$ resp.		
$\tilde{\nu}_l^* \tilde{\nu}_j$	$H_d^{(-)*}, H_d^{(-)}$	$-h^e_{ij} h^{e*}_{il}$
$\tilde{e}^*_{Ll}, \tilde{\nu}_j$	$h_d^{(0)*}, H_d^{(-)}$	$h^e_{ij} h^{e*}_{il}$
$\tilde{\nu}_l^* \tilde{e}_{Lj}$	$H_d^{(-)*}, h_d^{(0)}$	$h^e_{ij} h^{e*}_{il}$
$\tilde{e}^*_{Ll}, \tilde{e}_{Lj}$	$h_d^{(0)*}, h_d^{(0)}$	$-h^e_{ij} h^{e*}_{il}$
$\tilde{e}^*_{Ri}, \tilde{e}_{Rl}$	$h_d^{(0)*}, h_d^{(0)}$	$-h^e_{ij} h^{e*}_{jl}$
$\tilde{e}^*_{Ri}, \tilde{e}_{Rl}$	$H_d^{(-)*}, H_d^{(-)}$	$-h^e_{ij} h^{e*}_{jl}$
$\tilde{e}^*_{Ri}, \tilde{e}_{Rk}$	$\tilde{\nu}_l^*, \tilde{\nu}_j$	$-h^e_{ij} h^{e*}_{kl}$
$\tilde{e}^*_{Ri}, \tilde{e}_{Rk}$	$\tilde{e}^*_{Ll}, \tilde{e}_{Lj}$	$-h^e_{ij} h^{e*}_{kl}$
$\tilde{\nu}_k^*, \tilde{\nu}_j$	$h_u^{(0)*}, h_u^{(0)}$	$-h^u_{ij} h^{u*}_{ik}$
$\tilde{e}^*_{Lk}, \tilde{\nu}_j$	$H_u^{(+)*}, h_u^{(0)}$	$h^u_{ij} h^{u*}_{ik}$
$\tilde{e}^*_{Lj}, \tilde{\nu}_k$	$H_u^{(+)*}, h_u^{(0)}$	$h^u_{ij} h^{u*}_{ik}$
$\tilde{e}^*_{Lk}, \tilde{e}_{Lj}$	$H_u^{(+)*}, H_u^{(+)}$	$-h^u_{ij} h^{u*}_{ik}$
$\tilde{u}^*_{Ri}, \tilde{u}_{Rk}$	$H_u^{(+)*}, H_u^{(+)}$	$-h^u_{ij} h^{u*}_{kj}$
$\tilde{u}^*_{Ri}, \tilde{u}_{Rk}$	$\tilde{u}^*_{Ll}, \tilde{u}_{Lj}$	$-h^u_{ij} h^{u*}_{kl}$
$\tilde{u}^*_{Ri}, \tilde{u}_{Rk}$	$\tilde{d}^*_{Ll}, \tilde{d}_{Lj}$	$-h^u_{ij} h^{u*}_{kl}$
$\tilde{u}^*_{Lk}, \tilde{u}_{Lj}$	$H_d^{(-)*}, H_d^{(-)}$	$-h^d_{ij} h^{d*}_{ik}$
$\tilde{d}^*_{Lk}, \tilde{u}_{Lj}$	$h_d^{(0)*}, H_d^{(-)}$	$h^d_{ij} h^{d*}_{ik}$
$\tilde{u}^*_{Lk}, \tilde{d}_{Lj}$	$H_d^{(-)*}, h_d^{(0)}$	$h^d_{ij} h^{d*}_{ik}$
$\tilde{d}^*_{Lk}, \tilde{d}_{Lj}$	$h_d^{(0)*}, h_d^{(0)}$	$-h^d_{ij} h^{d*}_{ik}$
		continued below …



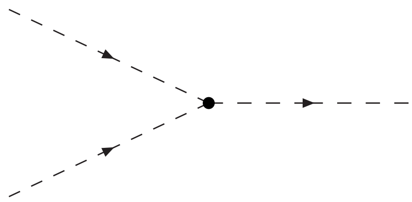
… continuation

in	out	vertex factor
$\tilde{d}^*_{Ri}, \tilde{d}_{Rk}$	$h_d^{(0)*}, h_d^{(0)}$	$-h^d_{ij} h^{d*}_{kj}$
$\tilde{d}^*_{Ri}, \tilde{d}_{Rk}$	$H_d^{(-)*}, H_d^{(-)}$	$-h^d_{ij} h^{d*}_{kj}$
$\tilde{d}^*_{Ri}, \tilde{d}_{Rk}$	$\tilde{u}^*_{Lk}, \tilde{u}_{Lj}$	$-h^d_{ij} h^{d*}_{kl}$
$\tilde{d}^*_{Ri}, \tilde{d}_{Rk}$	$\tilde{d}^*_{Ll}, \tilde{d}_{Lj}$	$-h^d_{ij} h^{d*}_{kl}$
$\tilde{e}^*_{Ri}, \tilde{\nu}_j$	$\tilde{u}^*_{Ll}, \tilde{d}_{Rk}$	$-h^e_{ij} h^{d*}_{kl}$
$\tilde{e}^*_{Ri}, \tilde{e}_{Lj}$	$\tilde{d}^*_{Ll}, \tilde{d}_{Rk}$	$-h^e_{ij} h^{d*}_{kl}$
$\tilde{u}^*_{Ri}, \tilde{d}_{Rk}$	$h_d^{(0)*}, H_u^{(+)}$	$-h^u_{ij} h^{d*}_{kl}$
$\tilde{u}^*_{Ri}, \tilde{d}_{Rk}$	$H_d^{(-)*}, h_u^{(0)}$	$-h^u_{ij} h^{d*}_{kl}$

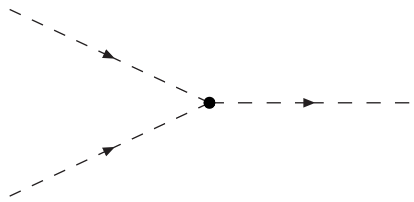


in	out	vertex factor
$h_u^{(0)}, H_u^{(+)}$	$H_u^{(+)*}$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right) v_u^*$
$h_u^{(0)}, h_u^{(0)}$	$h_u^{(0)*}$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right) v_u^*$
$h_d^{(0)}, H_d^{(-)}$	$H_d^{(-)}$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right) v_d^*$
$h_d^{(0)}, h_d^{(0)}$	$h_d^{(0)*}$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right) v_d^*$
$\tilde{\nu}_e, h_u^{(0)}$	$\tilde{\nu}_e^*$	$\frac{1}{4}\left(g_Y^2 + g_L^2\right) v_u^*$
$\tilde{e}_L, h_u^{(0)}$	$\tilde{e}_L^*$	$\frac{1}{4}\left(g_Y^2 - g_L^2\right) v_u^*$
$\tilde{e}_L, H_u^{(+)}$	$\tilde{\nu}_e^*$	$-\frac{1}{2} g_L^2$
$\tilde{e}_L, h_d^{(0)}$	$\tilde{e}_L^*$	$-\frac{1}{4}\left(g_Y^2 - g_L^2\right) v_d^*$
$\tilde{\nu}_e, h_d^{(0)}$	$\tilde{\nu}_e^*$	$-\frac{1}{4}\left(g_Y^2 + g_L^2\right) v_d^*$
$\tilde{\nu}_e, H_d^{(-)}$	$\tilde{e}_L^*$	$-\frac{1}{2} g_L^2 v_d^*$
$\tilde{e}_R, h_u^{(0)}$	$\tilde{e}_R^*$	$-\frac{1}{2} g_Y^2 v_u^*$
$\tilde{e}_R, h_d^{(0)}$	$\tilde{e}_R^*$	$\frac{1}{2} g_Y^2 v_d^*$
$\tilde{u}_L, h_u^{(0)}$	$\tilde{u}_L^*$	$-\frac{1}{4}\left(\frac{1}{3} g_Y^2 - g_L^2\right) v_u^*$
$\tilde{d}_L, h_u^{(0)}$	$\tilde{d}_L^*$	$-\frac{1}{4}\left(\frac{1}{3} g_Y^2 + g_L^2\right) v_u^*$
$\tilde{d}_L, H_u^{(+)}$	$\tilde{d}_L^*$	$-\frac{1}{2} g_L^2 v_u^*$

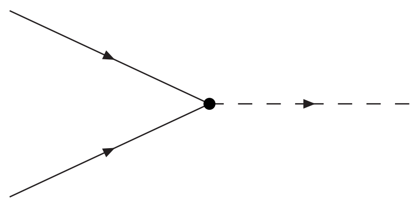
continued below …



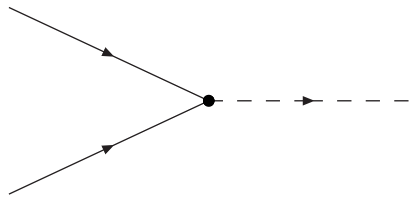
... continuation		
in	out	vertex factor
$\tilde{d}_L, h_d^{(0)}$	$\tilde{d}_L^*$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 + g_L^2\right) v_d^*$
$\tilde{u}_L, h_d^{(0)}$	$\tilde{u}_L^*$	$\frac{1}{4}\left(\frac{1}{3}g_Y^2 - g_L^2\right) v_d^*$
$\tilde{u}_L, H_d^{(-)}$	$\tilde{d}_L^*$	$-\frac{1}{2}g_L^2 v_d^*$
$\tilde{u}_R, h_u^{(0)}$	$\tilde{u}_R^*$	$\frac{1}{3}g_Y^2 v_u^*$
$\tilde{u}_R, h_d^{(0)}$	$\tilde{u}_R^*$	$-\frac{1}{3}g_Y^2 v_d^*$
$\tilde{d}_R, h_u^{(0)}$	$\tilde{d}_R^*$	$-\frac{1}{6}g_Y^2 v_u^*$
$\tilde{d}_R, h_d^{(0)}$	$\tilde{d}_R^*$	$\frac{1}{6}g_Y^2 v_d^*$
$h_u^{(0)}, h_d^{(0)}$	$h_u^{(0)*}$	$\frac{1}{4}\left(g_Y^2 + g_L^2\right) v_d^*$
$h_u^{(0)}, h_d^{(0)}$	$h_d^{(0)*}$	$\frac{1}{4}\left(g_Y^2 + g_L^2\right) v_u^*$
$h_d^{(0)}, H_u^{(+)}$	$H_u^{(+)*}$	$\frac{1}{4}\left(g_Y^2 - g_L^2\right) v_d^*$
$h_u^{(0)}, H_d^{(-)}$	$H_d^{(-)*}$	$\frac{1}{4}\left(g_Y^2 - g_L^2\right) v_u^*$
$H_u^{(+)}, H_d^{(-)}$	$h_d^{(0)*}$	$-\frac{1}{2}g_L^2 v_u^*$
$H_u^{(+)}, H_d^{(-)}$	$h_u^{(0)*}$	$-\frac{1}{2}g_L^2 v_d^*$
in addition to the couplings above there are those coming from the superpotential which mix among the different generations $i, j, k, l \in \{e, \mu\, \tau\}$ or $\{u, c, t\}$ resp.		
$\tilde{e}_{Ri}^*, \tilde{\nu}_j$	$H_d^{(-)}$	$-(A^e + \frac{1}{2})h_{ij}^e$
$\tilde{\nu}_j, H_d^{(-)}$	$\tilde{e}_{Ll}^*$	$\frac{1}{2}h_{ij}^e h_{il}^{e*} v_d^*$
$\tilde{e}_{Lj}, h_d^{(0)}$	$\tilde{e}_{Ll}^*$	$-\frac{1}{2}h_{ij}^e h_{il}^{e*} v_d^*$
$\tilde{e}_{Rl}, h_d^{(0)}$	$\tilde{e}_{Ri}^*$	$-\frac{1}{2}h_{ij}^e h_{jl}^{e*} v_d^*$
$\tilde{\nu}_j, h_u^{(0)}$	$\tilde{\nu}_k^*$	$-\frac{1}{2}h_{ij}^u h_{ik}^{u*} v_u^*$
$\tilde{e}_{Lj}, H_u^{(+)}$	$\tilde{\nu}_k^*$	$\frac{1}{2}h_{ij}^u h_{ik}^{u*} v_u^*$
$\tilde{u}_{Rk}, h_u^{(0)}$	$\tilde{u}_{Ri}^*$	$-\frac{1}{2}h_{ij}^u h_{kj}^{u*} v_u^*$
$\tilde{u}_{Lj}, H_d^{(-)}$	$\tilde{d}_{Lk}^*$	$\frac{1}{2}h_{ij}^d h_{ik}^{d*} v_d^*$
$\tilde{d}_{Lj}, h_d^{(0)}$	$\tilde{d}_{Lk}^*$	$-\frac{1}{2}h_{ij}^d h_{ik}^{d*} v_d^*$
$\tilde{d}_{Rk}, h_d^{(0)}$	$\tilde{d}_{Ri}^*$	$-\frac{1}{2}h_{ij}^d h_{kj}^{d*} v_d^*$
$\tilde{e}_{Ri}, H_u^{(+)}$	$\tilde{\nu}_j^*$	$-\frac{1}{2}h_{ij}^{e*} \mu^*$
$\tilde{e}_{Ri}, h_u^{(0)}$	$\tilde{e}_{Lj}^*$	$-\frac{1}{2}h_{ij}^{e*} \mu^*$
$\tilde{d}_{Ri}, H_u^{(+)}$	$\tilde{u}_{Lj}^*$	$-(A^{d*} + \frac{1}{2})h_{ij}^{d*} \mu^*$
$\tilde{d}_{Ri}, h_u^{(0)}$	$\tilde{d}_{Lj}^*$	$-\frac{1}{2}h_{ij}^{e*} \mu^*$
$\tilde{d}_{Rk}, H_u^{(+)}$	$\tilde{u}_{Ri}^*$	$-\frac{1}{2}h_{ij}^u h_{kl}^{d*} v_d^*$
continued below ...		



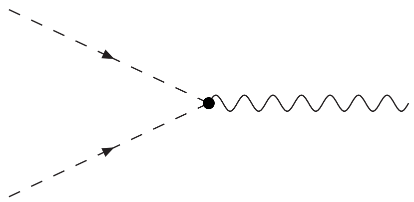
... continuation		
in	out	vertex factor
$\tilde{u}_{Ri}, H_d^{(-)}$	$\tilde{d}^*_{Rk}$	$-\frac{1}{2}h^{u*}_{ij}h^d_{kl}v^*_u$
$\tilde{u}_{Ri}, h_d^{(0)}$	$\tilde{u}^*_{Lj}$	$-(A^{u*}+\frac{1}{2})h^{u*}_{ij}\mu^*$
$\tilde{u}_{Ri}, H_d^{(-)}$	$\tilde{u}^*_{Lj}$	$-\frac{1}{2}h^{u*}_{ij}\mu^*$
$\tilde{e}_{Li}, h_d^{(0)}$	$\tilde{e}^*_{Rj}$	$h^e_{ij}A^e$
$\tilde{d}_{Li}, h_u^{(0)}$	$\tilde{u}^*_{Rj}$	$h^u_{ij}A^u$
$\tilde{d}_{Li}, h_d^{(0)}$	$\tilde{d}^*_{Rj}$	$h^d_{ij}A^d$



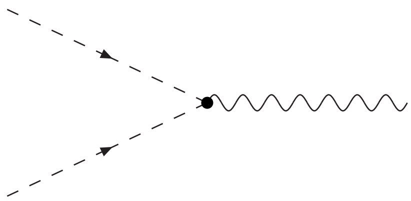
in	out	vertex factor
these interactions all come from the superpotential and mix among the different generations $i,j,k,l \in \{e,\mu\,\tau\}$ or $\{u,c,t\}$ resp.		
$\bar{e}^\alpha_{Ri}, \nu_{j\alpha}$	$H_d^{(-)}$	$-\frac{1}{2}h^e_{ij}$
$\bar{e}^\alpha_{Ri}, e_{Lj\alpha}$	$h_d^{(0)}$	$\frac{1}{2}h^e_{ij}$
$\bar{e}^\alpha_{Ri}, \tilde{H}^{(0)}_{d\alpha}$	$\tilde{e}_{Lj}$	$\frac{1}{2}h^e_{ij}$
$\nu^\alpha_j, \tilde{H}^{(-)}_{d\alpha}$	$\tilde{e}^*_{Ri}$	$-\frac{1}{2}h^e_{ij}$
$e^\alpha_{Lj}, \tilde{H}^{(0)}_{d\alpha}$	$\tilde{e}^*_{Ri}$	$\frac{1}{2}h^e_{ij}$
$\bar{u}^\alpha_{Ri}, u_{Lj\alpha}$	$h_u^{(0)}$	$-\frac{1}{2}h^u_{ij}$
$\bar{u}^\alpha_{Ri}, d_{Lj\alpha}$	$H_u^{(+)}$	$\frac{1}{2}h^u_{ij}$
$\bar{u}^\alpha_{Ri}, \tilde{H}^{(0)}_{u\alpha}$	$\tilde{u}_{Lj}$	$-\frac{1}{2}h^u_{ij}$
$\bar{u}^\alpha_{Ri}, \tilde{H}^{(+)}_{u\alpha}$	$\tilde{d}_{Lj}$	$\frac{1}{2}h^u_{ij}$
$u^\alpha_{Lj}, \tilde{H}^{(0)}_{u\alpha}$	$\tilde{u}^*_{Ri}$	$-\frac{1}{2}h^u_{ij}$
$d^\alpha_{Lj}, \tilde{H}^{(+)}_{u\alpha}$	$\tilde{u}^*_{Ri}$	$\frac{1}{2}h^u_{ij}$
$\bar{d}^\alpha_{Ri}, u_{Lj\alpha}$	$H_d^{(-)}$	$-\frac{1}{2}h^d_{ij}$
$\bar{d}^\alpha_{Ri}, d_{Lj\alpha}$	$h_d^{(0)}$	$\frac{1}{2}h^d_{ij}$
		continued below ...



... continuation		
in	out	vertex factor
$\bar{d}^{\alpha}_{Ri}, \tilde{H}^{(-)}_{d\alpha}$	$\tilde{u}_{Lj}$	$-\frac{1}{2} h^{d}_{ij}$
$\bar{d}^{\alpha}_{Ri}, \tilde{H}^{(0)}_{d\alpha}$	$\tilde{d}_{Lj}$	$\frac{1}{2} h^{d}_{ij}$
$u^{\alpha}_{Lj}, \tilde{H}^{(-)}_{d\alpha}$	$\tilde{d}^{*}_{Ri}$	$-\frac{1}{2} h^{d}_{ij}$
$d^{\alpha}_{Lj}, \tilde{H}^{(0)}_{d\alpha}$	$\tilde{d}^{*}_{Ri}$	$\frac{1}{2} h^{d}_{ij}$

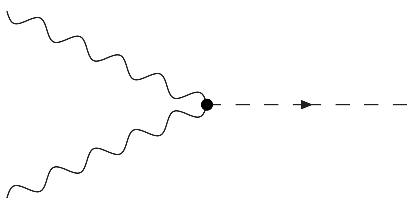


p_A with $A \in \{e, u, d, \dots\}$ is the momentum of the corresponding particle		
in	out	vertex factor
$\tilde{\nu}^{(1)}_{e}, \tilde{\nu}^{(2)*}_{e}$	Z^{0}_{μ}	$-\frac{1}{2}\left(g_Y s_W + g_L c_W\right)\left(p^{\mu}_{\nu^{(1)}} + p^{\mu}_{\nu^{(2)}}\right)$
$\tilde{\nu}^{*}_{e}, \tilde{e}_{L}$	W^{+}_{μ}	$\frac{1}{\sqrt{2}} g_L \left(p^{\mu}_{e} - p^{\mu}_{\nu}\right)$
$\tilde{e}^{(1)}_{L}, \tilde{e}^{(2)*}_{L}$	A_{μ}	$g_Y c_W \left(p^{\mu}_{e^{(1)}} + p^{\mu}_{e^{(2)}}\right)$
$\tilde{e}^{(1)}_{L}, \tilde{e}^{(2)*}_{L}$	Z^{0}_{μ}	$\frac{1}{2}\left(g_L c_W - g_Y s_W\right)\left(p^{\mu}_{e^{(1)}} + p^{\mu}_{e^{(2)}}\right)$
$\tilde{e}^{(1)}_{R}, \tilde{e}^{(2)*}_{R}$	A_{μ}	$-g_Y c_W \left(p^{\mu}_{e^{(1)}} + p^{\mu}_{e^{(2)}}\right)$
$\tilde{e}^{(1)}_{R}, \tilde{e}^{(2)*}_{R}$	Z^{0}_{μ}	$g_Y s_W \left(p^{\mu}_{e^{(1)}} + p^{\mu}_{e^{(2)}}\right)$
$\tilde{u}^{(1)}_{L}, \tilde{u}^{(2)*}_{L}$	A_{μ}	$-\frac{2}{3} g_Y c_W \left(p^{\mu}_{u^{(1)}} + p^{\mu}_{u^{(2)}}\right)$
$\tilde{u}^{(1)}_{L}, \tilde{u}^{(2)*}_{L}$	Z^{0}_{μ}	$\frac{1}{2}\left(\frac{1}{3} g_Y s_W - g_L c_W\right)\left(p^{\mu}_{u^{(1)}} + p^{\mu}_{u^{(2)}}\right)$
$\tilde{d}_{L}, \tilde{u}^{*}_{L}$	W^{+}_{μ}	$\frac{1}{\sqrt{2}} g_L \left(p^{\mu}_{d} - p^{\mu}_{u}\right)$
$\tilde{d}^{(1)}_{L}, \tilde{d}^{(2)*}_{L}$	A_{μ}	$\frac{1}{3} g_Y c_W \left(p^{\mu}_{d^{(1)}} + p^{\mu}_{d^{(2)}}\right)$
$\tilde{d}^{(1)}_{L}, \tilde{d}^{(2)*}_{L}$	Z^{0}_{μ}	$\frac{1}{2}\left(\frac{1}{3} g_Y s_W + g_L c_W\right)\left(p^{\mu}_{d^{(1)}} + p^{\mu}_{d^{(2)}}\right)$
$\tilde{u}^{(1)}_{L}, \tilde{u}^{(2)*}_{L}$	G^{a}_{μ}	$-\frac{1}{2} g_C \lambda^{a} \left(p^{\mu}_{u^{(1)}} + p^{\mu}_{u^{(2)}}\right)$
$\tilde{d}^{(1)}_{L}, \tilde{d}^{(2)*}_{L}$	G^{a}_{μ}	$-\frac{1}{2} g_C \lambda^{a} \left(p^{\mu}_{d^{(1)}} + p^{\mu}_{d^{(2)}}\right)$
$\tilde{u}^{(1)}_{R}, \tilde{u}^{(2)*}_{R}$	A_{μ}	$-\frac{2}{3} g_Y c_W \left(p^{\mu}_{u^{(1)}} + p^{\mu}_{u^{(2)}}\right)$
		continued below ...

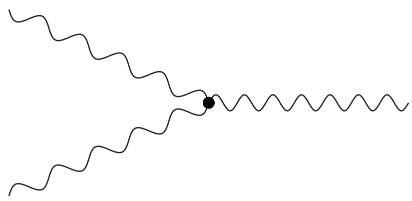


… continuation

in	out	vertex factor
$\tilde{u}_R^{(1)}, \tilde{u}_R^{(2)*}$	Z_μ^0	$\frac{2}{3} g_Y s_W \left(p_{u^{(1)}}^\mu + p_{u^{(2)}}^\mu \right)$
$\tilde{u}_R^{(1)}, \tilde{u}_R^{(2)*}$	G_μ^a	$-\frac{1}{2} g_C \lambda^a \left(p_{u^{(1)}}^\mu + p_{u^{(2)}}^\mu \right)$
$\tilde{d}_R^{(1)}, \tilde{d}_R^{(2)*}$	A_μ	$\frac{1}{3} g_Y c_W \left(p_{d^{(1)}}^\mu + p_{d^{(2)}}^\mu \right)$
$\tilde{d}_R^{(1)}, \tilde{d}_R^{(2)*}$	Z_μ^0	$-\frac{1}{3} g_Y s_W \left(p_{d^{(1)}}^\mu + p_{d^{(2)}}^\mu \right)$
$\tilde{d}_R^{(1)}, \tilde{d}_R^{(2)*}$	G_μ^a	$-\frac{1}{2} g_C \lambda^a \left(p_{d^{(1)}}^\mu + p_{d^{(2)}}^\mu \right)$
$H_u^{(+)1}, H_u^{(+)2*}$	A_μ	$-g_Y c_W \left(p_{H^1}^\mu + p_{H^2}^\mu \right)$
$H_u^{(+)1}, H_u^{(+)2*}$	Z_μ^0	$\frac{1}{2} \left(g_Y s_W - g_L c_W \right) \left(p_{H^1}^\mu + p_{H^2}^\mu \right)$
$h_u^{(0)}, H_u^{(+)}$	W_μ^+	$\frac{1}{\sqrt{2}} g_L \left(p_h^\mu - p_H^\mu \right)$
$h_u^{(0)1}, h_u^{(0)2*}$	Z_μ^0	$\frac{1}{2} \left(g_Y s_W + g_L c_W \right) \left(p_{h^1}^\mu + p_{h^2}^\mu \right)$
$h_d^{(0)1}, h_d^{(0)2*}$	Z_μ^0	$-\frac{1}{2} \left(g_Y s_W + g_L c_W \right) \left(p_{h^1}^\mu + p_{h^2}^\mu \right)$
$H_d^{(-)}, h_d^{(0)*}$	W_μ^+	$\frac{1}{\sqrt{2}} g_L \left(p_h^\mu - p_H^\mu \right)$
$H_d^{(-)1}, H_d^{(-)2*}$	A_μ	$g_Y c_W \left(p_{H^1}^\mu + p_{H^2}^\mu \right)$
$H_d^{(-)1}, H_d^{(-)2*}$	Z_μ^0	$\frac{1}{2} \left(g_L c_W - g_Y s_W \right) \left(p_{H^1}^\mu + p_{H^2}^\mu \right)$

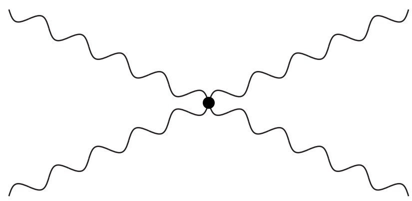


in	out	vertex factor
$W_\mu^+, W^{-\mu}$	$h_u^{(0)*}$	$\frac{1}{2} g_L^2 v_u$
$Z_\mu^0, Z^{0\mu}$	$h_u^{(0)*}$	$\frac{1}{4} \left(g_Y s_W + g_L c_W \right)^2 v_u$
W_μ^+, A_μ	$H_u^{(+)*}$	$\frac{1}{\sqrt{2}} g_L g_Y c_W v_u$
$W_\mu^+, Z^{0\mu}$	$H_u^{(+)*}$	$-\frac{1}{\sqrt{2}} g_L g_Y s_W v_u$
$Z_\mu^0, Z^{0\mu}$	$h_d^{(0)*}$	$\frac{1}{4} \left(g_Y s_W + g_L c_W \right)^2 v_d$
$W_\mu^+, W^{-\mu}$	$h_d^{(0)*}$	$\frac{1}{2} g_L^2 v_u$
$W_\mu^+, Z^{0\mu}$	$H_d^{(-)}$	$\frac{1}{2} g_L g_Y s_W v_d^*$
W_μ^+, A^μ	$H_d^{(-)}$	$-\frac{1}{\sqrt{2}} g_L g_Y c_W v_d^*$



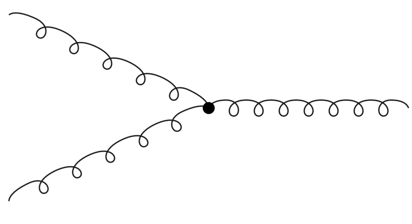
p_A with $A \in \{e, u, d, \dots\}$ is
the momentum of the corresponding particle

in	out	vertex factor
W^+_μ, W^-_ν	A_ρ	$\frac{1}{2} g_L s_W \big(p^\nu_{W^+} g^{\rho\mu} - p^\rho_{W^+} g^{\mu\nu} + p^\rho_{W^-} g^{\mu\nu} - p^\mu_{W^-} g^{\rho\nu} + p^\mu_A g^{\rho\nu} - p^\nu_A g^{\rho\mu}\big)$
W^+_μ, W^-_ν	Z^0_ρ	$-\frac{1}{2} g_L c_W \big(p^\nu_{W^+} g^{\rho\mu} - p^\rho_{W^+} g^{\mu\nu} + p^\rho_{W^-} g^{\mu\nu} - p^\mu_{W^-} g^{\rho\nu} + p^\mu_A g^{\rho\nu} - p^\nu_A g^{\rho\mu}\big)$



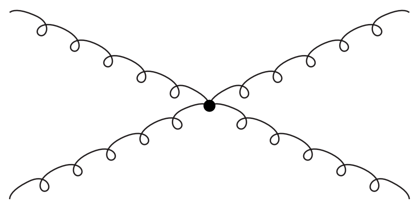
p_A with $A \in \{e, u, d, \dots\}$ is
the momentum of the corresponding particle

in	out	vertex factor
W^+_μ, W^+_ν	W^-_ρ, W^-_σ	$\frac{1}{2} g^2 \left(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}\right)$
W^+_μ, W^-_ν	A_ρ, A_σ	$g^2 s_W^2 \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\nu} g^{\rho\sigma}\right)$
W^+_μ, W^-_ν	A_ρ, Z^0_σ	$g^2 c_W^2 \left(g^{\mu\rho} g^{\nu\sigma} - g^{\mu\nu} g^{\rho\sigma}\right)$
W^+_μ, W^-_ν	A_ρ, Z^0_σ	$g^2 s_W c_W \left(2 g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}\right)$



p_A with $A \in \{e, u, d, \dots\}$ is
the momentum of the corresponding particle

in	out	vertex factor
G^d_μ, G^e_ν	G^f_ρ	$g f^{def} \big(g^{\mu\nu}(p^\rho_{G^d} - p_{G^e}) + g^{\mu\rho}(p_{G^f} - p_{G^d}) + g^{\nu\rho}(p_{G^e} - p_{G^f})\big)$



in	out	vertex factor
G^h_μ, G^i_ν	G^j_ρ, G^k_σ	$-g^2\Big(f^{ihc}f^{ckj}\,(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})$ $+\,f^{ijc}f^{chk}\,(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma})$ $+\,f^{ijc}f^{chk}\,(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})\Big)$

6.5 Summary

To calculate the matrix element for a given Feynman diagram write down the appropriate fields from section 6.1 for the external particles, attach the corresponding vertex factors from the tables in section 6.4 in the order given by the indices on the external particles and mulitply by i for every vertex except a pure gauge interaction. Invoke momentum conservation by multiplying with δ-functions. Finally insert the expression for the propagators from section 6.3 and integrate over every undetermined momentum.

Chapter 7

Sample Calculations

7.1 The Process $e^+e^- \xrightarrow{Z^0} \mu^+\mu^-$

For this process there are several different matrix elements contributing (fig. (7.1)). We begin by calculating the first matrix element, introducing $q_\mu = \frac{(k_1+k_2)_\mu}{m}$, $g_l = \frac{1}{2}(g_Y s_W - g_L c_W)$, $g_r = -g_Y s_W$. When squaring the amplitude we average over the spins of the incoming particles and sum over the spins of the outgoing particles, giving

$$\begin{aligned}
-iM_1 &= ig_l^2 \int d^4k \mathfrak{u}_L^{(r)}(p_1)\sigma^\mu \bar{\mathfrak{v}}_L^{(s)}(p_2) \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m^2}}{k^2 - m^2} \mathfrak{v}_L^{(t)}(k_2)\sigma^\nu \bar{\mathfrak{u}}_L^{(u)}(k_1) \\
&\quad \times \delta^{(4)}(k_1 + k_2 - k)\delta^{(4)}(p_1 + p_2 + k), \\
\frac{1}{4}\sum_{r,s,t,u=1}^{2} |M_1|^2 &= \frac{1}{4}\sum_{r,s,t,u=1}^{2} \frac{g_l^4}{(s-m^2)^2} \mathfrak{u}_L^{(r)}(p_1)\sigma^\mu \bar{\mathfrak{v}}_L^{(s)}(p_2)[g_{\mu\nu} - q_\mu q_\nu]\mathfrak{v}_L^{(t)}(k_2)\sigma^\nu \bar{\mathfrak{u}}_L^{(u)}(k_1) \\
&\quad \times \mathfrak{u}_L^{(u)}(k_1)\sigma^\rho \bar{\mathfrak{v}}_L^{(t)}(k_2)[g_{\rho\sigma} - q_\rho q_\sigma]\mathfrak{v}_L^{(s)}(p_2)\sigma^\sigma \bar{\mathfrak{u}}_R^{(r)}(p_1) \\
&= \frac{g_l^4}{4(s-m^2)^2} \mathrm{tr}(\sigma^\mu \bar{\sigma}^\eta \sigma^\sigma \bar{\sigma}^\omega) p_{2\eta} p_{1\omega} \mathrm{tr}(\sigma^\nu \bar{\sigma}^\epsilon \sigma^\rho \bar{\sigma}^\chi) k_{1\epsilon} k_{2\chi} [g_{\mu\nu} - q_\mu q_\nu][g_{\rho\sigma} - q_\rho q_\sigma] \\
&= \frac{g_l^4}{4(s-m^2)^2} \left(g^{\mu\eta}g^{\sigma\omega} - g^{\mu\sigma}g^{\eta\omega} + g^{\mu\omega}g^{\eta\sigma} - i\epsilon^{\mu\eta\sigma\omega}\right) \\
&\quad \times \left(g^{\nu\epsilon}g^{\rho\chi} - g^{\nu\rho}g^{\epsilon\chi} + g^{\nu\chi}g^{\rho\epsilon} - i\epsilon^{\nu\epsilon\rho\chi}\right)(g_{\mu\nu} - q_\mu q_\nu)(g_{\rho\sigma} - q_\rho q_\sigma) p_{1\omega} p_{2\eta} k_{1\epsilon} k_{2\chi} \\
&= \frac{g_l^4}{(s-m^2)^2}(k_1 p_2)(p_1 k_2), \qquad (7.1)
\end{aligned}$$

where s is the Mandelstamvariable $s = (p_1 + p_2)^2$ and we chose the momenta to be incoming in the left and right vertex except for the Z whose momentum points from the right to the left. We note that $\bar{\mathfrak{u}}_L(p) = \mathfrak{v}_R(p)$ and $\bar{\mathfrak{u}}_R(p) = \mathfrak{v}_L(p)$, so we can easily calculate the remaining matrix elements.

The second matrix element differs from the first by replacing $g_l^4 \leftrightarrow g_l^2 g_b^2$ and exchanging $k_1 \leftrightarrow k_2$:

$$\frac{1}{4}\sum_{\text{spins}} |M_2|^2 = \frac{g_l^2 g_b^2}{(s-m^2)^2}(p_1 k_1)(p_2 k_2). \qquad (7.2)$$

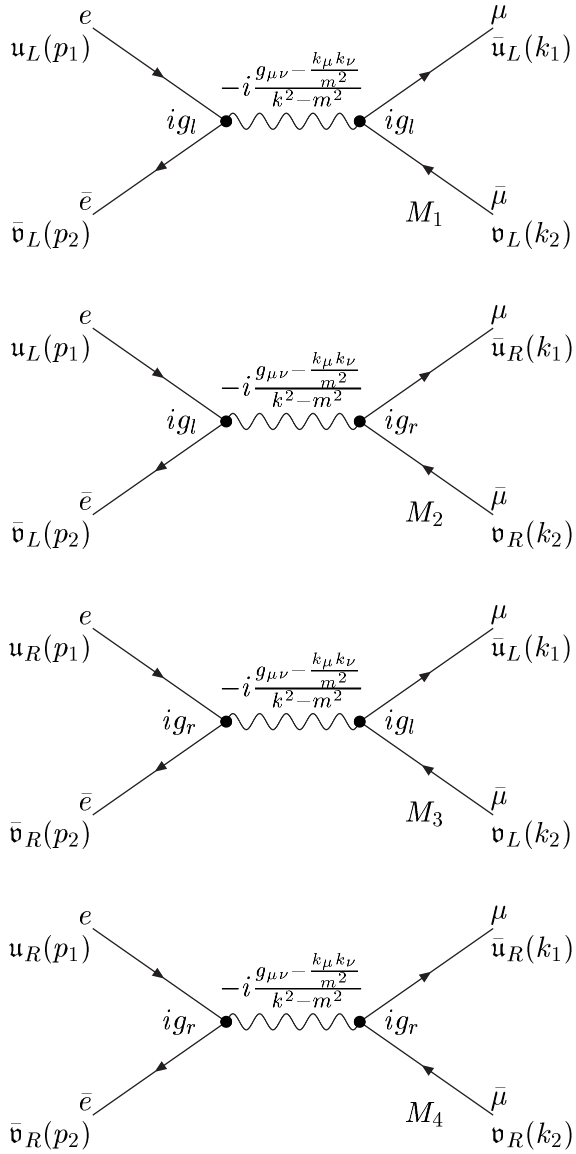


Figure 7.1: Processes contributing to $e^+e^- \to \mu^+\mu^-$

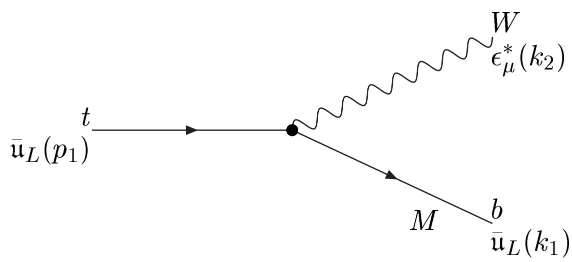


Figure 7.2: The process $t \to b + W$

The third matrix element differs from the first by replacing $g_l^4 \leftrightarrow g_b^2 g_l^2$ and exchanging $p_1 \leftrightarrow p_2$:

$$\frac{1}{4}\sum_{\text{spins}} |M_3|^2 = \frac{g_l^2 g_b^2}{(s-m^2)^2}(p_1k_1)(p_2k_2). \tag{7.3}$$

The fourth matrix element differs from the first by replacing $g_l^4 \leftrightarrow g_b^4$ and exchanging $p_1 \leftrightarrow p_2$ and $k_1 \leftrightarrow k_2$:

$$\frac{1}{4}\sum_{\text{spins}} |M_4|^2 = \frac{g_b^4}{(s-m^2)^2}(p_1k_2)(p_2k_1). \tag{7.4}$$

Finally we add up all the terms giving

$$\begin{aligned}
\frac{1}{4}\sum_{\text{spins}} |M|^2 &= \frac{1}{4}\sum_{\text{spins}}\sum_i |M_i|^2 \\
&= \frac{1}{(s-m^2)^2}(k_1p_2)(p_1k_2)(g_l^4+g_r^4) \\
&\quad + \frac{1}{(s-m^2)^2}(k_1p_1)(k_2p_2)(2g_l^2g_r^2) \\
&= \frac{g_Z^4}{8(s-m^2)^2}(k_1p_2)(p_1k_2)(c_V^4+6c_V^2c_A^2+c_A^4) \\
&\quad + \frac{g_Z^4}{8(s-m^2)^2}(p_1k_1)(p_2k_2)(c_V^2-c_A^2)^2.
\end{aligned} \tag{7.5}$$

We note that there are no interference terms because these are processes with different particles in the initial and final state. This is due to the fact that we neglect the masses of the incoming and outgoing particles, i.e. at high energy. Therefore helicity equals chirality. As it is not possible to 'convert' them into each other by performing a Lorentz-boost we can regard left-chiral and right-chiral fermions as a different particles.

7.2 The Process $t \to b+W$

The Feynman diagram for this process is given in figure (7.2). The matrix element is given by

$$-iM = \frac{ig_L}{2}\mathrm{u}_L^{(r)}(p_1)\sigma^\mu \bar{\mathrm{u}}_L^{(s)}(k_1)\epsilon_\mu^{(t)*}(k_2).$$

Summing over the spins we obtain

$$\begin{aligned}
\frac{1}{2}\sum_{\text{spins}}|M|^2 &= \frac{g_L^2}{4}\sum_{r,s,t,Pol} u_L^{(r)}(p_1)\sigma^\mu\bar{\mathfrak{u}}_L^{(s)}(k_1)\epsilon_\mu^{(t)*}(k_2)\epsilon_\nu^{(t)}(k_2)\mathfrak{u}_L^{(s)}(p_2)\sigma^\nu\bar{\mathfrak{u}}_L^{(r)}(p_1) \\
&= \frac{g_L^2}{4}\mathrm{tr}\left(\sigma^\nu\bar{\sigma}^\rho\sigma^\mu\bar{\sigma}^\sigma\right)\left(-g_{\mu\nu}+\frac{k_{2\mu}k_{2\nu}}{m_W^2}\right)p_{1\rho}k_{1\sigma} \\
&= \frac{g_L^2}{2}\left(g^{\nu\rho}g^{\mu\sigma}-g^{\nu\mu}g^{\rho\sigma}+g^{\nu\sigma}g^{\rho\mu}-i\epsilon^{\nu\rho\mu\sigma}\right)\left(-g_{\mu\nu}+\frac{k_{2\mu}k_{2\nu}}{m_W^2}\right)(p_{1\rho}k_{1\sigma}) \\
&= \frac{g_L^2}{2}\left((p_1k_1)+2\frac{(p_1k_2)(k_1k_2)}{m_W^2}\right), \qquad (7.6)
\end{aligned}$$

where we used

$$\sum_{\text{pol}}\epsilon_\mu^*(p)\epsilon_\nu(p) = -g_{\mu\nu}+\frac{p_\mu p_\nu}{m_W^2}. \qquad (7.7)$$

7.3 The Process $e^+e^- \xrightarrow{Z^0} \tilde{\chi}_i^0\tilde{\chi}_i^0$

The neutralinos $\tilde{\chi}_i^0$ are mass eigenstates of $\{\tilde{W}^3,\tilde{B}^0,\tilde{H}_u^{(0)},\tilde{H}_d^{(0)}\}$:

$$\tilde{\chi}_i^0 = N_{ij}\begin{pmatrix}\tilde{B}^0\\ \tilde{W}^3\\ \tilde{H}_u^{(0)}\\ \tilde{H}_d^{(0)}\end{pmatrix}_j, \qquad i=1\ldots 2, j=1\ldots 4. \qquad (7.8)$$

As the Z^0 couples only to the $\tilde{H}_{u,d}^{(0)}$ we have to consider the processes given in figure (7.3). Introducing $q_\mu = \frac{k_{1\mu}+k_{2\mu}}{m}$ and

$$\begin{aligned}
g_l &= \frac{1}{2}\left(-g_Y s_W + g_L c_W\right), \\
g_r &= g_Y s_W, \\
g_u &= \frac{1}{2}\left(g_Y s_W + g_L c_W\right), \\
g_d &= -\frac{1}{2}\left(g_Y s_W + g_L c_W\right), \qquad (7.9)
\end{aligned}$$

we find

$$-iM_1 = ig_l g_u \mathfrak{u}_L(p_1)\sigma^\mu\bar{\mathfrak{v}}_L(p_2)\frac{g_{\mu\nu}-q_\mu q_\nu}{s-m^2}\mathfrak{v}_L(k_2)\sigma^\nu\bar{\mathfrak{u}}_L(k_1). \qquad (7.10)$$

This has the same structure as eq.(7.1), so we can take the result from there. After summing and averaging over the spins we have

$$\frac{1}{4}\sum_{\text{spins}}|M_1|^2 = \frac{g_l^2 g_u^2}{(s-m^2)^2}(k_1p_2)(k_2p_1). \qquad (7.11)$$

The other diagrams give

$$
\begin{aligned}
\frac{1}{4}\sum_{\text{spins}} |M_2|^2 &= \frac{g_l^2 g_d^2}{(s-m^2)^2}(k_1p_2)(k_2p_1), \\
\frac{1}{4}\sum_{\text{spins}} |M_3|^2 &= \frac{g_r^2 g_u^2}{(s-m^2)^2}(k_1p_1)(k_2p_2), \\
\frac{1}{4}\sum_{\text{spins}} |M_4|^2 &= \frac{g_r^2 g_d^2}{(s-m^2)^2}(k_1p_1)(k_2p_2). \qquad (7.12)
\end{aligned}
$$

The total matrix element is given by the 'weighted' sum of the M_k

$$
\begin{aligned}
|M|^2 &= |N_{i3}|^2|M_1|^2 + |N_{i4}|^2|M_2|^2 + |N_{i3}|^2|M_3|^2 + |N_{i4}|^2|M_4|^2, \\
\frac{1}{4}\sum_{\text{spins}} |M|^2 &= \frac{1}{4(s-m^2)^2}\left(|N_{i3}|^2 + |N_{i4}|^2\right)\Big(\frac{1}{4}(k_1p_2)(k_2p_1)(g_Y^2 s_W^2 - g_L^2 c_W^2)^2 \\
&\quad +(k_1p_1)(k_2p_2) g_Y^2 s_W^2 (g_W s_W + g_L c_W)^2 \Big) . \qquad (7.13)
\end{aligned}
$$

We note that there are again no interference terms because there are different particles in the final state.

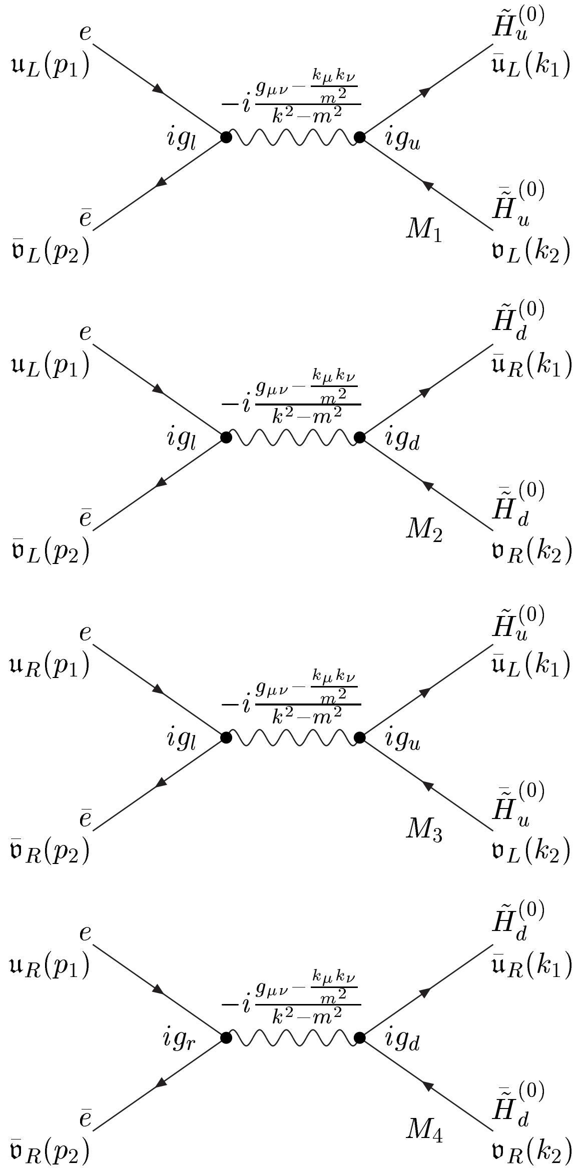


Figure 7.3: Processes contributing to $e^+e^- \to \tilde{\chi}^0\tilde{\chi}^0$

Appendix A

Calculations

A.1 Weyl Spinors

The Lagrangian density of a free Weyl spinor is given by

$$\mathcal{L} = i\bar{\psi}\bar{\sigma}^{\mu}\partial_{\mu}\psi. \tag{A.1}$$

Writing the spinors in the form

$$\begin{aligned}
\psi_{\alpha} &= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\left(u_{\alpha}a_{\vec{p}}e^{-ipx} + v_{\alpha}b^{\dagger}_{\vec{p}}e^{ipx}\right), \\
\bar{\psi}_{\dot{\alpha}} &= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\vec{p}}}}\left(\bar{u}_{\dot{\alpha}}a^{\dagger}_{\vec{p}}e^{ipx} + \bar{v}_{\dot{\alpha}}b_{\vec{p}}e^{-ipx}\right),
\end{aligned} \tag{A.2}$$

gives rise to the following solution of the equations of motion:

$$i\bar{\sigma}^{\mu\dot{\alpha}\alpha}\partial_{\mu}\psi_{\alpha} = 0. \tag{A.3}$$

Writing this explicitly in terms of the spinors u, v we obtain

$$\begin{aligned}
\begin{pmatrix} p_0 - p_3 & -p_1 + ip_2 \\ -p_1 - ip_2 & p_0 + p_3 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} &= 0, \\
(p_0 - p_3)u_1 + (-p_1 + ip_2)u_2 &= 0, \\
(-p_1 - ip_2)u_1 + (p_0 + p_3)u_2 &= 0, \\
u_1 &= \frac{p_1 - ip_2}{p_0 - p_3}u_2.
\end{aligned} \tag{A.4}$$

The solution for the anti-particle is given by

$$v_1 = \frac{p_1 - ip_2}{p_0 - p_3}v_2. \tag{A.5}$$

One can proceed analogously for the other equations giving the solutions for the different positions of the indices.

$$
\begin{aligned}
i\partial_\mu \bar{\psi}_{\dot{\alpha}} \bar{\sigma}^{\mu\dot{\alpha}\alpha} &= 0, \\
(\bar{u}_{\dot{1}}, \bar{u}_{\dot{2}}) \begin{pmatrix} p_0 - p_3 & -p_1 + ip_2 \\ -p_1 - ip_2 & p_0 + p_3 \end{pmatrix} &= 0, \\
\bar{u}_{\dot{1}} &= \frac{p_1 + ip_2}{p_0 - p_3} \bar{u}_{\dot{2}}, \\
\bar{v}_{\dot{1}} &= \frac{p_1 + ip_2}{p_0 - p_3} \bar{v}_{\dot{2}},
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
i\partial_\mu \psi^\alpha \sigma^\mu_{\alpha\dot{\alpha}} &= 0, \\
(u^1, u^2) \begin{pmatrix} p_0 + p_3 & p_1 - ip_2 \\ p_1 + ip_2 & p_0 - p_3 \end{pmatrix} &= 0, \\
u^1 &= -\frac{p_0 - p_3}{p_1 - ip_2} u^2, \\
v^1 &= -\frac{p_0 - p_3}{p_1 - ip_2} v^2.
\end{aligned}
\tag{A.7}
$$

A.2 SUSY Invariants

A.2.1 Chiral Superfield

The effect of a SUSY transformation on a chiral superfield is

$$
\begin{aligned}
\delta\Phi &= i\left(\zeta^\alpha Q_\alpha + \bar{Q}_{\dot{\alpha}}\bar{\zeta}^{\dot{\alpha}}\right)\Phi \\
&= -i\zeta^\alpha\Big(i\sqrt{2}\psi_\alpha(x) + 2i\theta_\alpha F(x) - i^2\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\varphi(x) + i^2\sqrt{2}\theta_\alpha\partial_\mu\psi(x)\sigma^\mu\bar{\theta} - \frac{i}{2}\theta_\alpha\bar{\theta}^2\Box\varphi(x) \\
&\quad -\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\varphi(x) - \sqrt{2}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\left(\theta\psi(x)\right) - \theta^2\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu F(x) + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\theta^\beta\sigma^\nu_{\beta\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\nu\partial_\mu\varphi(x) \\
&\quad -\frac{i}{\sqrt{2}}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\theta^2\partial_\mu\partial_\nu\psi^\beta(x)\sigma^\nu_{\beta\dot{\beta}}\bar{\theta}^{\dot{\beta}}\Big) \\
&\quad -i\Big(-(-i)^2\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu\varphi(x) + \frac{(-i)^2}{\sqrt{2}}\theta^2\partial_\mu\psi^\beta(x)\sigma^\mu_{\beta\dot{\alpha}} + \frac{i}{2}\theta^2\bar{\theta}_{\dot{\alpha}}\Box\varphi(x) + \theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\varphi(x) \\
&\quad +\sqrt{2}\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu(\theta^\beta\psi_\beta(x)) - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\theta^\beta\sigma^\nu_{\beta\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu\partial_\nu\varphi(x)\Big)\bar{\zeta}^{\dot{\alpha}} \\
&= \sqrt{2}\zeta\psi(x) + 2\zeta\theta F - i\zeta\sigma^\mu\bar{\theta}\partial_\mu\varphi(x) + i\sqrt{2}(\zeta\theta)\partial_\mu\psi(x)\sigma^\mu\bar{\theta} - \frac{1}{2}(\zeta\theta)\bar{\theta}^2\Box\varphi(x) \\
&\quad +i\zeta\sigma^\mu\bar{\theta}\partial_\mu\varphi(x) + i\sqrt{2}\zeta\sigma^\mu\bar{\theta}\partial_\mu(\theta\psi(x)) + i\theta^2\zeta\sigma^\mu\bar{\theta}\partial_\mu F(x) - \frac{1}{2}\bar{\theta}^2\zeta\sigma^\mu\bar{\sigma}^\nu\theta\partial_\mu\partial_\nu\varphi(x) \\
&\quad +\frac{1}{2\sqrt{2}}\theta^2\bar{\theta}^2\zeta\sigma^\mu\bar{\sigma}^\nu\partial_\mu\partial_\nu\psi(x) - i\theta\sigma^\mu\bar{\zeta}\partial_\mu\varphi(x) + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\zeta} + \frac{1}{2}\theta^2\bar{\theta}\bar{\zeta}\Box\varphi(x) \\
&\quad -i\theta\sigma^\mu\bar{\zeta}\partial_\mu\varphi(x) + \frac{i}{\sqrt{2}}\partial_\mu\psi(x)\sigma^\mu\bar{\zeta}\theta^2 + \frac{1}{2}\theta^2\partial_\mu\partial_\nu\varphi(x)\bar{\theta}\bar{\sigma}^\nu\sigma^\mu\bar{\zeta} \\
&\overset{!}{=} \delta\varphi(x) + \sqrt{2}\theta\delta\psi(x) + \theta^2\delta F(x) - i\theta\sigma^\mu\bar{\theta}\delta\varphi(x) + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\delta\psi(x)\sigma^\mu\bar{\theta} - \frac{1}{4}\theta^2\bar{\theta}^2\Box\delta\varphi(x).
\end{aligned}
\tag{A.8}
$$

We read off the transformations of the component fields

$$\begin{aligned}
\delta\varphi(x) &= \sqrt{2}\zeta\psi(x), \\
\delta\psi(x) &= \sqrt{2}\zeta F(x) - i\sqrt{2}\sigma^\mu\bar{\zeta}\partial_\mu\varphi(x), \\
\delta F(x) &= i\sqrt{2}\partial_\mu\psi(x)\sigma^\mu\bar{\zeta},
\end{aligned} \tag{A.9}$$

and see that $F(x)$ only changes by a total derivative.

A.2.2 Superfield Strength

First we calculate the superfield strength for an Abelian gauge field:

$$\begin{aligned}
D_\alpha V &= \left(\frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\right)\left(\theta\sigma^\mu\bar\theta V_\mu + i\theta^2\bar\theta\bar\lambda - i\bar\theta^2\theta\lambda + \frac{1}{2}\theta^2\bar\theta^2 D\right) \\
&= \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}V_\mu + 2i\theta_\alpha\bar\theta\bar\lambda - i\bar\theta^2\lambda_\alpha + \theta_\alpha\bar\theta^2 D - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu\theta^\beta\sigma^\nu_{\beta\dot\beta}\bar\theta^{\dot\beta}V_\nu \\
&\quad +\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu\bar\lambda_{\dot\beta}\bar\theta^{\dot\beta}\theta^2 \\
&= \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}V_\mu + 2i\theta_\alpha\bar\theta\bar\lambda - i\bar\theta^2\lambda_\alpha + \theta_\alpha\bar\theta^2 D + \frac{i}{2}\bar\theta^2(\sigma^\mu\bar\sigma^\nu)_\alpha^{\ \gamma}\theta_\gamma\partial_\mu V_\nu \\
&\quad -\frac{1}{2}\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\lambda^{\dot\alpha}\bar\theta^2\theta^2, \\
\bar D^2 D_\alpha V &= \left(\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\frac{\partial}{\partial\bar\theta^{\dot\beta}}\epsilon^{\dot\alpha\dot\beta} - 2i\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\frac{\partial}{\partial\bar\theta^{\dot\beta}}\epsilon^{\dot\alpha\dot\beta} - \theta^2\Box\right)D_\alpha V \\
&= 4i\lambda_\alpha - 4\theta_\alpha D - 2i(\sigma^\mu\bar\sigma^\nu)_\alpha^{\ \gamma}\theta_\gamma\partial_\mu V_\nu + 2\theta^2\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\lambda^{\dot\alpha} + 2i(\sigma^\mu\bar\sigma^\nu)_\alpha^{\ \gamma}\theta_\gamma\partial_\mu V_\nu \\
&\quad -4\theta\sigma^\mu\partial_\mu\bar\lambda\theta_\alpha + 4\partial_\mu\lambda_\alpha\theta\sigma^\mu\bar\theta + 4i\theta\sigma^\mu\bar\theta\theta_\alpha\partial_\mu D - 2\theta\sigma^\mu\bar\theta\partial_\mu\partial_\rho V_\nu(\sigma^\rho\bar\sigma^\nu)_\alpha^{\ \gamma}\theta_\gamma \\
&\quad -\theta^2\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\Box V_\mu + i\theta^2\bar\theta^2\Box\lambda_\alpha \\
&= 4i\lambda_\alpha - 4\theta_\alpha D - 2i(\sigma^\mu\bar\sigma^\nu)_\alpha^{\ \gamma}\theta_\gamma V_{\mu\nu} + 4\theta^2\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\lambda^{\dot\alpha} + 4\partial_\mu\lambda_\alpha\theta\sigma^\mu\bar\theta \\
&\quad +4i\theta\sigma^\mu\bar\theta\theta_\alpha\partial_\mu D - 2\theta\sigma^\mu\bar\theta\partial_\mu\partial_\rho V_\nu(\sigma^\rho\bar\sigma^\nu)_\alpha^{\ \gamma}\theta_\gamma - \theta^2\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\Box V_\mu \\
&\quad +i\theta^2\bar\theta^2\Box\lambda_\alpha,
\end{aligned} \tag{A.10}$$

which gives the kinetic term

$$\begin{aligned}
(W^\alpha W_\alpha)_F &= 32i\lambda\sigma^\mu\partial_\mu\bar\lambda + 16D^2 + 16iD(\sigma^\mu\bar\sigma^\nu)_\alpha^{\ \gamma}\frac{1}{2}\epsilon_{\alpha\gamma}V_{\mu\nu} \\
&\quad -4\left((\sigma^\mu\bar\sigma^\nu)_\alpha^{\ \beta}\theta_\beta(\sigma^\rho\bar\sigma^\eta)_\gamma^{\ \delta}\theta_\delta V_{\mu\nu}V_{\rho\eta}\right)_F \\
&= 32i\lambda\sigma^\mu\partial_\mu\bar\lambda + 16D^2 + 4\big(-V_{\mu\nu}V^{\mu\nu} + V_{\mu\nu}V^{\nu\mu} - i\underbrace{\epsilon^{\mu\nu\rho\eta}V_{\mu\nu}V_{\rho\eta}}_{\text{total divergence}}\big) \\
&= 32i\lambda\sigma^\mu\partial_\mu\bar\lambda + 16D^2 - 8V_{\mu\nu}V^{\mu\nu}.
\end{aligned} \tag{A.11}$$

Now we turn to the superfield strength of a non-Abelian gauge field as described in section 4.3.2.

$$\begin{aligned}
(D_\alpha V^b)V^c &= \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\theta^\beta\sigma^\rho_{\alpha\dot\alpha}\bar\theta^{\dot\beta}V^b_\mu V^c_\rho + \sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}V^b_\mu i\theta^2\bar\lambda^c_{\dot\beta}\bar\theta^{\dot\beta} + 2i\theta_\alpha\bar\lambda^b_{\dot\beta}\bar\theta^{\dot\beta}\theta^\beta\sigma^\rho_{\beta\dot\gamma}\bar\theta^{\dot\gamma}V^c_\rho \\
&= -\frac{1}{2}(\sigma^\mu\bar\sigma^\rho)_\alpha^{\ \gamma}\theta_\gamma V^b_\mu V^c_\rho\bar\theta^2 - \frac{i}{2}\sigma^\mu_{\alpha\dot\alpha}\bar\lambda^{c\dot\alpha}V^b_\mu\theta^2\bar\theta^2 + \frac{i}{2}\sigma^\mu_{\alpha\dot\beta}\bar\lambda^{b\dot\beta}V^c_\mu\theta^2\bar\theta^2 \\
&= -\frac{1}{2}(\sigma^\mu\bar\sigma^\nu)_\alpha^{\ \gamma}\theta_\gamma V^b_\mu V^c_\nu\bar\theta^2 + \frac{i}{2}\sigma^\mu_{\alpha\dot\alpha}(\bar\lambda^{b\dot\alpha}V^c_\mu - \bar\lambda^{c\dot\alpha}V^b_\mu)
\end{aligned}$$

$$\begin{aligned}
\bar{D}^2\left(\left(D_\alpha V^b\right)V^c\right) &= 2(\sigma^\mu\bar{\sigma}^\nu)_\alpha^{\ \beta}\theta_\beta V_\mu^b V_\nu^c - 2i\sigma^\mu_{\alpha\dot{\beta}}\left(\bar{\lambda}^{b\dot{\beta}}V_\mu^c - \bar{\lambda}^{c\dot{\beta}}V_\mu^b\right)\theta^2 \\
&\quad -2i\theta^\gamma\sigma^\rho_{\gamma\dot{\alpha}}\partial_\rho\bar{\theta}_{\dot{\beta}}(\sigma^\mu\bar{\sigma}^\nu)_\alpha^{\ \beta}\theta_\beta V_\mu^b V_\nu^c\epsilon^{\dot{\alpha}\dot{\beta}} \\
&= 2(\sigma^\mu\bar{\sigma}^\nu)_\alpha^{\ \beta}\theta_\beta V_\mu^b V_\nu^c - 2i\sigma^\mu_{\alpha\dot{\alpha}}\left(\bar{\lambda}^{b\dot{\alpha}}V_\mu^c - \bar{\lambda}^{c\dot{\alpha}}V_\mu^b\right)\theta^2 \\
&\quad +i\theta^2(\sigma^\mu\bar{\sigma}^\nu\sigma^\rho)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\rho(V_\mu^b V_\nu^c) \\
W_\alpha^a &= \bar{D}^2 D_\alpha V^a + igf^{abc}\bar{D}^2\left(\left(D_\alpha V^b\right)V^c\right) \\
&= 4i\lambda_\alpha^a - 4\theta_\alpha D^a - 2i(\sigma^\mu\bar{\sigma}^\nu)_\alpha^{\ \beta}\theta_\beta\underbrace{\left(V_{\mu\nu}^a - gf^{abc}V_\mu^b V_\nu^c\right)}_{=V'^a_{\mu\nu}} + 4\theta^2\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\theta}^{a\dot{\alpha}} \\
&\quad +4\partial_\mu\lambda_\alpha^a\theta\sigma^\mu\bar{\theta} + 4i\theta\sigma^\mu\bar{\theta}\theta_\alpha\partial_\mu D^a - 2\theta\sigma^\mu\bar{\theta}\partial_\mu\partial_\rho V_\nu^a(\sigma^\mu\bar{\sigma}^\nu)_\alpha^{\ \beta}\theta_\beta - \theta^2\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\Box V_\mu^a \\
&\quad +i\theta^2\bar{\theta}^2\Box\lambda_\alpha^a - 4gf^{abc}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\lambda}^{c\dot{\alpha}}V_\mu^b\theta^2 - gf^{abc}\theta^2(\sigma^\mu\bar{\sigma}^\nu\sigma^\rho)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\rho(V_\mu^b V_\nu^c)
\end{aligned} \tag{A.12}$$

which leads to the expression for the kinetic energy of the gauge field

$$\begin{aligned}
(W^{a\alpha}W_\alpha^a)_F &= 32i\lambda^a\sigma^\mu\partial_\mu\bar{\lambda}^a - 32ig\lambda^a\sigma^\mu\lambda^c V_\mu^b f^{abc} + 16D^aD^a - 8V'^a_{\mu\nu}V'^{a\mu\nu} \\
&\quad +8i\theta_\alpha D^a(\sigma^\mu\bar{\sigma}^\nu)_\alpha^{\ \beta}\theta_\beta V'^a_{\mu\nu} \\
&= 32i\lambda^a\sigma^\mu\tilde{D}_\mu^{ac}\bar{\lambda}^c + 16D^aD^a - 8V'^a_{\mu\nu}V'^{a\mu\nu},
\end{aligned} \tag{A.13}$$

with $\tilde{D}_\mu^{ac} = \partial_\mu\delta^{ac} - gf^{abc}V_\mu^b$ and $V'^a_{\mu\nu} = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a - gf^{abc}V_\mu^b V_\nu^c$.

A.2.3 Coupling Matter and Gauge Fields

The term containing the covariant derivative is given by

$$\begin{aligned}
(\phi^\dagger e^{2gt^aV^a}\phi)_D &= \Big((\varphi^\dagger + \sqrt{2}\bar{\theta}\bar{\psi} + \bar{\theta}^2F^\dagger + i\partial_\mu\varphi^\dagger\theta\sigma^\mu\bar{\theta} - \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi} - \frac{1}{4}\Box\varphi^\dagger\theta^2\bar{\theta}^2) \\
&\quad \times(1 + 2gt^a(\theta\sigma^\mu\bar{\theta}V_\mu^a + i\theta^2\bar{\theta}\bar{\lambda}^a - i\bar{\theta}^2\theta\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2D^a) \\
&\qquad + 2g^2t^at^b(\theta\sigma^\mu\bar{\theta}V_\mu^a + i\theta^2\bar{\theta}\bar{\lambda}^a - i\bar{\theta}^2\theta\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2D^a) \\
&\quad \times(\theta\sigma^\nu\bar{\theta}V_\nu^b + i\theta^2\bar{\theta}\bar{\lambda}^b - i\bar{\theta}^2\theta\lambda^b + \frac{1}{2}\theta^2\bar{\theta}^2D^b)) \\
&\quad \times(\varphi + \sqrt{2}\theta\psi + \theta^2F - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi + \frac{i}{\sqrt{2}}\theta^2\partial_\nu\psi\sigma^\nu\bar{\theta} - \frac{1}{4}\Box\varphi\theta^2\bar{\theta}^2)\Big)_D \\
&= \Big((\varphi^\dagger + \sqrt{2}\bar{\theta}\bar{\psi} + \bar{\theta}^2F^\dagger + i\partial_\mu\varphi^\dagger\theta\sigma^\mu\bar{\theta} - \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi} - \frac{1}{4}\Box\varphi^\dagger\theta^2\bar{\theta}^2) \\
&\quad \times(1 + 2gt^a(\theta\sigma^\mu\bar{\theta}V_\mu^a + i\theta^2\bar{\theta}\bar{\lambda}^a - i\bar{\theta}^2\theta\lambda^a + \frac{1}{2}\theta^2\bar{\theta}^2D^a) \\
&\qquad + 2g^2t^at^b\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}V_\mu^a\theta^\beta\sigma^\nu_{\beta\dot{\beta}}\bar{\theta}^{\dot{\beta}}V_\nu^b) \\
&\quad \times(\varphi + \sqrt{2}\theta\psi + \theta^2F - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi + \frac{i}{\sqrt{2}}\theta^2\partial_\nu\psi\sigma^\nu\bar{\theta} - \frac{1}{4}\Box\varphi\theta^2\bar{\theta}^2)\Big)_D
\end{aligned}$$

$$
\begin{aligned}
= \quad & -\frac{1}{4}\varphi^\dagger\Box\varphi - (\varphi^\dagger 2gt^aV_\mu^a i\partial_\nu\varphi\underbrace{\theta\sigma^\mu\bar\theta\theta\sigma^\nu\bar\theta}_{\frac{1}{2}g^{\mu\nu}\theta^2\bar\theta^2})_D \\
& +(\varphi 2gt^a(-i)\bar\theta^2\theta\lambda^a\sqrt{2}\theta\psi)_D + \varphi^\dagger gt^aD^a\varphi + \varphi^\dagger g^2t^at^bV_\mu^aV^{b\mu}\varphi \\
& +(\sqrt{2}\bar\psi\bar\theta\frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi\sigma^\mu\bar\theta)_D + (\sqrt{2}\bar\psi\bar\theta 2gt^a\theta\sigma^\mu\bar\theta V_\mu^a\sqrt{2}\theta\psi)_D \\
& +(\sqrt{2}\bar\psi\bar\theta 2gt^ai\theta^2\bar\lambda\bar\theta\varphi)_D + F^\dagger F + (i\partial_\mu\varphi^\dagger\theta\sigma^\mu\bar\theta 2gt^a\theta\sigma^\nu\bar\theta V_\nu^a\varphi)_D \\
& +(-i\partial_\mu\varphi^\dagger\theta\sigma^\mu\bar\theta i\partial_\nu\varphi\theta\sigma^\nu\bar\theta)_D + (\frac{-i}{\sqrt{2}}\bar\theta^2\theta\sigma^\mu\partial_\mu\bar\psi\sqrt{2}\theta\psi)_D - \frac{1}{4}(\Box\varphi^\dagger)\varphi \\
= \quad & -\frac{1}{4}\varphi^\dagger\Box\varphi - ig\varphi^\dagger t^aV_\mu^a\partial^\mu\varphi + ig\varphi^\dagger t^a\lambda^a\psi\sqrt{2} + g\varphi^\dagger t^aD^a\varphi \\
& +g^2\varphi^\dagger t^at^bV_\mu^aV^{b\mu}\varphi - \frac{i}{2}\partial_\mu\psi\sigma^\mu\bar\psi + gt^aV_\mu^a\psi\sigma^\mu\bar\psi - i\sqrt{2}gt^a\bar\psi\bar\lambda^a\varphi + F^\dagger F \\
& +ig\partial_\mu\varphi^\dagger t^aV^{a\mu}\varphi + \frac{1}{2}\partial_\mu\varphi^\dagger\partial^\mu\varphi + \frac{i}{2}\psi\sigma^\mu\partial_\mu\bar\psi - \frac{1}{4}\varphi\Box\varphi^\dagger \\
= \quad & \partial_\mu\varphi^\dagger\partial^\mu\varphi + ig(-\varphi^\dagger t^aV_\mu^a\partial^\mu\varphi + \partial^\mu\varphi^\dagger t^aV_\mu^a\varphi) \\
& +g^2\varphi^\dagger t^at^bV_\mu^aV^{b\mu}\varphi + ig\sqrt{2}(\varphi^\dagger t^a\lambda^a\psi - t^a\bar\psi\bar\lambda^a\varphi) \\
& +g\varphi^\dagger t^aD^a\varphi + F^\dagger F + i\psi\sigma^\mu\partial_\mu\bar\psi + gt^aV_\mu^a\psi\sigma^\mu\bar\psi \\
= \quad & [\underbrace{(\partial_\mu + igt^aV_\mu^a)}_{=:D_\mu}\varphi]^\dagger(\partial^\mu + igt^aV^{a\mu})\varphi + ig\sqrt{2}(\varphi^\dagger t^a\lambda^a\psi - t^a\varphi\bar\psi\bar\lambda) \\
& -i[(\partial_\mu + igt^aV_\mu^a)\psi]\sigma^\mu\bar\psi + g\varphi^\dagger t^aD^a\varphi + F^\dagger F \\
= \quad & (D_\mu\varphi)^\dagger(D^\mu\varphi) + ig\sqrt{2}(\varphi^\dagger t^a\lambda^a\psi - t^a\varphi\bar\lambda^a\bar\psi) - iD_\mu\psi\sigma^\mu\bar\psi \\
& +g\varphi^\dagger t^aD^a\varphi + F^\dagger F
\end{aligned} \tag{A.14}
$$

A.3 Feynman Rules

Reminder: The factor i appearing when we calculate time-ordered products stems from the fact that we actually wish to evaluate

$$\langle 0|T\phi_1\phi_2\phi_3 e^{i\int d^4z\mathcal{L}}|0\rangle \tag{A.15}$$

and when expanding the exponential function we get a factor of i.

A.3.1 Vertex Factors

Four Non-Abelian Gauge Bosons

First we examine $\mathcal{L}$ to find the expression corresponding to the interaction of 4 gluons using $G_\mu = G_\mu^a t^a$ with group index a

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{2}tr[G_{\mu\nu}G^{\mu\nu}] \\
&= -\frac{1}{2}tr[(\partial_\mu G_\nu - \partial_\nu G_\mu - ig[G_\mu, G_\nu])(\partial^\mu G^\nu - \partial^\nu G^\mu - ig[G^\mu, G^\nu])] \\
&= -\frac{1}{2}\mathrm{tr}[t^a, t^d]\left(\partial_\mu G_\mu^a - \partial_\nu G_\mu^a + gf^{abc}G_\mu^b G_\nu^c\right)\left(\partial^\mu G^{d\mu} - \partial^\nu G^{d\mu} + gf^{def}G^{e\mu}G^{f\nu}\right) \\
&= -\frac{1}{4}\left(\partial_\mu G_\mu^a - \partial_\nu G_\mu^a + gf^{abc}G_\mu^b G_\nu^c\right)\left(\partial^\mu G^{d\mu} - \partial^\nu G^{d\mu} + gf^{def}G^{e\mu}G^{f\nu}\right) \\
&= -\frac{1}{4}\big(2(\partial_\mu G_\nu^a \partial^\mu Ga\nu - \partial_\mu G_\nu^a \partial^\nu G^{a\mu}) + 2g(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a)f^{abc}G^{b\mu}G^{c\nu} \\
&\quad + g^2 f^{abc}f^{ade}G_\mu^b G_\nu^c G^{d\mu}G^{e\nu}\big) \\
&= -\frac{1}{2}(\partial_\mu G_\nu^a \partial^\mu Ga\nu - \partial_\mu G_\nu^a \partial^\nu G^{a\mu}) - gf^{abc}(\partial_\mu G_\nu^a)G^{b\mu}G^{c\nu} \\
&\quad -\frac{1}{4}g^2 f^{abc}f^{cde}G_\mu^a G_\nu^b G^{d\mu}G^{e\nu},
\end{aligned} \tag{A.16}$$

$$\begin{aligned}
L &= \int d^4x\mathcal{L} \\
&= \int \frac{d^4k_5 d^4k_6 d^4k_7 d^4k_8}{(2\pi)^{12}}\delta^{(4)}(k_5+k_6+k_7+k_8)\frac{-g^2}{4}f^{abc}f^{cde}\check{G}_{\mu 5}^a \check{G}_{\nu 6}^b \check{G}_7^{\mu d}\check{G}_8^{\nu e}.
\end{aligned} \tag{A.17}$$

Because of

$$\begin{aligned}
\phi(x)\phi^*(y) &= \int \frac{d^4p_1 d^4p_2}{(2\pi)^8}\,\tilde{\phi}(p_1)\tilde{\phi}^*(p_2)\,e^{ip_1x}e^{-ip_2y} \\
\Delta(x-y) &= \int \frac{d^4p_1}{(2\pi)^4}\check{\Delta}(p_1)e^{ip_1(x-y)}
\end{aligned} \tag{A.18}$$

we define for the sake of brevity

$$\check{\Delta}_{\mu\nu 12}^{ab} = (2\pi)^4\check{\Delta}_{\mu\nu}(p_1)\delta^{(4)}(p_1-p_2)\delta^{ab}, \tag{A.19}$$

with $\check{\Delta}(p_1)$ the Fourier transform of the propagator. Now we turn to the evaluation of the time-ordered product.

$$
\begin{aligned}
& -\frac{ig^2}{4}\langle 0|T\check{G}^h_\alpha(k_1)\check{G}^i_\beta(k_2)f^{abc}f^{cde}\check{G}^a_\mu(k_5)\check{G}^b_\nu(k_6)\check{G}^{d\mu}(k_7)\check{G}^{e\nu}(k_8)\check{G}^j_\gamma(k_3)\check{G}^k_\delta(k_4)|0\rangle \\
= \; & -\frac{ig^2}{4}f^{abc}f^{cde}\Big\{ \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
& + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} + \check{G}^h_{\alpha 1}\check{G}^i_{\beta 2}\check{G}^a_{\mu 5}\check{G}^b_{\nu 6}\check{G}^{d\mu}_7\check{G}^{e\nu}_8\check{G}^j_{\gamma 3}\check{G}^k_{\delta 4} \\
= \; & -\frac{g^2}{4}i^4 f^{abc}f^{cde}\Big[(\check{\Delta}^{bh}_{16}\check{\Delta}^{ej}_{83})_{\alpha\gamma}(\check{\Delta}^{ia}_{25}\check{\Delta}^{dk}_{74})_{\beta\delta} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{dk}_{74})_{\alpha\delta}(\check{\Delta}^{ib}_{26}\check{\Delta}^{je}_{83})_{\beta\gamma} \\
& +(\check{\Delta}^{hb}_{16}\check{\Delta}^{ek}_{84})_{\alpha\delta}(\check{\Delta}^{ia}_{25}\check{\Delta}^{dj}_{73})_{\beta\gamma} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{dj}_{73})_{\alpha\gamma}(\check{\Delta}^{ib}_{26}\check{\Delta}^{ek}_{84})_{\beta\delta} \\
& +(\check{\Delta}^{hd}_{17}\check{\Delta}^{ai}_{52})_{\alpha\beta}(\check{\Delta}^{jb}_{36}\check{\Delta}^{ek}_{84})_{\gamma\delta} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{di}_{72})_{\alpha\beta}(\check{\Delta}^{jb}_{36}\check{\Delta}^{ek}_{84})_{\gamma\delta} \\
& +(\check{\Delta}^{hd}_{17}\check{\Delta}^{ia}_{52})_{\alpha\beta}(\check{\Delta}^{je}_{38}\check{\Delta}^{bk}_{64})_{\gamma\delta} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{di}_{72})_{\alpha\beta}(\check{\Delta}^{je}_{38}\check{\Delta}^{bk}_{64})_{\gamma\delta} \\
& +(\check{\Delta}^{hd}_{17}\check{\Delta}^{aj}_{53})_{\alpha\gamma}(\check{\Delta}^{ib}_{26}\check{\Delta}^{ek}_{84})_{\beta\delta} + (\check{\Delta}^{hb}_{16}\check{\Delta}^{ek}_{84})_{\alpha\delta}(\check{\Delta}^{id}_{28}\check{\Delta}^{aj}_{53})_{\beta\gamma} \\
& +(\check{\Delta}^{hd}_{17}\check{\Delta}^{ak}_{54})_{\alpha\delta}(\check{\Delta}^{ib}_{26}\check{\Delta}^{ej}_{83})_{\beta\gamma} + (\check{\Delta}^{hb}_{16}\check{\Delta}^{ej}_{83})_{\alpha\gamma}(\check{\Delta}^{id}_{27}\check{\Delta}^{ak}_{54})_{\beta\delta} \\
& +(\check{\Delta}^{he}_{18}\check{\Delta}^{bj}_{63})_{\alpha\gamma}(\check{\Delta}^{ia}_{25}\check{\Delta}^{dk}_{74})_{\beta\delta} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{dk}_{74})_{\alpha\delta}(\check{\Delta}^{ie}_{28}\check{\Delta}^{bj}_{63})_{\beta\gamma} \\
& +(\check{\Delta}^{he}_{18}\check{\Delta}^{bk}_{64})_{\alpha\delta}(\check{\Delta}^{ia}_{25}\check{\Delta}^{dj}_{73})_{\beta\gamma} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{dj}_{73})_{\alpha\gamma}(\check{\Delta}^{ie}_{28}\check{\Delta}^{bk}_{64})_{\beta\delta} \\
& +(\check{\Delta}^{he}_{18}\check{\Delta}^{bi}_{62})_{\alpha\beta}(\check{\Delta}^{ja}_{35}\check{\Delta}^{dk}_{74})_{\gamma\delta} + (\check{\Delta}^{hb}_{16}\check{\Delta}^{ei}_{82})_{\alpha\beta}(\check{\Delta}^{ja}_{35}\check{\Delta}^{dk}_{74})_{\gamma\delta} \\
& +(\check{\Delta}^{he}_{18}\check{\Delta}^{bi}_{72})_{\alpha\beta}(\check{\Delta}^{jd}_{37}\check{\Delta}^{ak}_{54})_{\gamma\delta} + (\check{\Delta}^{hb}_{16}\check{\Delta}^{ei}_{82})_{\alpha\beta}(\check{\Delta}^{jd}_{37}\check{\Delta}^{ak}_{54})_{\gamma\delta} \\
& +(\check{\Delta}^{he}_{18}\check{\Delta}^{bk}_{64})_{\alpha\delta}(\check{\Delta}^{id}_{27}\check{\Delta}^{aj}_{53})_{\beta\gamma} + (\check{\Delta}^{hd}_{17}\check{\Delta}^{aj}_{53})_{\alpha\gamma}(\check{\Delta}^{ie}_{28}\check{\Delta}^{bk}_{64})_{\beta\delta} \\
& +(\check{\Delta}^{he}_{18}\check{\Delta}^{bj}_{63})_{\alpha\gamma}(\check{\Delta}^{id}_{27}\check{\Delta}^{ak}_{54})_{\beta\delta} + (\check{\Delta}^{hd}_{17}\check{\Delta}^{ak}_{54})_{\alpha\delta}(\check{\Delta}^{ie}_{28}\check{\Delta}^{bj}_{63})_{\beta\gamma}\Big] \qquad \text{(A.20)}
\end{aligned}
$$

in this expression each 2 rows differ by

- row 1/2 by $a \leftrightarrow b$ and $d \leftrightarrow e$,
- row 3/4 by $a \leftrightarrow d$ and $b \leftrightarrow e$,
- row 5/6 by $a \leftrightarrow e$ and $b \leftrightarrow d$,
- row 7/8 by $a \leftrightarrow e$ and $b \leftrightarrow d$,
- row 8/9 by $b \leftrightarrow e$ and $d \leftrightarrow a$,
- row 10/11 by $a \leftrightarrow b$ and $e \leftrightarrow d$,

and by exchanging the momenta k_5, k_6, k_7, k_8, but these are just integration variables. This leads to

$$\begin{aligned}
&-\frac{g^2}{2} f^{abc} f^{cde} \\
&\Big[(\check{\Delta}^{bh}_{16}\check{\Delta}^{ej}_{83})_{\alpha\gamma}(\check{\Delta}^{ia}_{25}\check{\Delta}^{dk}_{74})_{\beta\delta} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{dk}_{74})_{\alpha\delta}(\check{\Delta}^{ib}_{26}\check{\Delta}^{je}_{83})_{\beta\gamma} \\
&+(\check{\Delta}^{hd}_{17}\check{\Delta}^{ai}_{52})_{\alpha\beta}(\check{\Delta}^{jb}_{36}\check{\Delta}^{ek}_{84})_{\gamma\delta} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{di}_{72})_{\alpha\beta}(\check{\Delta}^{jb}_{36}\check{\Delta}^{ek}_{84})_{\gamma\delta} \\
&+(\check{\Delta}^{hd}_{17}\check{\Delta}^{aj}_{53})_{\alpha\gamma}(\check{\Delta}^{ib}_{26}\check{\Delta}^{ek}_{84})_{\beta\delta} + (\check{\Delta}^{hb}_{16}\check{\Delta}^{ek}_{z8})_{\alpha\delta}(\check{\Delta}^{id}_{27}\check{\Delta}^{aj}_{53})_{\beta\gamma} \\
&+(\check{\Delta}^{he}_{18}\check{\Delta}^{bj}_{63})_{\alpha\gamma}(\check{\Delta}^{ia}_{25}\check{\Delta}^{dk}_{74})_{\beta\delta} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{dk}_{74})_{\alpha\delta}(\check{\Delta}^{ie}_{28}\check{\Delta}^{bj}_{63})_{\beta\gamma} \\
&+(\check{\Delta}^{he}_{18}\check{\Delta}^{bi}_{62})_{\alpha\beta}(\check{\Delta}^{ja}_{35}\check{\Delta}^{dk}_{74})_{\gamma\delta} + (\check{\Delta}^{hb}_{16}\check{\Delta}^{ei}_{82})_{\alpha\beta}(\check{\Delta}^{ja}_{35}\check{\Delta}^{dk}_{74})_{\gamma\delta} \\
&+(\check{\Delta}^{he}_{18}\check{\Delta}^{bk}_{64})_{\alpha\delta}(\check{\Delta}^{id}_{27}\check{\Delta}^{aj}_{53})_{\beta\gamma} + (\check{\Delta}^{hd}_{17}\check{\Delta}^{aj}_{53})_{\alpha\gamma}(\check{\Delta}^{ie}_{28}\check{\Delta}^{bk}_{64})_{\beta\delta}\Big].
\end{aligned} \tag{A.21}$$

In this expression the rows again differ only by exchanging some indices and the momenta k_5, k_6, k_7, k_8

- row 1/6 by $b \leftrightarrow d$ and $a \leftrightarrow e$,
- row 2/5 by $a \leftrightarrow b$ and $d \leftrightarrow e$,
- row 1/6 by $a \leftrightarrow b$ and $d \leftrightarrow e$.

Now we evaluate the δ_{ij} from eq.(A.19) for the color-indices:

$$\begin{aligned}
&-g^2 f^{abc} f^{cde}\Big[(\check{\Delta}^{bh}_{16}\check{\Delta}^{ej}_{83})_{\alpha\gamma}(\check{\Delta}^{ia}_{25}\check{\Delta}^{dk}_{74})_{\beta\delta} + (\check{\Delta}^{ha}_{15}\check{\Delta}^{dk}_{74})_{\alpha\delta}(\check{\Delta}^{ib}_{26}\check{\Delta}^{je}_{83})_{\beta\gamma} + (\check{\Delta}^{hd}_{17}\check{\Delta}^{ai}_{52})_{\alpha\beta}(\check{\Delta}^{jb}_{36}\check{\Delta}^{ek}_{84})_{\gamma\delta} \\
&+(\check{\Delta}^{ha}_{15}\check{\Delta}^{di}_{72})_{\alpha\beta}(\check{\Delta}^{jb}_{36}\check{\Delta}^{ek}_{84})_{\gamma\delta} + (\check{\Delta}^{hd}_{17}\check{\Delta}^{aj}_{53})_{\alpha\gamma}(\check{\Delta}^{ib}_{26}\check{\Delta}^{ek}_{84})_{\beta\delta} + (\check{\Delta}^{hb}_{16}\check{\Delta}^{ek}_{84})_{\alpha\delta}(\check{\Delta}^{id}_{27}\check{\Delta}^{aj}_{53})_{\beta\gamma}\Big] \\
= \quad &-g^2\Big[f^{ihc} f^{ckj}(\check{\Delta}^{h}_{16}\check{\Delta}^{j}_{83})_{\alpha\gamma}(\check{\Delta}^{i}_{25}\check{\Delta}^{k}_{74})_{\beta\delta} + f^{hic} f^{ckj}(\check{\Delta}^{h}_{15}\check{\Delta}^{k}_{74})_{\alpha\delta}(\check{\Delta}^{i}_{26}\check{\Delta}^{j}_{83})_{\beta\gamma} \\
&+f^{ijc} f^{chk}(\check{\Delta}^{h}_{17}\check{\Delta}^{i}_{52})_{\alpha\beta}(\check{\Delta}^{j}_{36}\check{\Delta}^{k}_{84})_{\gamma\delta} + f^{hjc} f^{cik}(\check{\Delta}^{h}_{15}\check{\Delta}^{i}_{72})_{\alpha\beta}(\check{\Delta}^{j}_{36}\check{\Delta}^{k}_{84})_{\gamma\delta} \\
&+f^{jic} f^{chk}(\check{\Delta}^{h}_{17}\check{\Delta}^{j}_{53})_{\alpha\gamma}(\check{\Delta}^{i}_{26}\check{\Delta}^{k}_{84})_{\beta\delta} + f^{jhc} f^{cik}(\check{\Delta}^{h}_{16}\check{\Delta}^{k}_{84})_{\alpha\delta}(\check{\Delta}^{i}_{27}\check{\Delta}^{j}_{53})_{\beta\gamma}\Big].
\end{aligned} \tag{A.22}$$

Finally we can permute the f^{ijk} and integrate over the momenta k_5, k_6, k_7, k_8 giving

$$\begin{aligned}
-(2\pi)^4 g^2 \delta^{(4)}(k_1+k_2+k_3+k_4)\Big[& f^{ihc} f^{ckj}\left[(\check{\Delta}^{h}_{1}\check{\Delta}^{j}_{3})_{\alpha\gamma}(\check{\Delta}^{i}_{2}\check{\Delta}^{k}_{4})_{\beta\delta} - (\check{\Delta}^{h}_{1}\check{\Delta}^{k}_{4})_{\alpha\delta}(\check{\Delta}^{i}_{2}\check{\Delta}^{j}_{3})_{\beta\gamma}\right] \\
&+ f^{ijc} f^{chk}\left[(\check{\Delta}^{h}_{1}\check{\Delta}^{i}_{2})_{\alpha\beta}(\check{\Delta}^{j}_{3}\check{\Delta}^{k}_{4})_{\gamma\delta} - (\check{\Delta}^{h}_{1}\check{\Delta}^{j}_{3})_{\alpha\gamma}(\check{\Delta}^{i}_{2}\check{\Delta}^{k}_{4})_{\beta\delta}\right] \\
&+ f^{hjc} f^{cik}\left[(\check{\Delta}^{h}_{1}\check{\Delta}^{i}_{2})_{\alpha\beta}(\check{\Delta}^{j}_{3}\check{\Delta}^{k}_{4})_{\gamma\delta} - (\check{\Delta}^{h}_{1}\check{\Delta}^{k}_{4})_{\alpha\delta}(\check{\Delta}^{i}_{2}\check{\Delta}^{j}_{3})_{\beta\gamma}\right]\Big].
\end{aligned} \tag{A.23}$$

Three Non-Abelian Gauge Bosons

We take the relevant part of the Lagrangian density eq.(A.16)

$$\begin{aligned}
L &= \int d^4x\, \mathcal{L} \\
&= -\frac{g}{2} f^{abc} \int d^4x \Big(\partial_\nu \int \frac{d^4k_1}{(2\pi)^4} \check{G}^a_\mu(k_1) e^{ik_1\cdot x}\Big) \int \frac{d^4k_2}{(2\pi)^4} \check{G}^{b\mu}(k_2) e^{ik_2\cdot x} \int \frac{d^4k_3}{(2\pi)^4} \check{G}^{c\nu} e^{ik_3\cdot x} \\
&= -\frac{g}{2} f^{abc} \int \frac{d^4k_1\; d^4k_2\; d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3)\, ik_{1\nu} \check{G}^a_\mu(k_1) \check{G}^{b\mu}(k_2) \check{G}^{c\nu}(k_3). \qquad \text{(A.24)}
\end{aligned}$$

Now we evaluate $\langle 0|T\check{G}\check{G}\check{G}L|0\rangle$, using again eq.(A.19)

$$\begin{aligned}
&\langle 0|T\check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3|0\rangle \\
= \;& \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 \\
&+ \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 \\
&+ \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 \\
&+ \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 \\
&+ \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 + \check{G}^d_{\rho 4}\check{G}^e_{\sigma 5}\check{G}^f_{\eta 6}\check{G}^a_{\mu 1}\check{G}^{b\mu}_2\check{G}^{c\nu}_3 \\
= \;& \big[\check{\Delta}^{da}_{\rho\mu 41}\check{\Delta}^{eb\,\mu}_{\sigma\,52}\check{\Delta}^{fc\,\nu}_{\eta\,63} + \check{\Delta}^{da}_{\rho\mu 41}\check{\Delta}^{ec\,\nu}_{\sigma\,53}\check{\Delta}^{fb\,\mu}_{\eta\,62} + \check{\Delta}^{db\,\mu}_{\rho\,42}\check{\Delta}^{ea}_{\sigma\mu 51}\check{\Delta}^{fc\,\nu}_{\eta\,63} \\
&+ \check{\Delta}^{db\,\mu}_{\rho\,42}\check{\Delta}^{ec\,\nu}_{\sigma\,53}\check{\Delta}^{fa}_{\eta\mu 61} + \check{\Delta}^{dc\,\nu}_{\rho\,43}\check{\Delta}^{eb\,\mu}_{\sigma\,52}\check{\Delta}^{fa}_{\eta\mu 61} + \check{\Delta}^{dc\,\nu}_{\rho\,43}\check{\Delta}^{ea}_{\sigma\mu 51}\check{\Delta}^{fb\,\mu}_{\eta\,62}\big]. \qquad \text{(A.25)}
\end{aligned}$$

Combining this with eq.(A.24) we obtain

$$\begin{aligned}
&\langle 0|iT\check{G}\check{G}\check{G}\mathcal{L}|0\rangle \\
= \;& -\frac{ig}{2}\int \frac{d^4k_1\; d^4k_2\; d^4k_3}{(2\pi)^8}\delta^{(4)}(k_1+k_2+k_3) \\
&\Big[f^{def} ik_{1\nu}\check{\Delta}^d_{\rho\mu 41}\check{\Delta}^{e\,\mu}_{\sigma\,52}\check{\Delta}^{f\,\nu}_{\eta\,63} + f^{dfe} ik_{1\nu}\check{\Delta}^d_{\rho\mu 41}\check{\Delta}^{e\,\nu}_{\sigma\,53}\check{\Delta}^{f\,\mu}_{\eta\,62} \\
&+ f^{edf} ik_{1\nu}\check{\Delta}^{d\,\mu}_{\rho\,42}\check{\Delta}^e_{\sigma\mu 51}\check{\Delta}^{f\,\nu}_{\eta\,63} + f^{fde} ik_{1\nu}\check{\Delta}^{d\,\mu}_{\rho\,42}\check{\Delta}^{e\,\nu}_{\sigma\,53}\check{\Delta}^f_{\eta\mu 61} \\
&+ f^{fed} ik_{1\nu}\check{\Delta}^{d\,\nu}_{\rho\,43}\check{\Delta}^{e\,\mu}_{\sigma\,52}\check{\Delta}^f_{\eta\mu 61} + f^{efd} ik_{1\nu}\check{\Delta}^{d\,\nu}_{\rho\,43}\check{\Delta}^e_{\sigma\mu 51}\check{\Delta}^{f\,\mu}_{\eta\,62} \\
&- f^{def} ik_{1\mu}\check{\Delta}^d_{\rho\nu 41}\check{\Delta}^{e\,\mu}_{\sigma\,52}\check{\Delta}^{f\,\nu}_{\eta\,63} - f^{dfe} ik_{1\mu}\check{\Delta}^d_{\rho\nu 41}\check{\Delta}^{e\,\nu}_{\sigma\,53}\check{\Delta}^{f\,\mu}_{\eta\,62} \\
&- f^{edf} ik_{1\mu}\check{\Delta}^{d\,\mu}_{\rho\,42}\check{\Delta}^e_{\sigma\nu 51}\check{\Delta}^{f\,\nu}_{\eta\,63} - f^{fde} ik_{1\mu}\check{\Delta}^{d\,\mu}_{\rho\,42}\check{\Delta}^{e\,\nu}_{\sigma\,53}\check{\Delta}^f_{\eta\nu 61} \\
&- f^{fed} ik_{1\mu}\check{\Delta}^{d\,\nu}_{\rho\,43}\check{\Delta}^{e\,\mu}_{\sigma\,52}\check{\Delta}^f_{\eta\nu 61} - f^{efd} ik_{1\mu}\check{\Delta}^{d\,\nu}_{\rho\,43}\check{\Delta}^e_{\sigma\nu 51}\check{\Delta}^{f\,\mu}_{\eta\,62}\Big]. \qquad \text{(A.26)}
\end{aligned}$$

The right and the left half of the addends differ by interchanging the indices 2 and 3 of the $\check{\Delta}$ standing for the integration variables k_2 and k_3 which we are free to change. After permuting the f^{def} we get

$$
\begin{aligned}
& -i^2 g \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) f^{def} \\
& \Big[k_{1\nu} \check{\Delta}^d_{\rho\mu 41} \check{\Delta}^{e\ \mu}_{\sigma\ 52} \check{\Delta}^{f\ \nu}_{\eta\ 63} - k_{1\nu} \check{\Delta}^d_{\rho\mu 41} \check{\Delta}^{e\ \nu}_{\sigma\ 53} \check{\Delta}^{f\ \mu}_{\eta\ 62} - k_{1\nu} \check{\Delta}^{d\ \mu}_{\rho\ 42} \check{\Delta}^e_{\sigma\mu 51} \check{\Delta}^{f\ \nu}_{\eta\ 63} \\
& + k_{1\nu} \check{\Delta}^{d\ \mu}_{\rho\ 42} \check{\Delta}^{e\ \nu}_{\sigma\ 53} \check{\Delta}^f_{\eta\mu 61} - k_{1\nu} \check{\Delta}^{d\ \nu}_{\rho\ 43} \check{\Delta}^{e\ \mu}_{\sigma\ 52} \check{\Delta}^f_{\eta\mu 61} + k_{1\nu} \check{\Delta}^{d\ \nu}_{\rho\ 43} \check{\Delta}^e_{\sigma\mu 51} \check{\Delta}^{f\ \mu}_{\eta\ 62} \Big] \\
= \;& g \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) f^{def} \Big[\check{\Delta}^d_{\rho\mu 41} \check{\Delta}^{e\ \mu}_{\sigma\ 52} \check{\Delta}^{f\ \nu}_{\eta\ 63} (k_{1\nu} - k_{2\nu}) \\
& + \check{\Delta}^{d\ \mu}_{\rho\ 42} \check{\Delta}^{e\ \nu}_{\sigma\ 53} \check{\Delta}^f_{\eta\mu 61} (k_{1\nu} - k_{2\nu}) + \check{\Delta}^{d\ \nu}_{\rho\ 43} \check{\Delta}^{e\ \mu}_{\sigma\ 52} \check{\Delta}^f_{\eta\mu 61} (k_{2\nu} - k_{1\nu}) \Big] \\
= \;& g \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) f^{def} \Big[\check{\Delta}^d_{\rho\mu 41} \check{\Delta}^{e\ \mu}_{\sigma\ 52} \check{\Delta}^{f\ \nu}_{\eta\ 63} (k_{1\nu} - k_{2\nu}) \\
& + \check{\Delta}^{d\ \mu}_{\rho\ 41} \check{\Delta}^{e\ \nu}_{\sigma\ 52} \check{\Delta}^f_{\eta\mu 63} (k_{3\nu} - k_{1\nu}) + \check{\Delta}^{d\ \nu}_{\rho\ 41} \check{\Delta}^{e\ \mu}_{\sigma\ 52} \check{\Delta}^f_{\eta\mu 63} (k_{2\nu} - k_{3\nu}) \Big] \\
= \;& g (2\pi)^4 \delta^{(4)}(k_1+k_2+k_3) f^{def} \Big[\check{\Delta}^d_{\rho\mu}(k_1) \check{\Delta}^{e\ \mu}_{\sigma}(k_2) \check{\Delta}^{f\ \nu}_{\eta}(k_3) (k_{1\nu} - k_{2\nu}) \\
& + \check{\Delta}^{d\ \mu}_{\rho}(k_1) \check{\Delta}^{e\ \nu}_{\sigma}(k_2) \check{\Delta}^f_{\eta\mu}(k_3) (k_{3\nu} - k_{1\nu}) + \check{\Delta}^{d\ \nu}_{\rho}(k_1) \check{\Delta}^{e\ \mu}_{\sigma}(k_2) \check{\Delta}^f_{\eta\mu}(k_3) (k_{2\nu} - k_{3\nu}) \Big]. \quad \text{(A.27)}
\end{aligned}
$$

Fermion-Fermion-Gauge Boson

The part of the Lagrangian density involving a fermion-fermion-gauge boson interaction is given by

$$
\begin{aligned}
\mathcal{L} &= g G_{\mu z} \chi_z \sigma^\mu \bar{\psi}_z, \\
L &= \int d^4z \mathcal{L} \\
&= \int d^4z \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^12} e^{-iz(k_1+k_2+k_3)} g \check{G}_{\mu 1} \check{\chi}_2 \sigma^\mu \check{\bar{\psi}}_3 \\
&= \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) g t^a \check{G}_{\mu 1} \check{\chi}_2 \sigma^\mu \check{\bar{\psi}}_3, \qquad \text{(A.28)}
\end{aligned}
$$

with fermions ψ, χ and gauge boson G_μ. The time-ordered product gives

$$
\begin{aligned}
& \langle 0 | iT \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) g \check{G}^*_{\nu 4} \check{\psi}^\beta_5 \check{\bar{\chi}}^{\dot\beta}_6 \check{G}_{\mu 1} \check{\chi}^\alpha_2 \sigma^\mu_{\alpha\dot\alpha} \check{\bar{\psi}}^{\dot\alpha}_3 | 0 \rangle \\
= \;& i \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) g \check{G}^*_{\nu 4} \check{\psi}^\beta_5 \check{\bar{\chi}}^{\dot\beta}_6 \check{G}_{\mu 1} \check{\chi}^\alpha_2 \sigma^\mu_{\alpha\dot\alpha} \check{\bar{\psi}}^{\dot\alpha}_3 \\
= \;& -ig \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) \check{\Delta}^G_{\nu\mu 41} \check{\Delta}^{\chi\dot\beta}_{26} \sigma^\mu_{\alpha\dot\alpha} \check{\Delta}^{\psi\dot\alpha\beta}_{35} \\
= \;& (2\pi)^4 \delta^{(4)}(k_4+k_5+k_6)(-ig) \check{\Delta}^G_{\nu\mu}(k_4) \left(\check{\Delta}^{\chi\dot\beta\alpha}(k_6) \sigma^\mu_{\alpha\dot\alpha} \check{\Delta}^{\psi\dot\alpha\beta}(k_5) \right), \qquad \text{(A.29)}
\end{aligned}
$$

with $\check{\Delta}^G_{\nu\mu 41} = (2\pi)^4 \delta^{(4)}(k_4 - k_1) \check{\Delta}^G_{\nu\mu}$ and $\check{\Delta}^{\psi\dot\alpha\beta}_{35}$ analogously and external particles $G_{\nu 4}, \psi_5, \bar{\chi}_6$. We note that the additional contractions would give only disconnected diagrams if both fermions were identical.

Fermion-Sfermion-Gaugino

The relevant part of the Lagrangian density is given by

$$
\begin{aligned}
\mathcal{L} &= g\tilde{\phi}_z\psi_z\tilde{G}_z, \\
L &= \int d^4z\mathcal{L} \\
&= \int d^4z \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^12} e^{-iz(k_1+k_2+k_3)} g\left(\check{\tilde{\phi}}_2\check{\psi}_1\check{\tilde{G}}_3 + \text{h.c.}\right) \\
&= \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8}\delta^{(4)}(k_1+k_2+k_3)g\left(\check{\tilde{\phi}}_2\check{\psi}_1\check{\tilde{G}}_3 + \text{h.c.}\right), \qquad (\text{A.30}) \\
& \qquad (\text{A.31})
\end{aligned}
$$

with fermion ψ, sfermion $\tilde{\phi}$ and gaugino $\tilde{G}$. The time-ordered product gives

$$
\begin{aligned}
& \langle 0|iT \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8}\delta^{(4)}(k_1+k_2+k_3)g\check{\psi}_1\check{\tilde{\phi}}_2\check{\tilde{G}}_3\check{\bar{\tilde{\phi}}}{}^*_5\check{\bar{\psi}}_4\check{\bar{\tilde{G}}}_6|0\rangle \\
= \ & i\int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8}\delta^{(4)}(k_1+k_2+k_3)g\ \check{\psi}_1\check{\tilde{\phi}}_2\check{\tilde{G}}_3\check{\bar{\tilde{\phi}}}{}^*_5\check{\bar{\psi}}_4\check{\bar{\tilde{G}}}_6 \\
= \ & \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8}\delta^{(4)}(k_1+k_2+k_3)(-ig)\check{\Delta}^{\psi}_{14}\check{\Delta}^{\tilde{\phi}}_{25}\check{\Delta}^{\tilde{G}}_{36} \\
= \ & (2\pi)^4\delta^{(4)}(k_4+k_5+k_6)(-ig)\check{\Delta}^{\psi}(k_4)\check{\Delta}^{\tilde{\phi}}(k_5)\check{\Delta}^{\tilde{G}}(k_6), \qquad (\text{A.32})
\end{aligned}
$$

with external particles $\psi_4, \tilde{\phi}_5, \tilde{G}_6$.

Two Sfermions-Two Gauge Bosons

The relevant part of the Lagrangian density is

$$
\begin{aligned}
\mathcal{L} &= g\phi^*_z\chi_zA_{\mu z}G^{\mu}_z, \\
L &= \int \frac{d^4k_1\ d^4k_2\ d^4k_3\ d^4k_4}{(2\pi)^12}\delta^{(4)}(k_1+k_2+k_3+k_4)\check{\phi}^*_1\check{\chi}_2\check{A}_{\mu 3}\check{G}^{\mu}_4, \\
& \qquad (\text{A.33})
\end{aligned}
$$

with sfermions ϕ, χ and gauge bosons A_μ, G_μ. The time-ordered product gives

$$
\begin{aligned}
& \langle 0|T \int \frac{d^4k_1\ d^4k_2\ d^4k_3\ d^4k_4}{(2\pi)^12}\delta^{(4)}(k_1+k_2+k_3+k_4)\check{\phi}_5\check{\chi}^*_6\check{A}^*_{\rho 7}\check{G}^{*\nu}_8\check{\phi}^*_1\check{\chi}_2\check{A}_{\mu 3}\check{G}^{\mu}_4|0\rangle \\
= \ & ig\int \frac{d^4k_1\ d^4k_2\ d^4k_3\ d^4k_4}{(2\pi)^12}\delta^{(4)}(k_1+k_2+k_3+k_4)\ \check{\phi}_5\check{\chi}^*_6\check{A}^*_{\rho 7}\check{G}^{*\nu}_8\check{\phi}^*_1\check{\chi}_2\check{A}_{\mu 3}\check{G}^{\mu}_4 \\
= \ & (2\pi)^4\delta^{(4)}(k_5+k_6+k_7+k_8)ig\check{\Delta}^{\phi}(k_5)\check{\Delta}^{\chi}(k_6)\check{\Delta}^{A}_{\mu\rho}(k_7)\check{\Delta}^{G\nu\mu}, \qquad (\text{A.34})
\end{aligned}
$$

with external particles $\phi_5, \chi_6, A_{\nu 7}, G^{\nu}_8$. We note that an additional factor 2 appears in case the gauge bosons are real and identical.

Four Sfermions

The relevant part of the Lagrangian density is given by

$$\begin{aligned} \mathcal{L} &= g\phi_z^*\chi_z\xi_z^*\lambda_z, \\ L &= \int \frac{d^4k_1\ d^4k_2\ d^4k_3\ d^4k_4}{(2\pi)^{12}}\delta^{(4)}(k_1+k_2+k_3+k_4)g\check{\phi}_1^*\check{\chi}_2\check{\xi}_3^*\check{\lambda}_4, \end{aligned} \tag{A.35}$$

with sfermions ϕ, χ, ξ, λ The time-ordered product gives

$$\begin{aligned} &\langle 0|iT\int \frac{d^4k_1\ d^4k_2\ d^4k_3\ d^4k_4}{(2\pi)^{12}}\delta^{(4)}(k_1+k_2+k_3+k_4)g\check{\phi}_5\check{\chi}_6^*\check{\xi}_7\check{\lambda}_8^*\check{\phi}_1^*\check{\chi}_2\check{\xi}_3^*\check{\lambda}_4|0\rangle \\ &= ig\int \frac{d^4k_1\ d^4k_2\ d^4k_3\ d^4k_4}{(2\pi)^{12}}\delta^{(4)}(k_1+k_2+k_3+k_4)\ \check{\phi}_5\check{\chi}_6^*\check{\xi}_7\check{\lambda}_8^*\check{\phi}_1^*\check{\chi}_2\check{\xi}_3^*\check{\lambda}_4 \\ &= ig(2\pi)^4\delta^{(4)}(k_5+k_6+k_7+k_8)\check{\Delta}^{\phi}(k_5)\check{\Delta}^{\chi}(k_6)\check{\Delta}^{\xi}(k_7)\check{\Delta}^{\lambda}(k_8), \end{aligned} \tag{A.36}$$

with external particles $\phi_5, \xi_7, \chi_6, \lambda_8$.

Three Sfermions

The relevant part of the Lagrangian density is given by

$$\begin{aligned} \mathcal{L} &= g\phi_z^*\chi_z\xi_z^*, \\ L &= \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^{8}}\delta^{(4)}(k_1+k_2+k_3)g\check{\phi}_1^*\check{\chi}_2\check{\xi}_3^*, \end{aligned} \tag{A.37}$$

with sfermions ϕ, χ, ξ. The time-ordered product gives

$$\begin{aligned} &\langle 0|iT\int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^{8}}\delta^{(4)}(k_1+k_2+k_3)g\check{\phi}_5\check{\chi}_6^*\check{\xi}_7\phi_1^*\check{\chi}_2\check{\xi}_3^*|0\rangle \\ &= ig\int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^{8}}\delta^{(4)}(k_1+k_2+k_3+k_4)\ \check{\phi}_5\check{\chi}_6^*\check{\xi}_7\check{\lambda}_8^*\check{\phi}_1^*\check{\chi}_2\check{\xi}_3^*\check{\lambda}_4 \\ &= ig(2\pi)^4\delta^{(4)}(k_5+k_6+k_7)\check{\Delta}^{\phi}(k_5)\check{\Delta}^{\chi}(k_6)\check{\Delta}^{\xi}(k_7), \end{aligned} \tag{A.38}$$

with external particles ϕ_5, χ_6, ξ_7.

Two Sfermions-Gauge Boson

The relevant part of the Lagrangian density is given by

$$\begin{aligned} \mathcal{L} &= g(\partial_\mu\phi_z)\chi_z A_z^\mu, \\ L &= \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^{8}}\delta^{(4)}(k_1+k_2+k_3)g(-ik_{\mu 1})\check{\phi}_1\check{\chi}_2\check{A}_3^\mu, \end{aligned} \tag{A.39}$$

with sfermions ϕ, χ and gauge boson A^μ. The time-ordered product gives

$$\begin{aligned} &\langle 0|T\int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^{8}}\delta^{(4)}(k_1+k_2+k_3)ig\check{\phi}_4^*\check{\chi}_5^*\check{A}_6^{*\nu}(-ik_{\mu 1})\check{\phi}_1\check{\chi}_2\check{A}_3^\mu|0\rangle \\ &= g\int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^{8}}\delta^{(4)}(k_1+k_2+k_3)k_{\mu 1}\ \check{\phi}4^*\check{\chi}_5^*\check{A}_6^{*\nu}\check{\phi}_1\check{\chi}_2\check{A}_3^\mu \\ &= gk_{\mu 4}(2\pi)^4\delta^{(4)}(k_4+k_5+k_6)\check{\Delta}^{\phi}(k_4)\check{\Delta}^{\chi}(k_5)\check{\Delta}^{A\mu\nu}(k_6), \end{aligned} \tag{A.40}$$

with external particles ϕ_4, χ_5, A_6^ν.

Two Gauge Bosons-Boson

The relevant part of the Lagrangian density is given by

$$\begin{aligned}
\mathcal{L} &= g\phi_z A_{\mu z} G_z^\mu, \\
L &= \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) g \check{\phi}_1 \check{A}_{\mu 2} \check{G}_3^\mu,
\end{aligned} \tag{A.41}$$

with gauge bosons A_μ, G^μ and boson ϕ. The time-ordered product gives

$$\begin{aligned}
&\langle 0|iT \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) g \check{\phi}_4 \check{A}_{\rho 5} \check{G}_6^{*\nu} \check{\phi}_1 \check{A}_{\mu 2} \check{G}_3^\mu |0\rangle \\
&= \int \frac{d^4k_1\ d^4k_2\ d^4k_3}{(2\pi)^8} \delta^{(4)}(k_1+k_2+k_3) ig\, \check{\phi}_4 \check{A}_{\rho 5} \check{G}_6^{*\nu} \check{\phi}_1 \check{A}_{\mu 2} \check{G}_3^\mu \\
&= ig(2\pi)^4 \delta^{(4)}(k_4+k_5+k_6) \check{\Delta}^\phi(k_4) \check{\Delta}^A_{\rho\mu}(k_5) \check{\Delta}^{G\mu\nu}(k_6),
\end{aligned} \tag{A.42}$$

with external particles $\phi_4, A_{\rho 5}, G_6^\nu$. We note that an additional factor 2 appears in case the gauge bosons are real and identical.

Mass Term

The mass term in the Lagrangian density is given by

$$\begin{aligned}
\mathcal{L} &= -m\psi\chi - m\bar{\psi}\bar{\chi}, \\
L &= -m \int \frac{d^4p_1\ d^4p_2}{(2\pi)^4} \delta^{(4)}(p_1+p_2) \left(\check{\psi}_1 \check{\chi}_2 + \text{h.c.}\right)
\end{aligned} \tag{A.43}$$

with Weyl spinors χ, ψ. The time-ordered product then gives

$$\begin{aligned}
&\langle 0|iT \check{\bar{\psi}}_3 \check{\bar{\chi}}_4 L|0\rangle \\
&= -im \int \frac{d^4k_1\ d^4k_2}{(2\pi)^4} \delta^{(4)}(k_1+k_2)\, \check{\bar{\chi}}_{4\dot\alpha} \check{\bar{\psi}}_4^{\dot\alpha} \check{\psi}_1^\beta \check{\chi}_{2\beta} \\
&= im(2\pi)^4 \delta^{(4)}(p_3+p_4) \check{\Delta}^\chi(p_3) \check{\Delta}^\psi(p_4).
\end{aligned} \tag{A.44}$$

We note that in the case $\chi = \psi$ we get an additional factor of 2.

A.3.2 Propagators

Free Weyl-Spinors

The generating functional for the propagator of a free Weyl-spinor is

$$Z[\eta,\bar{\eta}] = N \int D\psi D\bar{\psi} e^{i\int d^4x \left(i\psi\sigma^\mu\partial_\mu\bar{\psi} + \eta\psi + \bar{\eta}\bar{\psi}\right)}. \tag{A.45}$$

We need to rewrite it in order to extract some information. So we take a look at the equations of motion for a free right-chiral Weyl-spinor.

$$i\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}^{\dot{\alpha}} = 0 \tag{A.46}$$

Its Green's function $\Delta^{r\dot{\alpha}\beta}_{xy}$ is defined by

$$i\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_\mu\Delta^{r\dot{\alpha}\beta}_{xy} \quad = \quad i\delta^{(4)}(x-y)\delta^{\beta}_{\alpha}. \tag{A.47}$$

In momentum space this gives

$$\begin{aligned} \sigma^{\mu}_{\alpha\dot{\alpha}}p_\mu\check{\Delta}^{r\dot{\alpha}\beta}_{p} &= i\delta^{\beta}_{\alpha} \\ p_\nu\bar{\sigma}^{\nu\dot{\beta}\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}p_\mu\check{\Delta}^{r\dot{\alpha}\beta}_{p} &= ip_\nu\bar{\sigma}^{\nu\dot{\beta}\beta}. \end{aligned} \tag{A.48}$$

With $\bar{\sigma}^\nu\sigma^\mu = g^{\mu\nu} + 2\bar{\sigma}^{\nu\mu}$, $\sigma^\mu p_\mu\sigma^\nu p_\nu = p^2$ and the antisymmetry of $\bar{\sigma}^{\nu\mu}$ we get

$$\check{\Delta}^{r\dot{\beta}\beta}_{p} \quad = \quad i\frac{p_\nu\bar{\sigma}^{\nu\dot{\beta}\beta}}{p^2}, \tag{A.49}$$

which we translate back to position space

$$\Delta^{r\dot{\alpha}\alpha}_{xy} \quad = \quad \int\frac{d^4p}{(2\pi)^4}ie^{-ip(x-y)}\frac{p_\nu\bar{\sigma}^{\nu\dot{\alpha}\alpha}}{p^2}. \tag{A.50}$$

One has to be careful about the position of the spinor indices of Δ because of

$$\mathcal{L} = i\psi^\alpha\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}^{\dot{\alpha}} = i\bar{\psi}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\partial_\mu\psi_\alpha = -i\partial_\mu\psi^\alpha\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = -i\partial_\mu\bar{\psi}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\alpha}\psi_\alpha, \tag{A.51}$$

leading to the equations for the Green's functions

$$\begin{aligned} i\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_\mu\Delta^{r\dot{\alpha}\beta} = i\partial_\mu\Delta^{r}_{\alpha\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta} = i\delta^{(4)}(x-y)\delta^{\beta}_{\alpha}, \\ i\bar{\sigma}^{\mu\dot{\beta}\beta}\partial_\mu\Delta^{l}_{\beta\dot{\alpha}} = i\partial_\mu\Delta^{l\dot{\beta}\beta}\sigma^{\mu}_{\beta\dot{\alpha}} = i\delta^{(4)}(x-y)\delta^{\dot{\beta}}_{\dot{\alpha}}, \end{aligned} \tag{A.52}$$

which in momentum space are solved by

$$\check{\Delta}^{l}_{\alpha\dot{\alpha}} = i\frac{p_\mu\sigma^{\mu}_{\alpha\dot{\alpha}}}{p^2}, \quad \check{\Delta}^{l\dot{\alpha}\alpha} = i\frac{p^\mu\bar{\sigma}^{\dot{\alpha}\alpha}_{\mu}}{p^2}, \quad \check{\Delta}^{r}_{\alpha\dot{\alpha}} = i\frac{p_\mu\sigma^{\mu}_{\alpha\dot{\alpha}}}{p^2}, \quad \check{\Delta}^{r\dot{\alpha}\alpha} = i\frac{p^\mu\bar{\sigma}^{\dot{\alpha}\alpha}_{\mu}}{p^2}. \tag{A.53}$$

Now we turn to the calculation of the 2-point function via the generating functional

$$Z[\eta,\bar{\eta}] = N\int D\psi D\bar{\psi}e^{iL}, \tag{A.54}$$

where we defined L to contain the source terms. We only manipulate the exponent by using $\Delta^{l\dot{\alpha}\alpha} = \Delta^{r\dot{\alpha}\alpha}$ and shifting the integrands $\psi^\alpha_x = \psi'^\alpha_x + i\int d^4y\ \bar{\eta}_{y\dot{\alpha}}\Delta^{l\dot{\alpha}\alpha}_{xy}$, $\bar{\psi}^{\dot{\alpha}}_x = \bar{\psi}'^{\dot{\alpha}}_x + i\int d^4y\ \Delta^{r\dot{\alpha}\alpha}_{xy}\eta_{x\alpha}$

which does not affect the measure.

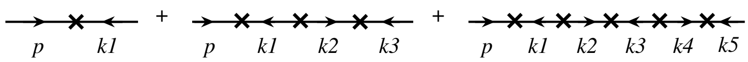
$$\begin{aligned}
L &= \int d^4x \Big(i\psi_x^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \bar\psi_x^{\dot\alpha} + \psi_x^\alpha \eta_{x\alpha} + \bar\eta_{\dot\alpha}\bar\psi^{\dot\alpha} \Big) \\
&= \int d^4x \Big(i\psi_x'^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \bar\psi_x'^{\dot\alpha} + i^2 \int d^4y \; \bar\eta_{y\dot\alpha} \Delta^{l\dot\alpha\alpha}_{xy} \sigma^\mu_{\alpha\dot\beta} \partial_\mu \bar\psi_x'^{\dot\beta} \\
&\quad + i^2 \int d^4y \; \psi_x'^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \Delta^{r\dot\alpha\beta}_{xy} \eta_{x\beta} \\
&\quad + i^3 \int d^4y \int d^4z \; \bar\eta_{y\dot\alpha} \Delta^{l\dot\alpha\alpha}_{xy} \sigma^\mu_{\alpha\dot\beta} \partial_\mu \Delta^{r\dot\beta\beta}_{xz} \eta_{z\beta} \\
&\quad + \psi_x'^\alpha \eta_{x\alpha} + i \int d^4y \; \bar\eta_{y\dot\alpha} \Delta^{l\dot\alpha\alpha}_{xy} \eta_{x\alpha} \\
&\quad + \bar\eta_{x\dot\alpha} \bar\psi_x'^{\dot\alpha} + i \int d^4y \; \bar\eta_{\dot\alpha x} \Delta^{\dot\alpha\alpha}_{xy} \eta_{y\alpha} \Big) \\
&= \int d^4x \Big(i\psi_x^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \bar\psi_x^{\dot\alpha} + i \int d^4y \; \bar\eta_{\dot\alpha y} \Delta^{l\dot\alpha\alpha}_{xy} \eta_{\alpha x} \\
&\quad - i \int d^4y \; \bar\eta_{\dot\alpha y} \Delta^{l\dot\alpha\alpha}_{xy} \eta_{\alpha x} + i \int d^4y \; \bar\eta_{y\dot\alpha} \Delta^{r\dot\alpha\alpha}_{xy} \eta_{x\alpha} \Big) \\
&= \int d^4x \Big(i\psi_x'^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \bar\psi_x'^{\dot\alpha} + i \int d^4z \; \bar\eta_{y\dot\alpha} \Delta^{r\dot\alpha\alpha}_{xy} \eta_{x\alpha} \Big). \qquad (A.55)
\end{aligned}$$

Now we can compute the 2-point function[1]:

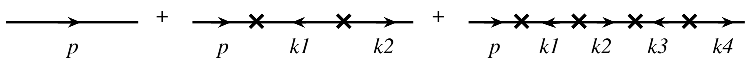
$$\begin{aligned}
\langle 0 | T \bar\psi_1^{\dot\beta} \psi_2^\beta | 0 \rangle &= \frac{1}{Z[0]} \frac{1}{i} \frac{\delta}{\delta\bar\eta_{\dot\beta 1}} \frac{1}{i} \frac{\delta}{\delta\eta_{\beta 2}} \\
&\quad \int D\psi D\bar\psi e^{i\int d^4x (i\psi^\alpha \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \bar\psi^{\dot\alpha})} e^{-\int d^4x \int d^4y \; \bar\eta_{\dot\alpha x} \Delta^{r\dot\alpha\alpha}_{xy} \eta_{\alpha y}} \Big|_{\eta=\bar\eta=0} \\
&= \frac{\delta}{\delta\bar\eta_{\dot\beta 1}} \int d^4y \; \bar\eta_{\dot\alpha x} \Delta^{r\dot\alpha\beta}_{x2} e^{-\int d^4x \int d^y \; \bar\eta_{\dot\alpha x} \Delta^{r\dot\alpha\alpha}_{xy} \eta_{\alpha y}} \Big|_{\eta=\bar\eta=0} \\
&= \Delta^{r\dot\beta\beta}_{12} e^{-\int d^4x \int d^4y \; \bar\eta_{\dot\alpha x} \Delta^{r\dot\alpha\alpha}_{xy} \eta_{\alpha y}} \Big|_{\eta=\bar\eta=0} \\
&= \Delta^{r\dot\beta\beta}_{12}. \qquad (A.56)
\end{aligned}$$

Everything seems fine. Now back to the pole in the expression for the propagator (A.53). We can modify its structure by adding up the contributions of the clashing mass terms of the fermions. We only take an even number of mass insertions in order not to interchange left to

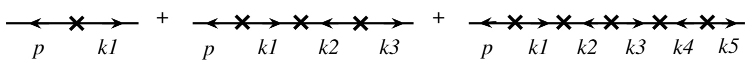
[1] It is important to remember that $\bar\psi^{\dot\beta}$ is a right chiral spinor and ψ^β an anti right-chiral spinor. So the superficial contradiction to the usual order vanishes, the ordering of the operators is OK.



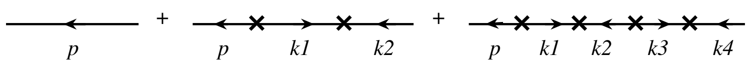
(a) incoming left-chiral/ outgoing right-chiral



(b) incoming left-chiral/ outgoing left-chiral



(c) incoming right-chiral/ outgoing left-chiral



(d) incoming right-chiral/ outgoing right-chiral

Figure A.1: Contributions of the mass terms to the propagator right-chiral fermions (figure(A.3.2)). We suppress the spinor indices and obtain:

$$\begin{aligned}
\check{\Delta}^l_{\text{tot},p} &= \check{\Delta}^l_p + \int d^4k_1 d^4k_2 (im)^2 \delta^{(4)}(p+k_1)\delta^{(4)}(k_1+k_2)\check{\Delta}^l_p\check{\Delta}^r_1\check{\Delta}^l_2 \\
&\quad + \int \sum_{i=1}^{4} dk_i (im)^4 \delta^{(4)}(p+k_1)\delta^{(4)}(k_1+k_2)\delta^{(4)}(k_2+k_3)\delta^{(4)}(k_3+k_4)\check{\Delta}^l_p\check{\Delta}^r_1\check{\Delta}^l_2\check{\Delta}^r_3\check{\Delta}^l_4 \\
&\quad + \ldots \\
&= \check{\Delta}^l_p - m^2\check{\Delta}^l_p \underbrace{\check{\Delta}^r_p\check{\Delta}^l_p}_{=-\frac{1}{p^2}} + m^4\check{\Delta}^l_p\check{\Delta}^r_p\check{\Delta}^l_p\check{\Delta}^r_p\check{\Delta}^l_p + \ldots \\
&= i\frac{\bar{\sigma}\cdot p}{p^2}\sum_{n=0}^{\infty}\left(\frac{m^2}{p^2}\right)^n \\
&= i\frac{\bar{\sigma}\cdot p}{p^2}\frac{1}{1-\frac{m^2}{p^2}} \\
&= i\frac{\bar{\sigma}\cdot p}{p^2-m^2}. \qquad \text{(A.57)}
\end{aligned}$$

Now the pole lies on the real axis only, however we note that the geometric series converges only for $p > m$. The correct way of handling the poles is to add $+i\epsilon$ in the denominator giving[2]

$$\check{\Delta}^l_{\text{tot}} = i\frac{\bar{\sigma}\cdot p}{p^2-m^2+i\epsilon}. \qquad \text{(A.58)}$$

An analogous calculation for $\check{\Delta}^r_{\text{tot}}$ leads to

$$\check{\Delta}^r_{\text{tot}} = i\frac{\bar{\sigma}\cdot p}{p^2-m^+ i\epsilon}. \qquad \text{(A.59)}$$

We can also try to sum the contributions of an odd number of mass terms, describing an incoming left-chiral and outgoing right-chiral fermion.

$$\begin{aligned}
\check{\Delta}^{lr}_{\text{tot},p} &= \int d^4k_1 (im)\delta^{(4)}(p+k_1)\check{\Delta}^l_p\check{\Delta}^r_1 \\
&\quad + \int d^4k_1\, d^4k_2\, d^4k_3 (im)^3\delta^{(4)}(p+k_1)\delta^{(4)}(k_1+k_2)\delta^{(4)}(k_2+k_3)\check{\Delta}^l_p\check{\Delta}^r_1\check{\Delta}^l_2\check{\Delta}^r_3 \\
&\quad + \int d^4k_1\, d^4k_2\, d^4k_3\, d^4k_4\, d^4k_5 (im)^5\delta^{(4)}(p+k_1)\delta^{(4)}(k_1+k_2)\delta^{(4)}(k_2+k_3) \\
&\qquad \times \delta^{(4)}(k_3+k_4)\delta^{(4)}(k_4+k_5)\check{\Delta}^l_p\check{\Delta}^r_1\check{\Delta}^l_2\check{\Delta}^r_3\check{\Delta}^l_4\check{\Delta}^r_5 \\
&\quad + \ldots \\
&= -im\frac{\bar{\sigma}\cdot p}{p^2}\frac{\sigma\cdot p}{p^2} - im^3\frac{\bar{\sigma}\cdot p}{p^2}\frac{\sigma\cdot p}{p^2}\frac{\bar{\sigma}\cdot p}{p^2}\frac{\sigma\cdot p}{p^2} - im^5\frac{\bar{\sigma}\cdot p}{p^2}\frac{\sigma\cdot p}{p^2}\frac{\bar{\sigma}\cdot p}{p^2}\frac{\sigma\cdot p}{p^2}\frac{\bar{\sigma}\cdot p}{p^2}\frac{\sigma\cdot p}{p^2} - \ldots \\
&= \frac{-im}{p^2}\left(1+\frac{m^2}{p^2}+\frac{m^4}{p^4}+\ldots\right) \\
&= \frac{-im}{p^2}\frac{1}{1-\frac{m^2}{p^2}} \\
&= \frac{-im}{p^2-m^2+i\epsilon}. \qquad \text{(A.60)}
\end{aligned}$$

[2]The $i\epsilon$ in the denominator comes from adding $+i\epsilon\times$ (term quadratic in the field) to $\mathcal{L}$ to make the path integral converge. For the sake of brevity however we will omit this when writing down $\mathcal{L}$.

In a similiar matter we get for $\check{\Delta}^{rl}_{\mathrm{tot},p}$:

$$\check{\Delta}^{rl}_{\mathrm{tot},p} = \frac{-im}{p^2 - m^2 + i\epsilon}. \tag{A.61}$$

Free Bosons

The generating functional for the propagator of a free boson ϕ with mass m is

$$Z[J, J^*] = N \int D\phi D\phi^* e^{i\int d^4x\left(-\phi^*(\Box+m^2)\phi - J\phi - J^*\phi^*\right)}. \tag{A.62}$$

In analogy to the propagator for the free Weyl spinor we shift the integration variables in the exponent by

$$\begin{aligned} \phi_x &= \phi'_x - i\int d^4z \Delta_{xz} J^*_z \\ \phi^*_x &= \phi^{*\prime}_x - i\int d^4z \Delta_{xz} J_z, \end{aligned} \tag{A.63}$$

with Δ_{xz} solving

$$\begin{aligned} (\Box + m^2)\Delta_{xz} &= -i\delta^{(4)}(x - z) \\ \Rightarrow \check{\Delta} &= \frac{i}{k^2 - m^2 + i\epsilon}. \end{aligned} \tag{A.64}$$

This gives

$$Z[J, J^*] = N \int D\phi D\phi^* e^{i\int d^4x\left(-\phi'^*_x(\Box+m^2)\phi_x + i\int d^4z J_x \Delta_{xz} J^*_z\right)}, \tag{A.65}$$

so that we can calculate the 2-point function

$$\begin{aligned} \langle 0|T\phi_1\phi^*_2|0\rangle &= \frac{1}{Z[0]} i\frac{\delta}{\delta J_1} i\frac{\delta}{\delta J^*_2} N \int D\phi D\phi^* e^{i\int d^4x\left(-\phi'^*_x(\Box+m^2)\phi'_x + i\int d^4z J_x \Delta_{xz} J^*_z\right)}\Big|_{J=J^*=0} \\ &= \Delta_{12}. \end{aligned} \tag{A.66}$$

Free Massless Gauge Bosons

When trying to evaluate the 2-point function for massless gauge bosons we discover that we cannot calculate the propagator by simply inverting the equation of motion in Fourier space[3]. In the path integral formalism the origin of this problem is visiblemore clearly. Because of gauge invariance one has to be careful not to perform the functional integration over equivalent field configurations. We take care of this by inserting a factor $1 = \Upsilon_G[A^a_\mu]\Upsilon_G[A^a_\mu]^{-1}$ with Υ to be defined below into the generating functional and hiding the infinity which originates from the

[3]For example in the case of the electro-magnetic field the equation of motion is $(-k^2 g_{\mu\nu} + k_\mu k_\nu)\check{\Delta}^{\nu\rho}_k = i\delta^\rho_\mu$ where the operator on the left hand side is singular.

equivalent field configurations inside the normalization N''

$$\begin{aligned}
Z[J^a_\mu] &= N\int DA^a_\mu \Upsilon_G[A^a_\mu] \underbrace{\int DU^a\ \delta[G[A^{aU}_\mu]]}_{=\Upsilon_G[A^a_\mu]^{-1}} e^{iS[A^{aU}_\mu]+i\int d^4z\ J^{a\nu}_z A^{aU}_{\nu z}}, \\
&\Rightarrow \Upsilon_G[A^a_\mu] = \int DG\ \delta[G] \det\left|\frac{\delta U^a}{\delta G[A^{aU}_\mu]}\right| = \det\left|\frac{\delta U^a}{\delta G[A^{aU}_\mu]}\right|_{G=0}, \\
&\Rightarrow \Upsilon_G[A^a_\mu]^{-1} = \det\left|\frac{\delta G[A^{aU}_\mu]}{\delta U^a}\right|_{G=0} \\
&= N\underbrace{\int DU}_{=\infty}\int DA^a_\mu \det\left|\frac{\delta G[A^{aU}_\mu]}{\delta U^a}\right|_{G=0}\delta[G[A^a_\mu]]e^{iS[A^a_\mu]+\int d^4z\ J^{a\nu}_z A^a_{\nu z}} \\
&= N'\int DA^a_\mu \det\left|\frac{\delta G[A^{aU}_\mu]}{\delta U^a}\right|_{G=0}\delta[G[A^a_\mu]]e^{iS[A^a_\mu]+i\int d^4z\ J^{a\nu}_z A^a_{\nu z}}, \qquad \text{(A.67)}
\end{aligned}$$

with $A^{aU}_\mu = A^a_\mu - \partial_\mu U^a + f^{abc}U^b A^c_\mu$ and the gauge fixing condition $G[A^a_\mu] = \partial^\mu A^a_\mu = 0$. We note that from line 1 to line 4 we only shifted and rescaled the integration variable A^a_μ and then defined $N' = N\int DU$, hiding the infinity. As the gauge is arbitrary we may proceed introducing $G' = G + c$ and integrating over all functions c with 'Gaussian' weight[4], changing only the normalization of Z. This gives

$$\begin{aligned}
Z[J^a_\mu] &= N''\int DA^a_\mu \int Dc\, e^{-\frac{i}{2\xi}\int d^4x\ c_x c_x}\delta[G'[A^a_\mu]-c]\det\left|\frac{\delta G'}{\delta U^a}\right|_{G=0} e^{iS[A^a_\mu]+i\int d^4z\ J^{a\nu}_z A^a_{\nu z}} \\
&= N''\int DA^a_\mu \det\left|\frac{\delta G'}{\delta U^a}\right|_{G=0} e^{iS[A^a_\mu]+i\int d^4\ zJ^{a\nu}_z A^a_{\nu z}-\frac{i}{2\xi}\int d^4x\ G'[A^a_{\mu x}]G'[A^a_{\mu x}]} \\
&= N''\int DA^a_\mu \int D\bar\eta D\eta\, e^{iS[A^a_\mu]+i\int d^4z\ J^{a\nu}_z A^a_{\nu z}+i\int d^4x d^4y\ \bar\eta^a_x \frac{\delta G'[A^{aU}_{\mu x}]}{\delta U^a_y}\big|_{G=0}\eta^b_y - \frac{i}{2\xi}\int d^4x\ G'[A^a_{\mu x}]G'[A^a_{\mu x}]}, \qquad \text{(A.68)}
\end{aligned}$$

with

$$\det\left|\frac{\delta G'[A^{aU}_\mu]}{\delta U^a}\right|_{G=0} = \int D\bar\eta D\eta\, e^{i\int d^4x d^4y\ \bar\eta^a_x \frac{\delta G'[A^{aU}_{\mu x}]}{\delta U^a_y}\big|_{G=0}\eta^b_y} \quad \text{with anticommuting}\,\eta.$$

We have to find an expression for $\frac{\delta G'_x[A^{aU}_\mu]}{\delta U^a_y}\Big|_{G=0}$

$$\begin{aligned}
G[A^{aU}_\mu] &= \partial^\mu\left(A^a_\mu - \partial_\mu U^a + f^{abc}U^b A^c_\mu\right) \\
&= G[A^a_\mu] - \left(\Box\delta^{ab} - f^{abc}\left(A^c_\mu\partial^\mu + (\partial^\mu A^c_\mu)\right)\right)U^b \\
\Rightarrow \frac{\delta G'_x[A^{aU}_\mu]}{\delta U^a_y}\Big|_{G=0} &= \left(f^{abc}\left(A^c_\mu\partial^\mu + (\partial^\mu A^c_\mu)\right)\right)\delta(x-y), \qquad \text{(A.69)}
\end{aligned}$$

[4]Here ξ is an arbitrary real parameter

where we used $\frac{\delta G'_x[A^{aU}_\mu]}{\delta U^a_y}\Big|_{G=0} = \frac{\delta G_x[A^{aU}_\mu]}{\delta U^a_y}\Big|_{G=0}$. Plugging this into (A.68) gives

$$\begin{aligned} Z[J^a_\mu] &= \int DA^a_\mu D\eta^* D\eta \\ &\quad e^{i\int d^4x\ J^{a\mu}_x A^a_{\mu x} + iS[A^a_\mu] + \int d^4x\left(-\frac{i}{2\xi}\partial^\nu A^a_{x\mu}\partial^\rho A^a_{x\rho} + if^{abc}\bar\eta^a_x A^c_{\mu x}\partial^\mu\eta^b_x + if^{abc}\bar\eta^a_x(\partial^\mu A^c_{\mu x})\eta^b_x \right.} \\ &\qquad\qquad e^{\left. + i\bar\eta^a_x\Box\eta^a_x\right)}. \end{aligned} \tag{A.70}$$

The anti-commuting fields η, $\bar\eta$ are the so-called Faddeev-Popov ghosts [FP67]. Now we are able to calculate the 2-point functions.

$U(1)$ **Gauge Boson** The kinetic term for the $U(1)$ gauge boson is given by

$$\mathcal{L} = -\frac{1}{4}A^{\mu\nu}A_{\mu\nu} - \frac{1}{2\xi_Y}(\partial^\mu A^0_\mu)^2 = \frac{1}{2}A^0_\mu\left(\Box g^{\mu\nu} - (1-\frac{1}{\xi_Y})\partial^\mu\partial^\nu\right)A^0_\nu, \tag{A.71}$$

leading to the equation of motion

$$\left(\Box g^{\mu\nu} - (1-\frac{1}{\xi_Y})\partial^\mu\partial^\nu\right)A^0_\nu = 0.$$

We transform the equation to Fourier space and get

$$\left(-k^2 g^{\mu\nu} + (1-\frac{1}{\xi_Y})k^\mu k^\nu\right)\check{A}^0_\nu = 0,$$

and thus

$$\left(-k^2 g^{\mu\nu} + (1-\frac{1}{\xi_Y})k^\mu k^\nu\right)\check\Delta_{\nu\rho} = i\delta^\mu_\rho.$$

We invert the equation to find

$$\check\Delta_{\nu\rho} = \frac{-i}{k^2+i\epsilon}\left(g_{\nu\rho} - (1-\xi_Y)\frac{k_\nu k_\rho}{k^2}\right). \tag{A.72}$$

As usual we complete the square in the exponent of the generating functional $W[J]$ by shifting $A^0_\mu = A^{0\prime}_\mu + i\int d^4y\ \Delta_{xy\mu\nu}J^\nu_y$

$$\begin{aligned} &\frac{1}{2}A^0_{\mu x}\left(\Box g^{\mu\nu} - (1-\frac{1}{\xi_Y})\partial^\mu\partial^\nu\right)A^0_{\nu x} + J^\mu_x A_{\mu x} \\ = &\ \frac{1}{2}A^{0\prime}_{\mu x}\left(\Box g^{\mu\nu} - (1-\frac{1}{\xi_Y})\partial^\mu\partial^\nu\right)A^{0\prime}_{\nu x} + \frac{i}{2}\int d^4y\ J^\mu_x\Delta_{\mu\nu xy}J^\nu_y. \end{aligned} \tag{A.73}$$

So we get for the 2 point function

$$\begin{aligned} \langle 0|TA^0_{\rho 1}A^0_{\sigma 2}|0\rangle &= \frac{(-i)^2 N\delta^2}{Z[0]\delta J_{\rho 1}\delta J_{\sigma 2}}\int DA^0_\mu e^{i\int d^4x\left(\frac{1}{2}A^{0\prime}_{\mu x}\left(\Box g^{\mu\nu} - (1-\frac{1}{\xi_Y})\partial^\mu\partial^\nu\right)A^{0\prime}_{\nu x} + \frac{i}{2}\int d^4y\ J^\mu_x\Delta_{\mu\nu xy}J^\nu_y\right)}\Big|_{J=0} \\ &= \Delta_{\rho\sigma 12}, \end{aligned} \tag{A.74}$$

which is what we expected.

$SU(2)$ Gauge Bosons The $SU(2)_L$ gauge bosons can be treated the same way as the $U(1)$ gauge boson giving

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}W^{+\mu\nu}W_{-\mu\nu} - \frac{1}{2\xi_L}(\partial^\mu W^a_\mu)^2 + m^2 W^+_\mu W^{\mu-}, \\
\Rightarrow \mathcal{L}_{\text{kin}} &= -\frac{1}{2}tr\left[[\partial_\mu W^{a+}_\mu \tau^a - \partial_\nu W^{a-}_\mu \tau^a][\partial^\mu W^{c\nu+}\tau^c - \partial^\nu W^{c\mu-}\tau^c]\right] \\
&\quad + \frac{1}{2\xi_L}W^{a+}_\mu \partial^\mu\partial^\nu W^{a-}_\nu + m^2 W^+_\mu W^{\mu-} \\
&= \frac{1}{2}W^{a+}_\mu\left((\Box + m^2)g^{\mu\nu} - (1-\frac{1}{\xi_G})\partial^\mu\partial^\nu\right)W^{a-}_\nu. \qquad \text{(A.75)}
\end{aligned}$$

This leads to the following equation of motion and the propagator

$$\begin{aligned}
\left((\Box + m^2)g^{\mu\nu} - (1-\frac{1}{\xi_L})\partial^\mu\partial^\nu\right)\check{\Delta}_{\nu\rho} &= i\delta^\mu_\rho, \\
\Rightarrow \check{\Delta}_{\nu\rho} &= \frac{-i}{k^2 - m^2 + i\epsilon}\left(g_{\nu\rho} - (1-\xi_L)\frac{k_\nu k_\rho}{k^2}\right). \qquad \text{(A.76)}
\end{aligned}$$

$SU(3)_C$ Gauge Bosons Similarily, for the $SU(3)_C$ gauge bosons we have

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}G^{\mu\nu}G_{\mu\nu} - \frac{1}{2\xi_C}(\partial^\mu G^a_\mu)^2, \\
\Rightarrow \mathcal{L}_{\text{kin}} &= -\frac{1}{2}tr\left[[\partial_\mu G^a_\mu t^a - \partial_\nu G^a_\mu t^a][\partial^\mu G^{c\nu}t^c - \partial^\nu G^{c\mu}t^c]\right] + \frac{1}{2\xi_C}G^a_\mu\partial^\mu\partial^\nu G^a_\nu \\
&= \frac{1}{2}G^a_\mu\left(\Box g^{\mu\nu} - (1-\frac{1}{\xi_C})\partial^\mu\partial^\nu\right)G^a_\nu. \qquad \text{(A.77)}
\end{aligned}$$

This leads to the following equation of motion and the propagator

$$\begin{aligned}
\left(\Box g^{\mu\nu} - (1-\frac{1}{\xi_C})\partial^\mu\partial^\nu\right)\check{\Delta}_{\nu\rho} &= i\delta^\mu_\rho, \\
\Rightarrow \check{\Delta}_{\nu\rho} &= \frac{-i}{k^2 + i\epsilon}\left(g_{\nu\rho} - (1-\xi_C)\frac{k_\nu k_\rho}{k^2}\right). \qquad \text{(A.78)}
\end{aligned}$$

A.4 MSSM Lagrangian

A.4.1 Kinetic Terms

We have kinetic terms for the matter and gauge fields:

$$
\begin{aligned}
\mathcal{L}_{\text{kin}} &= \mathcal{L}_{\text{kin,matter}} + \mathcal{L}_{\text{kin,gauge}} \\
&= iD_\mu L\sigma^\mu \bar{L} + i(D_\mu \tilde{L})^\dagger (D^\mu L) + i\sqrt{2}[\tilde{L}^\dagger (\frac{-g_Y}{2}\tilde{B}^0 + g_L \tilde{W}^a \tau^a) L - \tilde{L}(\frac{g_Y}{2}\bar{\tilde{B}}^0 + g_L \bar{\tilde{W}}^a \tau^a)\bar{L}] \\
&\quad + \frac{-g_Y}{2}\tilde{L}^\dagger D_Y \tilde{L} + g_L \tilde{L}^\dagger D_L^a \tau^a \tilde{L} + F_{e_L}^\dagger F_{e_L} + F_{\nu_e}^\dagger F_{\nu_e} \\
&\quad + iD_\mu \bar{e}_R \sigma^\mu e_R + (D_\mu \tilde{e}_R)^* (D^\mu \tilde{e}_R) + i\sqrt{2}[\tilde{e}_R \frac{-g_Y}{2}\tilde{B}^0 \bar{e}_R - \tilde{e}_R^* \frac{-g_Y}{2}\bar{\tilde{B}}^0 e_R] \\
&\quad + g_Y \tilde{e}_R^* D_Y \tilde{e}_R + F_{e_R}^\dagger F_{e_R} + iD_\mu Q_L \sigma^\mu \bar{Q}_L + (D_\mu \tilde{Q}_L)^\dagger (D^\mu \tilde{Q}_L) \\
&\quad + i\sqrt{2}[\tilde{Q}_L^\dagger (\frac{g_Y}{6}\tilde{B}^0 + g_L \tilde{W}^a \tau^a + g_C \tilde{G}^a t^a) Q_L - \tilde{Q}(\frac{g_Y}{6}\bar{\tilde{B}}^0 + g_L \bar{\tilde{W}}^a \tau^a + g_C \bar{\tilde{G}}^a t^a)\bar{Q}_L] \\
&\quad + \frac{g_Y}{6}\tilde{Q}_L^\dagger D_Y \tilde{Q}_L + g_L \tilde{Q}_L^\dagger D_L^a \tau^a \tilde{Q}_L + g_C \tilde{Q}_L^\dagger D_C^a t^a \tilde{Q}_L + F_{u_L}^\dagger F_{u_L} + F_{d_L}^\dagger F_{d_L} \\
&\quad + iD_\mu u_R \sigma^\mu \bar{u}_R + (D_\mu \tilde{u}_R)^* (D^\mu \tilde{u}_R) \\
&\quad + i\sqrt{2}[\tilde{u}_R (\frac{-2g_Y}{3}\tilde{B}^0 + g_C \tilde{G}^a t^a)\bar{u}_R - \tilde{u}_R^* (\frac{-2g_Y}{3}\bar{\tilde{B}}^0 + g_C \bar{\tilde{G}}^a t^a) u_R] \\
&\quad + \frac{-2g_Y}{3}\tilde{u}_R^* D_Y \tilde{u}_R + g_C \tilde{u}_R^* D_C^a t^a \tilde{u}_R + F_{u_R}^\dagger F_{u_R} + iD_\mu d_R \sigma^\mu \bar{d}_R + (D_\mu \tilde{d}_R)^* (D^\mu \tilde{d}_R) \\
&\quad + i\sqrt{2}[\tilde{d}_R (\frac{g_Y}{3}\tilde{B}^0 + g_C \tilde{G}^a t^a)\bar{d}_R - \tilde{d}_R^* (\frac{g_Y}{3}\bar{\tilde{B}}^0 + g_C \bar{\tilde{G}}^a t^a) d_R] \\
&\quad + \frac{g_Y}{3}\tilde{d}_R^* D_Y \tilde{d}_R + g_C \tilde{d}_R^* D_c^a t^a \tilde{d}_R + F_{d_R}^\dagger F_{d_R} \\
&\quad + iD_\mu \tilde{H}_u \sigma^\mu \bar{\tilde{H}}_u + (D_\mu H_u)^\dagger (D^\mu H_u) \\
&\quad + i\sqrt{2}[H_u^\dagger (\frac{g_Y}{2}\tilde{B}^0 + g_L \tilde{W}^a \tau^a)\tilde{H}_u - H_u (\frac{g_Y}{2}\bar{\tilde{B}}^0 + g_L \bar{\tilde{W}}^a \tau^a)\bar{\tilde{H}}_u] \\
&\quad + \frac{g_Y}{2}H_u^\dagger D_Y H_u + g_L H_u^\dagger D_L^a \tau^a H_u + F_{H_u^+}^\dagger F_{H_u^+} + F_{H_u^0}^\dagger F_{H_u^0} \\
&\quad + iD_\mu \tilde{H}_d \sigma^\mu \bar{\tilde{H}}_d + (D_\mu H_d)^\dagger (D^\mu H_d) \\
&\quad + i\sqrt{2}[H_d^\dagger (\frac{-g_Y}{2}\tilde{B}^0 + g_L \tilde{W}^a \tau^a)\tilde{H}_d - H_d (\frac{-g_Y}{2}\bar{\tilde{B}}^0 + g\bar{\tilde{W}}^a \tau^a)\bar{\tilde{H}}_d] \\
&\quad + \frac{-g_Y}{2}H_d^\dagger D_Y H_d + g_L H_d^\dagger D_L^a \tau^a H_d + F_{H_d^-}^\dagger F_{H_d^-} + F_{H_d^0}^\dagger F_{H_d^0} \\
&\quad - \frac{1}{4}A_{\mu\nu}^0 A^{0\mu\nu} + i\tilde{A}^0 \sigma^\mu \partial_\mu \bar{\tilde{A}}^0 + \frac{1}{2}D_Y D_Y - \frac{1}{4}B_{\mu\nu}^a B^{a\mu\nu} + i\tilde{B}^a \sigma^\mu \tilde{D}_\mu^{ac} \bar{\tilde{B}}^c + \frac{1}{2}D_L^a D_L^a \\
&\quad - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} + i\tilde{G}^a \sigma^\mu \tilde{D}_\mu^{ac} \bar{\tilde{G}}^c + \frac{1}{2}D_C^a D_C^a.
\end{aligned}
\tag{A.79}
$$

The equations of motion for the auxiliary fields D_i give

$$
\begin{aligned}
D_Y &= -g_Y \Big(\frac{-1}{2}\tilde{L}^\dagger \tilde{L} + \tilde{e}_R^* \tilde{e}_R + \frac{1}{6}\tilde{Q}_L^\dagger \tilde{Q}_L \\
&\quad + \frac{1}{3}(-2\tilde{u}_R^* \tilde{u}_R + \tilde{d}_R^* \tilde{d}_R) + \frac{1}{2}(H_u^\dagger H_u - H_d^\dagger H_d) \Big) \\
D_L^a &= -g_L \Big(\tilde{L}^\dagger \tau^a \tilde{L} + \tilde{Q}_L^\dagger \tau^a \tilde{Q}_L + H_u^\dagger \tau^a H_u + H_d^\dagger \tau^a H_d \Big) \\
D_C^a &= -g_C \Big(\tilde{Q}_L^\dagger t^a \tilde{Q}_L + \tilde{u}_R^* t^a \tilde{u}_R + \tilde{d}_R^* t^a \tilde{d}_R \Big).
\end{aligned}
\tag{A.80}
$$

We note that the expressions in $\mathcal{L}_{\text{kin,matter}}$ containing the D_i are equal to $-D_iD_i$ so that we can combine this with the term $+\frac{1}{2}D_iD_i$ coming from $\mathcal{L}_{\text{kin,gauge}}$, hence a factor of $-\frac{1}{2}$ will remain. Making use of (B.16) and remembering $\tau^a = \frac{\sigma^a}{2}, t^a = \frac{\lambda^a}{2}$ we get

$$
\begin{aligned}
(D_Y)^2 \quad = \quad & \frac{g_Y^2}{4}\left(\tilde{L}^\dagger\tilde{L}\right)^2 + g_Y^2\left(\tilde{e}_R^*\tilde{e}_R\right)^2 + \frac{g_Y^2}{36}\left(\tilde{Q}_L^\dagger\tilde{Q}_L\right)^2 \\
& +\frac{4g_Y^2}{9}\left(\tilde{u}_R^*\tilde{u}_R\right)^2 + \frac{g_Y^2}{9}\left(\tilde{d}_R^*\tilde{d}_R\right)^2 + \frac{g_Y^2}{4}\left(H_u^\dagger H_u\right)^2 \\
& +\frac{g_Y^2}{4}\left(H_d^\dagger H_d\right)^2 \\
& -g_Y^2\tilde{L}^\dagger\tilde{L}\tilde{e}_R^*\tilde{e}_R - \frac{g_Y^2}{6}\tilde{L}^\dagger\tilde{L}\tilde{Q}_L^\dagger\tilde{Q}_L + \frac{2g_Y^2}{3}\tilde{L}^\dagger\tilde{L}\tilde{u}_R^*\tilde{u}_R \\
& -\frac{g_Y^2}{3}\tilde{L}^\dagger\tilde{L}\tilde{d}_R^*\tilde{d}_R - \frac{g_Y^2}{2}\tilde{L}^\dagger\tilde{L}H_u^\dagger H_u + \frac{g_Y^2}{2}\tilde{L}^\dagger\tilde{L}H_d^\dagger H_d \\
& +\frac{g_Y^2}{3}\tilde{e}_R^*\tilde{e}_R\tilde{Q}_L^\dagger\tilde{Q}_L - \frac{4g_Y^2}{3}\tilde{e}_R^*\tilde{e}_R\tilde{u}_R^*\tilde{u}_R - \frac{2g_Y^2}{3}\tilde{e}_R^*\tilde{e}_R\tilde{d}_R^*\tilde{d}_R \\
& +g_Y^2\tilde{e}_R^*\tilde{e}_RH_u^\dagger H_u - g_Y^2\tilde{e}_R^*\tilde{e}_RH_d^\dagger H_d \\
& -\frac{2g_Y^2}{9}\tilde{Q}_L^\dagger\tilde{Q}_L\tilde{u}_R^*\tilde{u}_R + \frac{g_Y^2}{9}\tilde{Q}_L^\dagger\tilde{Q}_L\tilde{d}_R^*\tilde{d}_R + \frac{g_Y^2}{6}\tilde{Q}_L^\dagger\tilde{Q}_LH_u^\dagger H_u \\
& -\frac{g_Y^2}{6}\tilde{Q}_L^\dagger\tilde{Q}_LH_d^\dagger H_d \\
& -\frac{4g_Y^2}{9}\tilde{u}_R^*\tilde{u}_R\tilde{d}_R^*\tilde{d}_R - \frac{2g_Y^2}{3}\tilde{u}_R^*\tilde{u}_RH_u^\dagger H_u + \frac{2g_Y^2}{3}\tilde{u}_R^*\tilde{u}_RH_d^\dagger H_d \\
& +\frac{g_Y^2}{3}\tilde{d}_R^*\tilde{d}_RH_u^\dagger H_u - \frac{g_Y^2}{3}\tilde{d}_R^*\tilde{d}_RH_d^\dagger H_d \\
& -\frac{g_Y^2}{2}H_u^\dagger H_uH_d^\dagger H_d \\
4(D_L)^2 \quad = \quad & g_L^2\left(\tilde{L}^\dagger\tilde{L}\right)^2 + g_L^2\left(\tilde{Q}_L^\dagger\tilde{Q}_L\right)^2 + g_L^2\left(H_u^\dagger H_u\right)^2 + g_L^2\left(H_d^\dagger H_d\right)^2 \\
& +4g_L^2\tilde{L}^\dagger\tilde{Q}_L\tilde{Q}_L^\dagger\tilde{L} - 2g_L^2\tilde{L}^\dagger\tilde{L}\tilde{Q}_L^\dagger\tilde{Q}_L \\
& +4g_L^2\tilde{L}^\dagger H_uH_u^\dagger\tilde{L} - 2g_L^2H_u^\dagger H_u\tilde{L}^\dagger\tilde{L} \\
& +4g_L^2\tilde{L}^\dagger H_dH_d^\dagger\tilde{L} - 2g_L^2H_d^\dagger H_d\tilde{L}^\dagger\tilde{L} \\
& +4g_L^2\tilde{Q}_L^\dagger H_uH_u^\dagger\tilde{Q}_L - 2g_L^2\tilde{Q}_L^\dagger\tilde{Q}_LH_u^\dagger H_u \\
& +4g_L^2\tilde{Q}_L^\dagger H_dH_d^\dagger\tilde{Q}_L - 2g_L^2\tilde{Q}_L^\dagger\tilde{Q}_LH_d^\dagger H_d \\
& +4g_L^2H_u^\dagger H_dH_d^\dagger H_u - 2g_L^2H_u^\dagger H_uH_d^\dagger H_d \\
4(D_C)^2 \quad = \quad & \frac{4g_C^2}{3}\left(\tilde{Q}_L^\dagger\tilde{Q}_L\right)^2 + \frac{4g_C^2}{3}\left(\tilde{u}_R^*\tilde{u}_R\right)^2 + \frac{4g_C^2}{3}\left(\tilde{d}_R^*\tilde{d}_R\right)^2 \\
& +4g_C^2\tilde{Q}_L^\dagger\tilde{u}_R\tilde{u}_R^*\tilde{Q}_L - \frac{4g_C^2}{3}\tilde{Q}_L^\dagger\tilde{Q}_L\tilde{u}_R^*\tilde{u}_R \\
& +4g_C^2\tilde{Q}_L^\dagger\tilde{d}_R\tilde{d}_R^*\tilde{Q}_L - \frac{4g_C^2}{3}\tilde{Q}_L^\dagger\tilde{Q}_L\tilde{d}_R^*\tilde{d}_R \\
& +4g_C^2\tilde{u}_R^*\tilde{d}_R\tilde{d}_R^*\tilde{u}_R - \frac{4g_C^2}{3}\tilde{u}_R^*\tilde{u}_R\tilde{d}_R^*\tilde{d}_R.
\end{aligned} \tag{A.81}
$$

A.4.2 Superpotential

The superpotential for the MSSM is given by

$$W = h^e_{ij}\tilde{e}^*_{Ri}\tilde{L}^r_j H^s_d \epsilon^{rs} + h^u_{ij}\tilde{u}^*_{Ri}\tilde{Q}^r_j H^s_u \epsilon^{rs} + h^d_{ij}\tilde{d}^*_{Ri}\tilde{Q}^r_j H^s_d \epsilon^{rs} + \mu H^r_u H^s_d \epsilon^{rs}. \tag{A.82}$$

Using

$$\begin{aligned}
\mathcal{L}_{\rm int} &= -\frac{1}{2}W^{ij}(\phi,\phi^\dagger)\psi_i\psi_j + W^i(\phi,\phi^\dagger)F_i + \text{h.c.},\\
F^\dagger_k &= -\frac{\delta W}{\delta\phi_k},\\
F^\dagger_{\tilde{L}^r_j} &= -h^e_{ij}\tilde{e}^*_{Ri}H^s_d\epsilon^{rs},\ F^\dagger_{H^s_d} = -[h^e_{ij}\tilde{e}^*_{Ri}\tilde{L}^r_j\epsilon^{rs} + h^d_{ij}\tilde{d}^*_{Ri}\tilde{Q}^r_j\epsilon^{rs} + \mu H^r_u\epsilon^{rs}],\\
F^\dagger_{\tilde{Q}^r_j} &= -[h^u_{ij}\tilde{u}^*_{Ri}H^s_u\epsilon^{rs} + h^d_{ij}\tilde{d}^*_{Ri}H^s_d\epsilon^{rs}],\ F^\dagger_{H^s_u} = -[h^u_{ij}\tilde{u}^*_{Ri}\tilde{Q}^r_j\epsilon^{rs} + \mu H^s_d\epsilon^{rs}],\\
F^\dagger_{\tilde{d}^*_{Ri}} &= -h^d_{ij}\tilde{Q}^r_j H^s_d\epsilon^{rs},\ F^\dagger_{\tilde{e}^*_{Ri}} = -h^e_{ij}\tilde{L}^r_j H^s_d\epsilon^{rs},\ F^\dagger_{\tilde{u}^*_{Ri}} = -h^u_{ij}\tilde{Q}^r_j H^s_u\epsilon^{rs},
\end{aligned} \tag{A.83}$$

and $\epsilon^{rs} = \begin{cases} +1 & \text{for } r=1, s=2 \\ -1 & \text{for } r=2, s=1 \end{cases}$ this gives

$$\begin{aligned}
\mathcal{L}_{\rm int} = & -\frac{1}{2}[h^e_{ij}(\bar{e}_{Ri}L^r_j H^s_d\epsilon^{rs} + \bar{e}_{Ri}\tilde{L}^r_j\tilde{H}^s_d\epsilon^{rs} + \tilde{e}^*_{Ri}L^r_j\tilde{H}^s_d\epsilon^{rs})\\
& +h^u_{ij}(\bar{u}_{Ri}Q^r_j H^s_u\epsilon^{rs} + \bar{u}_{Ri}\tilde{Q}^r_j\tilde{H}^s_u\epsilon^{rs} + \tilde{u}^*_{Ri}Q^r_j\tilde{H}^s_u\epsilon^{rs})\\
& +h^d_{ij}(\bar{d}_{Ri}Q^r_j H^s_d\epsilon^{rs} + \bar{d}_{Ri}\tilde{Q}^r_j\tilde{H}^s_d\epsilon^{rs} + \tilde{d}^*_{Ri}Q^r_j\tilde{H}^s_d\epsilon^{rs})\\
& +\mu\tilde{H}^r_u\tilde{H}^s_d\epsilon^{rs}]\\
& -[h^e_{ij}\tilde{L}^r_j H^s_d\epsilon^{rs}h^{e*}_{il}\tilde{L}^t_l H^q_d\epsilon^{tq} + h^e_{ij}\tilde{e}^*_{Ri}H^s_d\epsilon^{rs}h^{e*}_{jl}\tilde{e}_{Rl}H^{t*}_d\epsilon^{rt}\\
& +h^e_{ij}h^{e*}_{kl}\tilde{e}^*_{Ri}\tilde{L}^r_j\epsilon^{rs}\tilde{e}_{Rk}\tilde{L}^{t*}_l\epsilon^{ts} + h^u_{ij}\tilde{Q}^r_j H^s_u\epsilon^{rs}h^{u*}_{ik}\tilde{Q}^{t*}_k H^{q*}_u\epsilon^{tq}\\
& +h^u_{ij}\tilde{u}^*_{Ri}H^s_u\epsilon^{rs}h^{u*}_{kj}\tilde{u}_{Rk}H^{t*}_u\epsilon^{rt} + h^u_{ij}\tilde{u}^*_{Ri}\tilde{Q}^r_j\epsilon^{rs}h^{u*}_{kl}\tilde{u}_{Rk}\tilde{Q}^{t*}_l\epsilon^{ts}\\
& +h^d_{ij}\tilde{Q}^r_j H^s_d\epsilon^{rs}h^{d*}_{ik}\tilde{Q}^{t*}_k H^{q*}_d\epsilon^{tq} + h^d_{ij}\tilde{d}^*_{Ri}H^s_d\epsilon^{rs}h^{d*}_{kj}\tilde{d}_{Rk}H^{t*}_d\epsilon^{rt}\\
& +h^d_{ij}\tilde{d}^*_{Ri}\tilde{Q}^r_j\epsilon^{rs}h^{d*}_{kl}\tilde{d}_{Rk}\tilde{Q}^{t*}_l\epsilon^{ts} + \mu^2 H^s_u\epsilon^{rs}H^{t*}_u\epsilon^{rt}\\
& +\mu^2 H^s_d\epsilon^{rs}H^{t*}_d\epsilon^{rt} + h^e_{ij}\tilde{e}^*_{Ri}\tilde{L}^r_j\epsilon^{rs}h^{d*}_{kl}\tilde{d}_{Rk}\tilde{Q}^{t*}_j\epsilon^{ts}\\
& +h^e_{ij}\tilde{e}^*_{Ri}\tilde{L}^r_j\epsilon^{rs}\mu H^{t*}_u\epsilon^{ts} + h^d_{ij}\tilde{d}^*_{Ri}\tilde{Q}^r_j\epsilon^{rs}\mu H^{t*}_u\epsilon^{ts}\\
& +h^u_{ij}\tilde{u}^*_{Ri}H^s_u\epsilon^{rs}h^{d*}_{kl}\tilde{d}_{Rk}H^{t*}_d\epsilon^{rt} + h^u_{ij}\tilde{u}^*_{Ri}\tilde{Q}^r_j\epsilon^{rs}\mu H^{t*}_d\epsilon^{ts}] + \text{h.c.}\\
= & -\frac{1}{2}[h^e_{ij}\bar{e}_{Ri}\nu_j H^{(-)}_d - h^e_{ij}\bar{e}_{Ri}e_{Lj}v_d - h^e_{ij}\bar{e}_{Ri}e_{Lj}h^{(0)}_d\\
& +h^e_{ij}\bar{e}_{Ri}\tilde{\nu}_j H^{(-)}_d - h^e_{ij}\bar{e}_{Ri}e_{Lj}\tilde{H}^{(0)}_d + h^e_{ij}\tilde{e}^*_{Ri}\nu_j\tilde{H}^{(-)}_d\\
& -h^e_{ij}\tilde{e}^*_{Ri}e_{Lj}\tilde{H}^{(0)}_d + h^u_{ij}\bar{u}_{Ri}u_{Lj}v_u + h^u_{ij}\bar{u}_{Ri}u_{Lj}h^{(0)}_u\\
& -h^u_{ij}\bar{u}_{Ri}d_{Lj}H^{(+)}_u + h^u_{ij}\bar{u}_R\tilde{u}_{Lj}\tilde{H}^{(0)}_u - h^u_{ij}\bar{u}_{Ri}\tilde{d}_{Lj}\tilde{H}^{(+)}_u\\
& +h^u_{ij}\tilde{u}^*_{Ri}u_{Lj}\tilde{H}^{(0)}_u - h^u_{ij}\tilde{u}^*_{Ri}d_{Lj}\tilde{H}^{(+)}_u + h^d_{ij}\bar{d}_{Ri}u_{Lj}H^{(-)}_d\\
& -h^d_{ij}\bar{d}_{Ri}d_{Lj}v_d - h^d_{ij}\bar{d}_{Ri}d_{Lj}h^{(0)}_d + h^d_{ij}\bar{d}_{Ri}\tilde{u}_{Lj}\tilde{H}^{(-)}_d\\
& -h^d_{ij}\bar{d}_{Ri}\tilde{d}_{Lj}\tilde{H}^{(0)}_d + h^d_{ij}\tilde{d}^*_{Ri}u_{Lj}\tilde{H}^{(+)}_d - h^d_{ij}\tilde{d}^*_{Ri}d_{Lj}\tilde{H}^{(0)}_d\\
& +\mu\tilde{H}^{(+)}_u\tilde{H}^{(-)}_d - \mu\tilde{H}^{(0)}_u\tilde{H}^{(0)}_d]
\end{aligned}$$

$$
\begin{aligned}
&-[h^e_{ij}h^{e*}_{il}\tilde{\nu}_j\tilde{\nu}^*_l H^{(-)*}_d H^{(-)}_d - h^e_{ij}h^{e*}_{il}\tilde{\nu}_j H^{(-)}_d \tilde{e}^*_{Ll} v^*_d - h^e_{ij}h^{e*}_{il}\tilde{\nu}_j H^{(-)}_d \tilde{e}^*_{Ll} h^{(0)*}_d \\
&-h^e_{ij}h^{e*}_{il}\tilde{e}_{Lj} v_d \tilde{\nu}^*_l H^{(-)*}_d + h^e_{ij}h^{e*}_{il}\tilde{e}_{Lj} v_d \tilde{e}^*_{Ll} v^*_d + h^e_{ij}h^{e*}_{il}\tilde{e}_{Lj} v_d \tilde{e}^*_{Ll} h^{(0)}_d \\
&-h^e_{ij}h^{e*}_{il}\tilde{e}_{Lj} h^{(0)}_d \tilde{\nu}^*_e H^{(-)*}_d + h^e_{ij}h^{e*}_{il}\tilde{e}_{Lj} h^{(0)}_d \tilde{e}^*_{Ll} v^*_d + h^e_{ij}h^{e*}_{il}\tilde{e}_{Lj} h^{(0)}_d \tilde{e}^*_{Ll} h^{(0)*}_d \\
&+h^e_{ij}h^{e*}_{jl}\tilde{e}^*_{Ri}\tilde{e}_{Rl} v^*_d v_d + h^e_{ij}h^{e*}_{jl}\tilde{e}^*_{Ri}\tilde{e}_{Rl} v_d h^{(0)*}_d + h^e_{ij}h^{e*}_{jl}\tilde{e}^*_{Ri}\tilde{e}_{Rl} v^*_d h^{(0)}_d \\
&+h^e_{ij}h^{e*}_{jl}\tilde{e}^*_{Ri}\tilde{e}_{Rl} h^{(0)*}_d h^{(0)}_d + h^e_{ij}h^{e*}_{jl}\tilde{e}^*_{Ri}\tilde{e}_{Rl} H^{(-)*}_d H^{(-)}_d + h^e_{ij}h^{e*}_{kl}\tilde{e}^*_{Ri}\tilde{e}_{Rk}\tilde{\nu}_j\tilde{\nu}^*_l \\
&+h^e_{ij}h^{e*}_{kl}\tilde{e}^*_{Ri}\tilde{e}_{Rk}\tilde{e}_{Lj}\tilde{e}^*_{Ll} \\
&+h^u_{ij}h^{u*}_{ik}\tilde{\nu}_j v_u \tilde{\nu}^*_k v^*_u + h^u_{ij}h^{u*}_{ik}\tilde{\nu}_j v_u \tilde{\nu}^*_k h^{(0)*}_u - h^u_{ij}h^{u*}_{ik}\tilde{\nu}_j v_u \tilde{e}^*_{Lk} H^{(+)}_u \\
&+h^u_{ij}h^{u*}_{ik}\tilde{\nu}_j h^{(0)}_u \tilde{\nu}^*_k v^*_u + h^u_{ij}h^{u*}_{ik}\tilde{\nu}_j h^{(0)}_u \tilde{\nu}^*_k h^{(0)*}_u - h^u_{ij}h^{u*}_{ik}\tilde{\nu}_j h^{(0)}_u \tilde{e}^*_{Lk} H^{(+)*}_u \\
&-h^u_{ij}h^{u*}_{ik}\tilde{e}_{Lj} H^{(+)}_u \tilde{\nu}^*_k v^*_u - h^u_{ij}h^{u*}_{ik}\tilde{e}_{Lj} H^{(+)}_u \tilde{\nu}^*_k h^{(0)*}_u + h^u_{ij}h^{u*}_{ik}\tilde{e}_{Lj} H^{(+)}_u \tilde{e}^*_{Lk} H^{(+)*}_u \\
&+h^u_{ij}h^{u*}_{kj}\tilde{u}^*_{Ri}\tilde{u}_{Rk} v^*_u v_u + h^u_{ij}h^{u*}_{kj}\tilde{u}^*_{Ri}\tilde{u}_{Rk} v_u h^{(0)*}_u + h^u_{ij}h^{u*}_{kj}\tilde{u}^*_{Ri}\tilde{u}_{Rk} v^*_u h^{(0)}_u \\
&+h^u_{ij}h^{u*}_{kj}\tilde{u}^*_{Ri}\tilde{u}_{Rk} H^{(+)*}_u H^{(+)}_u + h^u_{ij}h^{u*}_{kl}\tilde{u}^*_{Ri}\tilde{u}_{Rk}\tilde{u}_{Lj}\tilde{u}^*_{Ll} + h^u_{ij}h^{u*}_{kl}\tilde{u}^*_{Ri}\tilde{u}_{Rk}\tilde{d}_{Lj}\tilde{d}^*_{Ll} \\
&+h^d_{ij}h^{d*}_{ik}\tilde{u}_{Lj} H^{(-)}_d \tilde{u}^*_{Lk} H^{(-)*}_d - h^d_{ij}h^{d*}_{ik}\tilde{u}_{Lj} H^{(-)}_d \tilde{d}^*_{Lk} v^*_d - h^d_{ij}h^{d*}_{ik}\tilde{u}_{Lj} H^{(-)}_d \tilde{d}^*_{Lk} h^{(0)*}_d \\
&-h^d_{ij}h^{d*}_{ik}\tilde{d}_{Lj} v_d \tilde{u}^*_{Lk} H^{(-)*}_d + h^d_{ij}h^{d*}_{ik}\tilde{d}_{Lj} v_d \tilde{d}^*_{Lk} v^*_d + h^d_{ij}h^{d*}_{ik}\tilde{d}_{Lj} v_d \tilde{d}^*_{Lk} h^{(0)*}_d \\
&-h^d_{ij}h^{d*}_{ik}\tilde{d}_{Lj} h^{(0)}_d \tilde{u}^*_{Lk} H^{(-)*}_d + h^d_{ij}h^{d*}_{ik}\tilde{d}_{Lj} h^{(0)}_d \tilde{d}^*_{Lk} v^*_d + h^d_{ij}h^{d*}_{ik}\tilde{d}_{Lj} h^{(0)}_d \tilde{d}^*_{Lk} h^{(0)*}_d \\
&+h^d_{ij}h^{d*}_{kj}\tilde{d}^*_{Ri}\tilde{d}_{Rk} v^*_d v_d + h^d_{ij}h^{d*}_{kj}\tilde{d}^*_{Ri}\tilde{d}_{Rk} v_d h^{(0)*}_d + h^d_{ij}h^{d*}_{kj}\tilde{d}^*_{Ri}\tilde{d}_{Rk} v^*_d h^{(0)}_d \\
&+h^d_{ij}h^{d*}_{kj}\tilde{d}^*_{Ri}\tilde{d}_{Rk} h^{(0)*}_d h^{(0)}_d + h^d_{ij}h^{d*}_{kj}\tilde{d}^*_{Ri}\tilde{d}_{Rk} H^{(-)*}_d H^{(-)}_d + h^d_{ij}h^{d*}_{kl}\tilde{d}^*_{Ri}\tilde{d}_{Rk}\tilde{u}_{Lj}\tilde{u}^*_{Lk} \\
&+h^d_{ij}h^{d*}_{kl}\tilde{d}^*_{Ri}\tilde{d}_{Rk}\tilde{d}_{Lj}\tilde{d}^*_{Ll} \\
&+\mu^2 v^*_u v_u + \mu^2 v^*_u h^{(0)}_u + \mu^2 v_u h^{(0)*}_u + \mu^2 h^{(0)}_u h^{(0)*}_u + \mu^2 H^{(+)*}_u H^{(+)}_u \\
&+\mu^2 v^*_d v_d + \mu^2 v^*_d h^{(0)}_d + \mu^2 v_d h^{(0)*}_d + \mu^2 h^{(0)}_d h^{(0)*}_d + \mu^2 H^{(-)*}_d H^{(-)}_d \\
&+h^e_{ij}h^{d*}_{kl}\tilde{e}^*_{Ri}\tilde{d}_{Rk}\tilde{\nu}_j\tilde{u}_{Ll} + h^e_{ij}h^{d*}_{kl}\tilde{e}^*_{Ri}\tilde{d}_{Rk}\tilde{e}_{Lj}\tilde{d}_{Ll} \\
&+h^e_{ij}\mu\tilde{e}^*_{Ri}\tilde{\nu}_j H^{(+)*}_u + h^e_{ij}\mu v_u \tilde{e}^*_{Ri}\tilde{e}_{Lj} + h^e_{ij}\mu\tilde{e}^*_{Ri}\tilde{e}_{Lj} h^{(0)*}_u \\
&+h^d_{ij}\mu\tilde{d}^*_{Ri}\tilde{u}_{Lj} H^{(+)*}_u + h^e_{ij}\mu v_u \tilde{d}^*_{Ri}\tilde{d}_{Lj} + h^e_{ij}\mu\tilde{d}^*_{Ri}\tilde{d}_{Lj} h^{(0)*}_u \\
&+h^u_{ij}h^{d*}_{kl} v_d \tilde{u}^*_{Ri}\tilde{d}_{Rk} H^{(+)}_u + h^u_{ij}h^{d*}_{kl}\tilde{u}^*_{Ri}\tilde{d}_{Rk} H^{(+)}_u h^{(0)*}_d + h^u_{ij}h^{d*}_{kl} v_u \tilde{u}^*_{Ri}\tilde{d}_{Rk} H^{(-)*}_d \\
&+h^u_{ij}h^{d*}_{kl}\tilde{u}^*_{Ri}\tilde{d}_{Rk} h^{(0)}_u H^{(-)*}_d \\
&+h^u_{ij}\mu v_d \tilde{u}^*_{Ri}\tilde{u}_{Lj} + h^u_{ij}\mu\tilde{u}^*_{Ri}\tilde{u}_{Lj} h^{(0)*}_d + h^u_{ij}\mu\tilde{u}^*_{Ri}\tilde{u}_{Lj} H^{(-)*}_d] \\
&+\text{h.c.,}
\end{aligned}
\tag{A.84}
$$

where we made use of $\sum_s \epsilon^{rs}\epsilon^{ts} = \delta^{rt}$ and gave vevs to the Higgs bosons $H_u = (H^{(+)}_u, v_u + h^{(0)}_u), H_d = (v_d + h^{(0)}_d, H^{(-)}_d)$. The term in the second square brackets is equal to $-\sum_i W^\dagger_i W_i$. The term coming from the Hermitian conjugation is canceled by the one coming from $\mathcal{L}_{\rm kin}$.

A.4.3 Higgs Mechanism

The covariant derivative acting on the Higgs doublets is given by

$$
\begin{aligned}
D_\mu &= \partial_\mu + \sum_{a=1}^{3} \frac{ig_L}{2}\sigma^a W_\mu^a + ig_Y Y B_\mu^0 \\
D_\mu H_u &= \begin{pmatrix} \left(\partial_\mu + \frac{ig_L}{2}W_\mu^3 + \frac{ig_Y}{2}B_\mu^0\right) H_u^{(+)} + \frac{ig_Y}{2}\left(W_\mu^1 - iW_\mu^2\right)\left(v_u + h_u^{(0)}\right) \\ \frac{ig_L}{2}\left(W_\mu^1 + iW_\mu^2\right) H_u^{(+)} + \left(\partial_\mu - \frac{ig_L}{2}W_\mu^3 + \frac{ig_Y}{2}B_\mu^0\right)\left(v_u + h_u^{(0)}\right) \end{pmatrix}, \\
(D_\mu H_u)^\dagger &= \begin{pmatrix} \left(\partial_\mu - \frac{ig_L}{2}W_\mu^3 - \frac{ig_Y}{2}B_\mu^0\right) H_u^{(+)*} - \frac{ig_Y}{2}\left(W_\mu^1 + iW_\mu^2\right)\left(v_u^* + h_u^{(0)*}\right) \\ -\frac{ig_L}{2}\left(W_\mu^1 - iW_\mu^2\right) H_u^{(+)*} + \left(\partial_\mu + \frac{ig_L}{2}W_\mu^3 - \frac{ig_Y}{2}B_\mu^0\right)\left(v_u^* + h_u^{(0)*}\right) \end{pmatrix}^T, \\
D_\mu H_d &= \begin{pmatrix} \left(\partial_\mu + \frac{ig_L}{2}W_\mu^3 - \frac{ig_Y}{2}B_\mu^0\right)\left(v_d + h_d^{(0)}\right) + \frac{ig_L}{2}\left(W_\mu^1 - iW_\mu^2\right) H_d^{(-)} \\ \frac{ig_L}{2}\left(W_\mu^1 + iW_\mu^2\right)\left(v_d + h_d^{(0)}\right) + \left(\partial_\mu - \frac{ig_L}{2}W_\mu^3 - \frac{ig_Y}{2}B_\mu^0\right) H_d^{(-)} \end{pmatrix}, \\
(D_\mu H_d)^\dagger &= \begin{pmatrix} \left(\partial_\mu - \frac{ig_L}{2}W_\mu^3 + \frac{ig_Y}{2}B_\mu^0\right)\left(v_d^* + h_d^{(0)*}\right) - \frac{ig_L}{2}\left(W_\mu^1 + iW_\mu^2\right) H_d^{(-)*} \\ -\frac{ig_L}{2}\left(W_\mu^1 - iW_\mu^2\right)\left(v_d^* + h_d^{(0)*}\right) + \left(\partial_\mu + \frac{ig_L}{2}W_\mu^3 + \frac{ig_Y}{2}B_\mu^0\right) H_d^{(-)*} \end{pmatrix}^T.
\end{aligned}
\tag{A.85}
$$

If we define

$$
W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right), \tag{A.86}
$$

the mass matrix for $W^\pm$ is already diagonal with entry

$$
m_W^2 = \frac{g_L^2}{2}\left(|v_u|^2 + |v_d|^2\right). \tag{A.87}
$$

The mass matrix for $\left(B_\mu^0, W_\mu^3\right)$ is given by

$$
(W^3, B^0)\begin{pmatrix} \frac{g_L^2}{4}\left(|v_u|^2 + |v_d|^2\right) & \frac{g_L g_Y}{4}\left(|v_u|^2 + |v_d|^2\right) \\ \frac{g_L g_Y}{4}\left(|v_u|^2 + |v_d|^2\right) & \frac{g_Y^2}{4}\left(|v_u|^2 + |v_d|^2\right) \end{pmatrix}\begin{pmatrix} W^3 \\ B^0 \end{pmatrix}. \tag{A.88}
$$

Its eigenvalues λ_1, λ_2 and eigenvectors are calculated to

$$
\begin{aligned}
A_\mu &= \frac{1}{\sqrt{g_L^2+g_Y^2}}\left(g_L B_\mu^0 + g_Y W_\mu^3\right), \quad m_A = 2\lambda_1 = 0, \\
Z_\mu^0 &= \frac{1}{\sqrt{g_L^2+g_Y^2}}\left(g_L W_\mu^3 - g_Y B_\mu^0\right), \quad m_Z^2 = 2\lambda_2 = \frac{g_L^2 + g_Y^2}{2}\left(|v_u|^2 + |v_d|^2\right),
\end{aligned}
\tag{A.89}
$$

where we can define the weak mixing angle to be

$$
\begin{aligned}
\tan\theta_W &= \frac{g_Y}{g_L}, \\
\sin\theta_W &= \frac{g_Y}{\sqrt{g_Y^2 + g_L^2}}, \\
\cos\theta_W &= \frac{g_L}{\sqrt{g_Y^2 + g_L^2}},
\end{aligned}
\tag{A.90}
$$

leading to the mass eigenstate fields

$$\begin{aligned} Z^0_\mu &= \cos\theta_W W^3_\mu - \sin\theta_W B_\mu, \\ A_\mu &= \cos\theta_W B_\mu + \sin\theta_W W^3_\mu, \end{aligned} \tag{A.91}$$

The gauge eigenstates in terms of the mass eigenstates are given by

$$\begin{aligned} B_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z^0_\mu, \\ W^3_\mu &= \sin\theta_W A_\mu + \cos\theta_W Z^0_\mu. \end{aligned} \tag{A.92}$$

Defining $\tau^\pm = \frac{1}{2}\left(\sigma^1 \pm i\sigma^2\right)$ and $s_W = \sin\theta_W, c_W = \cos\theta_W$ the covariant derivative of the broken theory is then given by

$$\begin{aligned} D_\mu &= \partial_\mu + \frac{ig_L}{\sqrt{2}}\left(W^+_\mu\tau^+ + W^-_\mu\tau^-\right) + iA_\mu\left(g_L s_W\tau^3 + g_Y c_W Y\right) \\ &\quad + iZ_\mu\left(g_Y s_W Y - g_L c_W \tau^3\right) \\ &= \begin{pmatrix} \begin{array}{c} \partial_\mu + \frac{i}{2}A_\mu\left(g_L s_W + 2g_Y c_W Y\right) \\ -\frac{i}{2}Z_\mu\left(2g_Y s_W Y - g_L c_W\right) \end{array} & \frac{ig_L}{\sqrt{2}}W^+_\mu \\ \frac{ig_L}{\sqrt{2}}W^-_\mu & \begin{array}{c} \partial_\mu + \frac{i}{2}A_\mu\left(-g_L s_W + 2g_Y c_W Y\right) \\ -\frac{i}{2}Z_\mu\left(2g_Y s_W Y + g_L c_W\right) \end{array} \end{pmatrix}. \end{aligned} \tag{A.93}$$

Appendix B

Notation and Conventions

First of all, we write the variable on which a field depends as a subscript to the lower right, e.g. ϕ_1 means $\phi(x_1)$ or ϕ_z means $\phi(z)$. When drawing Feynman diagrams the arrow of time points towards the right side.

B.1 Indices

- Indices from the beginning of the Latin alphabet $\{a, b, c, d, e, f\}$ enumerate the generators of the gauge group, e.g. σ^a
- Indices from the middle of the Latin alphabet $\{i, j, k, l\}$ denote the generation, e.g. $e_i = (e, \mu, \tau)$.
- Indices from the last third of the Latin alphabet $\{r, s, t\}$ denote the weak $SU(2)_L$ component, e.g. $L^r = (\nu, e)$.
- Undotted indices from the beginning of the Greek alphabet $\{\alpha, \beta, \gamma, \delta\}$ are left-chiral Weyl spinor indices, e.g.θ^α.
- Dotted indices from the beginning of the Greek alphabet $\{\dot{\alpha}, \dot{\beta}, \dot{\gamma}, \dot{\delta}\}$ are indices of right-chiral Weyl spinors, e.g.$\bar{\theta}^{\dot{\alpha}}$.
- Indices from the end of the Greek alphabet $\{\mu, \nu, \rho, \sigma\}$ are Lorentz indices, e.g. p_μ.

B.2 Fourier-Transform

The Fourier transform is defined by

$$\begin{aligned} f_1 &= \int \frac{d^4 k_1}{(2\pi)^4} e^{-ik_1 x_1} \check{f}_1, \\ \check{f}_1 &= \int d^4 x_1 e^{ix_1 k_1} f_1. \end{aligned} \tag{B.1}$$

B.3 Gauge Covariant Derivative

We define the gauge covariant derivative to be

$$D_\mu = \partial_\mu + igt^a G^a_\mu, \tag{B.2}$$

where t^a are the generators of the gauge group (obeying $[t^a, t^b] = if^{abc}t^c$), G^a_μ the gauge fields and g the coupling constant. This leads to the field strength tensor

$$\begin{aligned}G_{\mu\nu} &= \frac{i}{g}[D_\mu, D_\nu] \\ &= \frac{i}{g}[\partial_\mu - igt^aG^a_\mu, d_\nu - igt^bG^b_\nu] \\ &= \frac{1}{g}[\partial_\mu, gt^bG^b_\nu] + \frac{1}{g}[gt^aG^a_\mu, \partial_\nu] - \frac{i}{g}[gt^aG^a_\mu, gt^bG^b_\nu] \\ &= \partial_\mu G_\nu - \partial_\nu G_\mu - ig[G_\mu, G_\nu], \end{aligned} \tag{B.3}$$

with $G_\mu = t^aG^a_\mu$.

B.4 Definition of Green's Function

We define the Green's function $\Delta_{12} = \Delta(x_1 - x_2)$ for an operator A to solve

$$A\Delta_{12} = i\delta^{(n)}(x_1 - x_2), \tag{B.4}$$

with n = number of dimensions.

B.5 Tools for Weyl Spinors

We define

$$\sigma^\mu = (1, \sigma^i), \qquad \bar{\sigma}^\mu = (1, -\sigma^i) \tag{B.5}$$

with $\sigma^i, i = 1 \ldots 3$ the Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{B.6}$$

A useful definition is

$$\sigma^{\mu\nu} = \frac{1}{4}\{\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu\}. \tag{B.7}$$

We present some traces of products of σ-matrices

$$\begin{aligned} tr\,(\sigma^\mu\bar{\sigma}^\nu\sigma^\rho\bar{\sigma}^\eta) &= 2\,(g^{\mu\nu}g^{\rho\eta} - g^{\mu\rho}g^{\nu\eta} + g^{\mu\eta}g^{\nu\rho} - i\epsilon^{\mu\nu\rho\eta})\,, \\ tr\,(\sigma^\mu\bar{\sigma}^\nu) &= 2g^{\mu\nu}. \end{aligned} \tag{B.8}$$

We multiply two left-chiral Weyl spinors χ, ψ by

$$\chi\psi = \chi^\alpha\psi_\alpha = -\psi_\alpha\chi^\alpha = -\epsilon_{\alpha\beta}\psi^\beta\chi^\alpha = \psi^\beta\epsilon_{\beta\alpha}\chi^\alpha = \psi^\beta\chi_\beta = \psi\chi, \tag{B.9}$$

and two right-chiral Weyl spinors $\bar{\chi}, \bar{\psi}$ by

$$\bar{\theta}\bar{\psi} = \bar{\theta}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\beta}}\bar{\psi}^{\dot{\alpha}} = -\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \epsilon_{\dot{\beta}\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \bar{\psi}_{\dot{\beta}}\bar{\theta}^{\dot{\beta}} = \bar{\psi}\bar{\theta}, \tag{B.10}$$

with

$$\begin{aligned} \epsilon_{\alpha\beta} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\ \epsilon_{\dot\alpha\dot\beta} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon^{\dot\alpha\dot\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \end{aligned} \tag{B.11}$$

We note that in spite of the convention for the multiplication of two Weyl spinors we raise and lower the spinor indices by multiplying with ϵ always from the left

$$\begin{aligned} \theta^{\alpha} &= \epsilon^{\alpha\beta}\theta_{\beta}, \quad \theta_{\alpha} = \epsilon_{\alpha\beta}\theta^{\beta}, \\ \bar\theta^{\dot\alpha} &= \epsilon^{\dot\alpha\dot\beta}\bar\theta_{\beta}, \quad \bar\theta_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar\theta^{\dot\beta}. \end{aligned} \tag{B.12}$$

The following useful identities hold

$$\begin{aligned} -\frac{1}{2}\epsilon^{\alpha\beta}\theta\theta &= \theta^{\alpha}\theta^{\beta}, \\ \frac{1}{2}\epsilon^{\dot\alpha\dot\beta}\bar\theta\bar\theta &= \bar\theta^{\dot\alpha}\bar\theta^{\dot\beta}. \end{aligned} \tag{B.13}$$

B.6 Generators of $SU(N)$

Throughout this text we use

$$t^a = \frac{\lambda^a}{2}, \quad \tau^a = \frac{\sigma^a}{2}, \tag{B.14}$$

with λ^a the usual Gell-Mann matrices and σ^a the Pauli matrices

$$\begin{aligned} &\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ &\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \\ &\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \end{aligned} \tag{B.15}$$

The generators $\frac{\lambda^a}{2}$ of the group $SU(N)$ satisfy the identity ([CL84], eq.(4.134))

$$\sum_{a=1}^{N^2-1} (\lambda^a)_{\alpha\beta}(\lambda^a)_{\gamma\delta} = 2\left(\delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{1}{N}\delta_{\alpha\beta}\delta_{\gamma\delta}\right). \tag{B.16}$$

When multiplying the kinetic terms for the sfermions we need ([HK85], eq.(C.95))

$$4(t^a t^b)_{ij} = \frac{1}{6}\delta^{ab}\delta_{ij} + \frac{1}{2}(d^{abc} + i f^{abc})T^c_{ij}. \tag{B.17}$$

The additional factor 4 is due to eq.(B.14).

B.7 Useful Formulae

When multiplying two totally antisymmetric ϵ-tensors it is useful to note that ([SU76], eq.(5.45))

$$\begin{aligned}
\epsilon_{ijkl}\epsilon^{ijkl} &= -4!, \\
\epsilon_{ikjl}\epsilon^{ijkm} &= -3!\delta_l^m, \\
\epsilon_{ijkl}\epsilon^{ijmn} &= -2!\delta_{kl}^{mn}, \\
\epsilon_{ijkl}\epsilon^{imnp} &= -1!\delta_{jkl}^{mnp}, \\
\epsilon_{ijkl}\epsilon^{mnpq} &= -0!\delta_{ijkl}^{mnpq},
\end{aligned} \tag{B.18}$$

with

$$\delta_{ijkl\ldots}^{mnpq\ldots} = \det \begin{vmatrix} \delta_i^m & \delta_j^m & \delta_k^m & \ldots \\ \delta_i^n & \delta_j^n & \delta_k^n & \ldots \\ \delta_i^p & \delta_j^p & \delta_k^p & \ldots \\ \ldots & & & \end{vmatrix}. \tag{B.19}$$

This gives e.g.

$$\begin{aligned}
\epsilon_{\mu\nu}{}^{\rho\sigma}\epsilon^{\mu\nu\kappa\eta} &= 2g^{\eta\rho}g^{\kappa\sigma} - 2g^{\rho\kappa}g^{\sigma\eta}, \\
\epsilon_{\mu}{}^{\nu\rho\sigma}\epsilon^{\mu\kappa\eta\chi} &= g^{\nu\chi}g^{\eta\rho}g^{\kappa\sigma} + g^{\chi\rho}g^{\sigma\eta}g^{\kappa\nu} + g^{\sigma\chi}g^{\nu\eta}g^{\rho\kappa} \\
&\quad -g^{\kappa\nu}g^{\eta\rho}g^{\chi\sigma} - g^{\kappa\rho}g^{\eta\sigma}g^{\nu\chi} - g^{\kappa\sigma}g^{\eta\nu}g^{\chi\rho}.
\end{aligned} \tag{B.20}$$

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